Kinematics of Machines

UNIT-I
Mechanism and Machines

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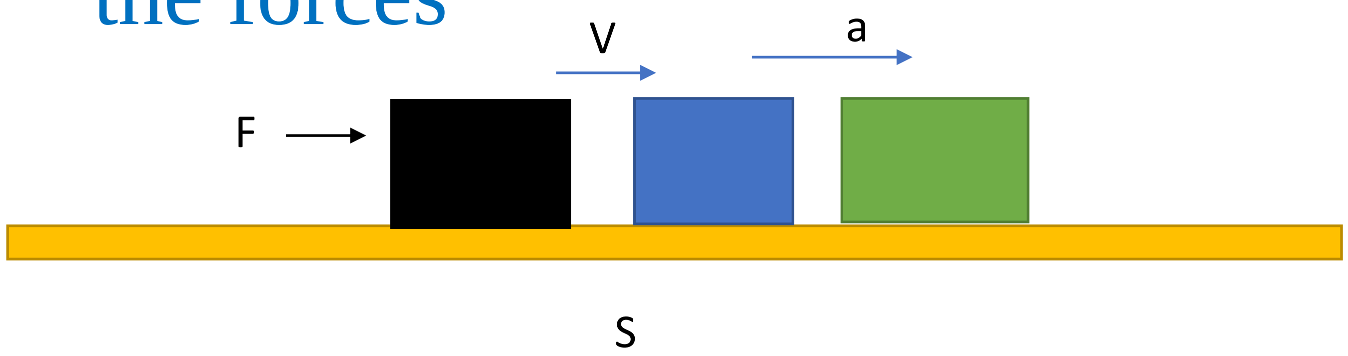
- **UNIT-I Mechanisms:** Elements or Links – Classification – Rigid Link, flexible and fluid link – Types of kinematics pairs – sliding, turning, rolling, screw and spherical pairs – lower and higher pairs – closed and open pairs – constrained motion – completely, partially or successfully and incompletely constrained .
- **Mechanism and Machines –** Mobility of Mechanisms : Grubler's criterion, classification of machines – kinematics chain – inversions of mechanism – inversions of four bar chain, single and double slider crank chains, Mechanical Advantage

Kinematics of Machines

It deals with the study of motion of objects without considering the forces

It is a device which receives one form of energy and converts into useful work

E.g: Engine , pump, boiler etc



It deals with study of motion characteristics of various parts of mechanism without considering the forces causing the motion and their masses

THEORY OF MACHINES

KINEMATICS OF MACHINES

DYNAMICS OF MACHINES

KINETICS

STATICS

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Kinematics:

- It deals with the relative motions of different parts of a mechanism without taking into consideration the forces producing the motions.
- Thus, it is the study, from a geometric point of view, to know the displacement, velocity and acceleration of a part of a Mechanism.

Dynamics:

- It involves the calculations of forces impressed upon different parts of a mechanism.
- The forces can be either static or dynamic.
- Dynamics is further subdivided into kinetics and statics.
- Kinetics is the study of forces when the body is in motion E.g Machine

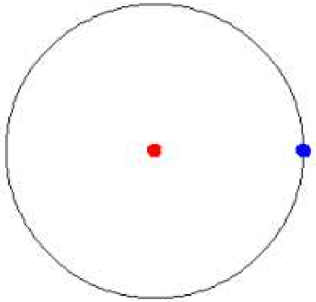
whereas Statics deals with forces when the body is stationary E.g structure

Structure – a single body with no motion /combination of bodies with no relative motion

Introduction

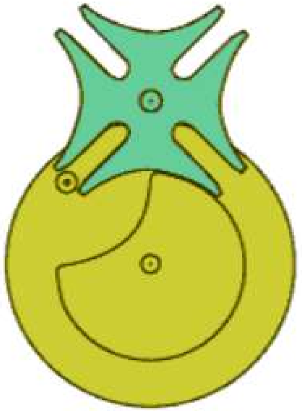
- If a number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others, it is known as a Mechanism
- A mechanism transmits and modifies a motion.
- A Machine is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work.
- Thus, a mechanism is a fundamental unit and one has to start with its study.
- The study of a mechanism involves its analysis as well as synthesis.
- Analysis is the study of motions and forces concerning different parts of an existing mechanism, whereas synthesis involves the design of its different parts.
- In a mechanism, the various parts are so proportioned and related that the motion of one imparts requisite motions to the others and the parts are able to withstand the forces impressed upon them

Classification of Motion



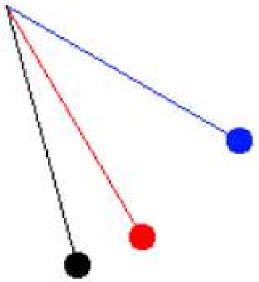
- Continuous rotation motion
- Linear motion / Rectilinear Motion / *translatory motion*

- Intermittent motion



Geneva Wheel

- Angular Oscillations



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Basic Definitions

- Link or element
- Kinematic pair
- Kinematic chain
- Mechanism, Machine & Structure
- Degrees of freedom (DOF)
- Grubler's criterion
- Inversion

Kinematic Chains and Inversions

- Inversions of Four bar chain
- Single slider crank chain and
- Double slider crank chain

Basic Definitions

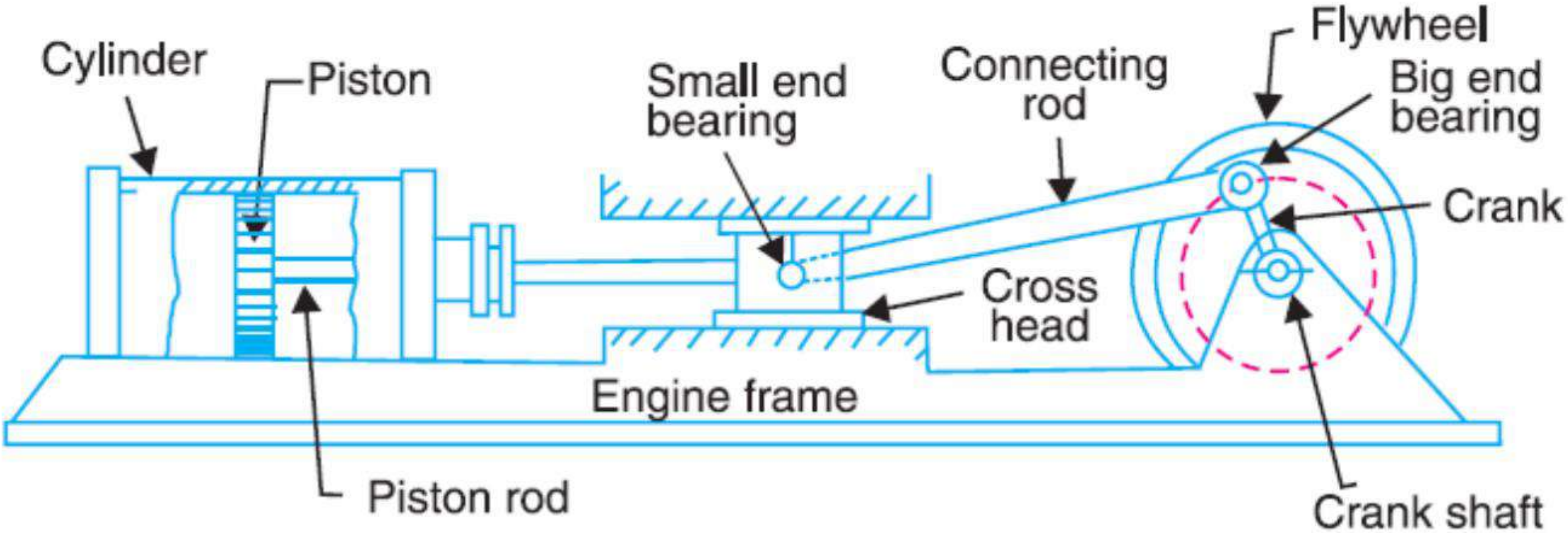
Resistant body: A body is said to be resistant if it is capable of transmitting the required force with negligible deformation

Rigid body : body that do not go under any deformation

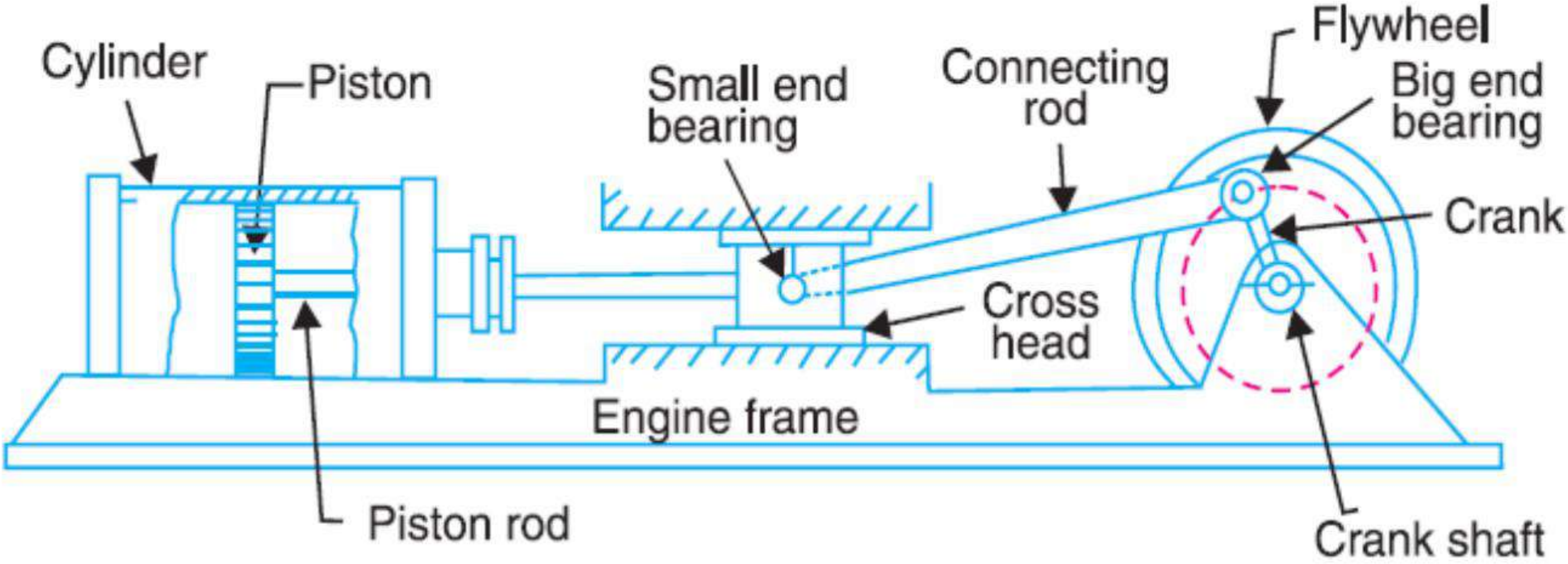
A link need not necessarily be a rigid link, but it must be a resistant body

For example- Springs, belts and oil used in hydraulic press are not rigid links but they are resistant bodies

- **Kinematic Link or Element**
  - Each part of a machine, which moves relative to some other part, is known as a kinematic link (or simply link) or element.
  - A link may consist of several parts, which are rigidly fastened together, so that they do not move relative to one another.



- For example, in a reciprocating steam engine, as shown in Fig. 1,piston, piston rod and crosshead constitute one link ; connecting rod with big and small end bearings constitute a second link ; crank, crankshaft and flywheel a third link and the cylinder, engine frame and main bearings a fourth link



Types of Links

- **Rigid link :** Link which does not undergo any deformation while transmitting motion.

- **Flexible link:** Link which is partly deformed in a manner not to affect the transmission of motion.

  e.g.: belts, ropes, chains and wires

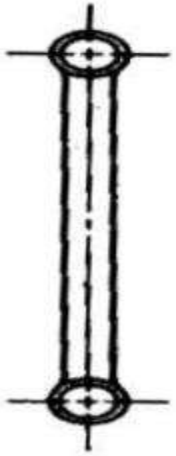
- **Fluid link:** A link formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure.

  e.g.: hydraulic presses, jacks and brakes.

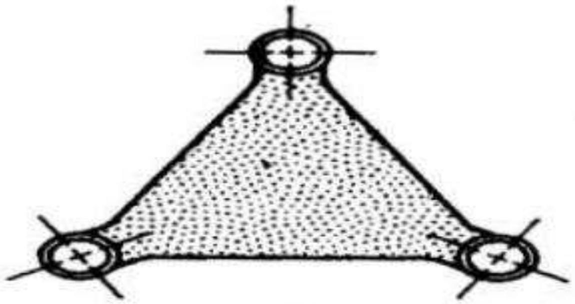
CLASSIFICATION OF LINKS

Link or element: It is the name given to any body which has motion relative to another. All materials have some elasticity. A rigid link is one, whose deformations are so small that they can be neglected in determining the motion parameters of the link

- **Unary link** : Link with one node (bucket of an excavator)
- **Binary link:** Link which is connected to other links at two points. (Fig.a)
- **Ternary link:** Link which is connected to other links at three points. (Fig.b)
- **Quaternary link:** Link which is connected to other links at four points. (Fig. c)



(a)



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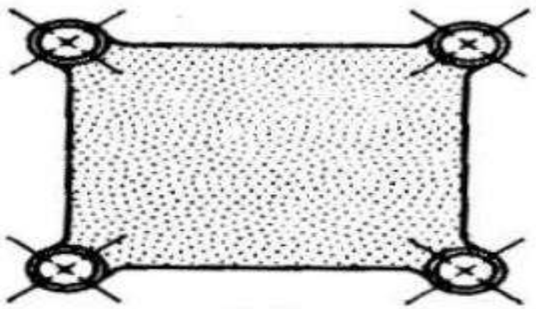

(c)

- **Structure :** It is an assemblage of a number of resistant bodies (members) having no relative motion between them and meant for carrying loads having straining action.
- Examples : A railway bridge, a roof truss, machine frames etc.,

Difference Between a Machine and a Structure

- The parts of a machine move relative to one another, whereas the members of a structure do not move relative to one another

- A machine transforms the available energy into some useful work, whereas in a structure no energy is transformed into useful work

- The links of a machine may transmit both power and motion, while the members of a structure transmit forces only (load bearing member).

Kinematic Pair

The two links or elements of a machine, when in contact with each other, are said to form a pair, If the *relative motion between them is completely or successfully constrained* (*i.e. in a definite* direction)

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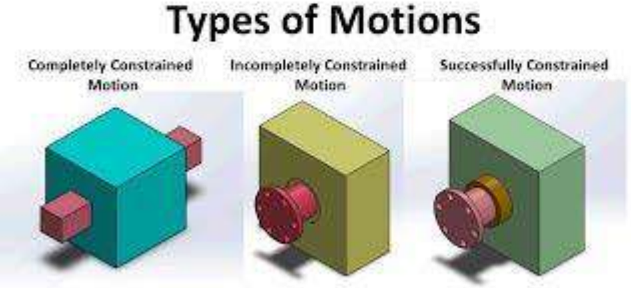
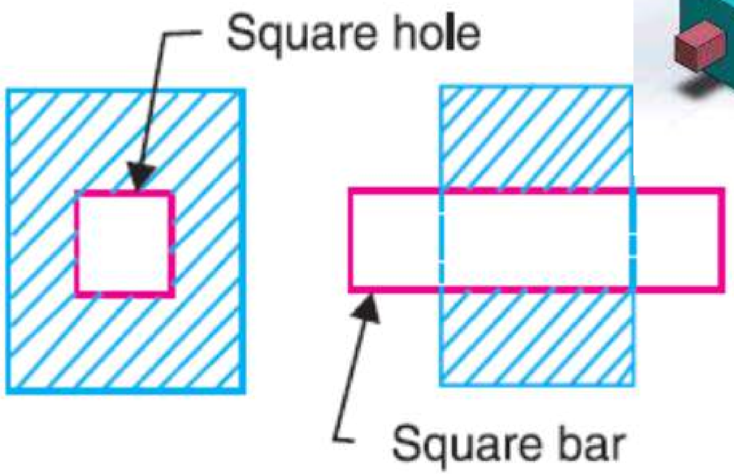
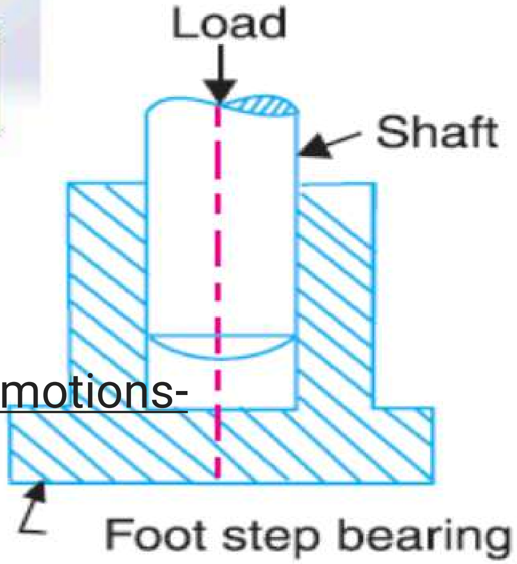




Completely constrained

Types of constrained motions-



Successfully constrained

Types of Constrained Motions

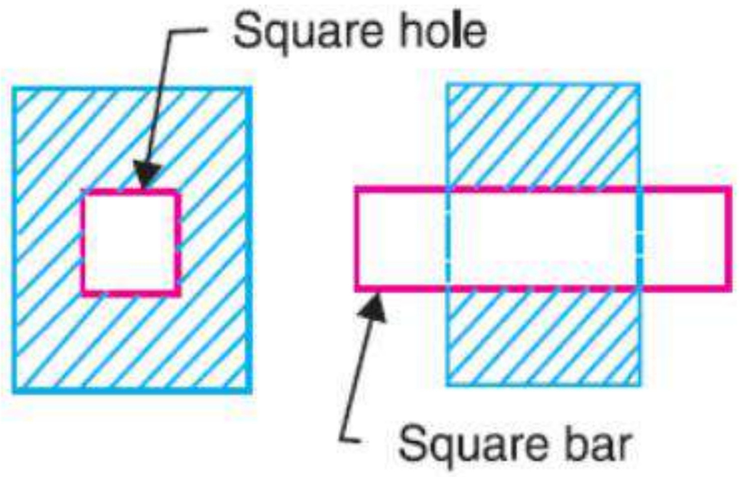
Following are the three types of constrained motions :

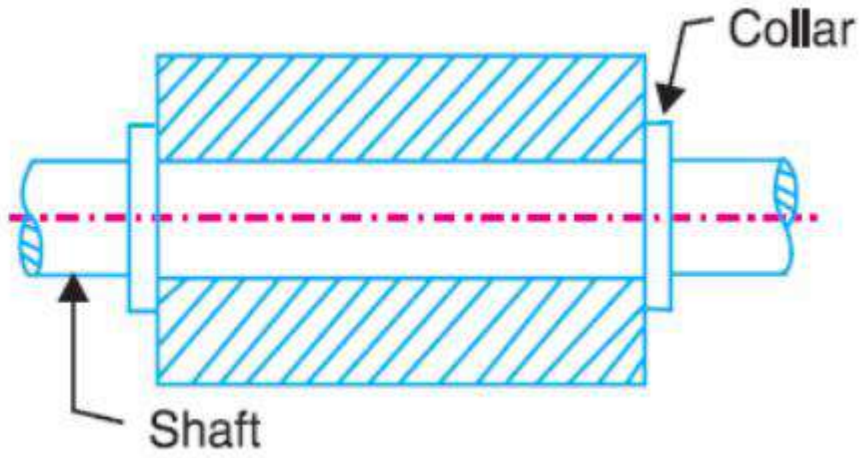
- **Completely constrained motion**

  When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion.

  For Example, the piston and cylinder (in a steam engine) form a pair and the motion of the piston is limited to a definite direction (i.e. it will only reciprocate) relative to the cylinder irrespective of the direction of motion of the crank.

  The motion of a square bar in a square hole, as shown in Fig. 2, and The motion of a shaft with collars at each end in a circular hole, as shown in Fig. 3,





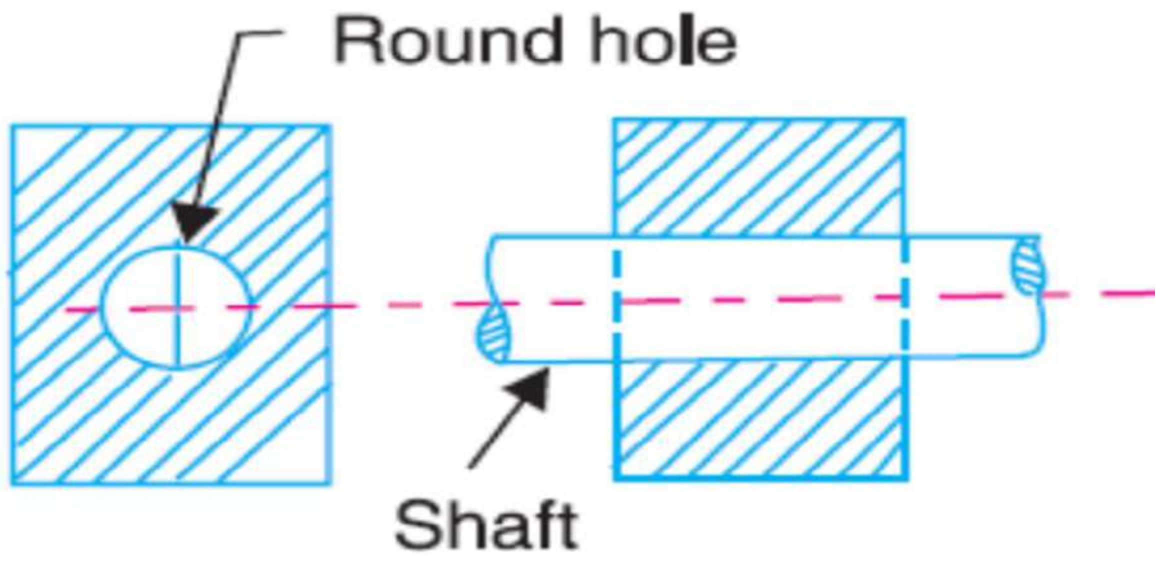
2.Incompletely constrained motion :

When the motion between a pair can take place in more than one direction, then the motion is called an incompletely constrained motion.

The change in the direction of impressed force may alter the direction of relative motion between the pair.

E.g: A circular bar or shaft in a circular hole, as shown in Fig.,

it may either rotate or slide in a hole. These both motions have no relationship with the other (Automobile wheel)



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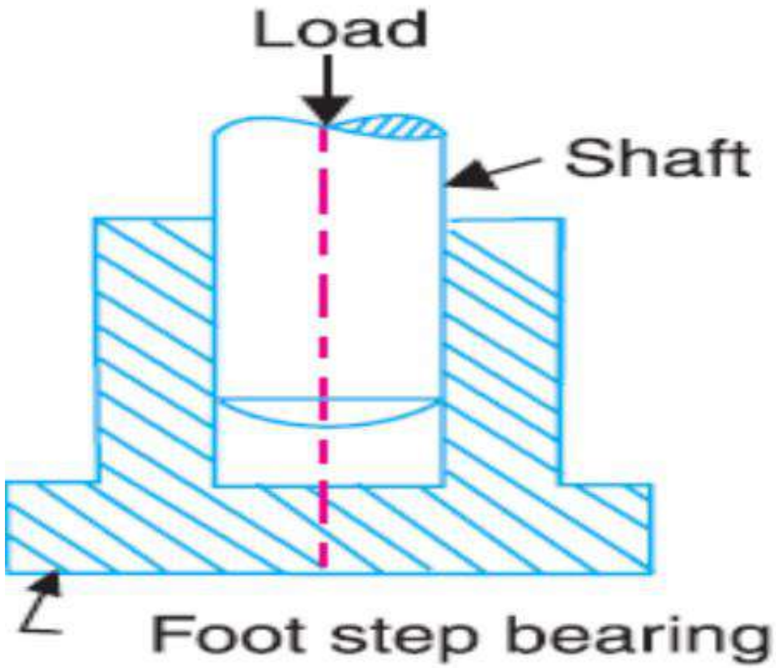
3. Successfully constrained motion

When the motion between the elements, forming a pair, is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion.

- Consider a shaft in a foot-step bearing as shown in Fig.
- The shaft may rotate in a bearing or it may move upwards.
- This is a case of incompletely constrained motion.
- But if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion of the pair is said to be successfully constrained motion.

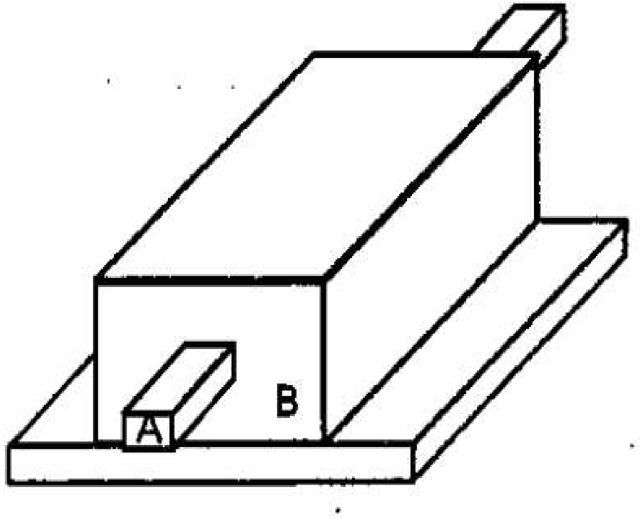
- The motion of an I.C. engine valve and the piston reciprocating inside an engine cylinder is not completed by itself but due to the rotation of crank



• Classification of Kinematic Pairs

1. According to the type of relative motion between the elements.

1. Sliding pair / Prismatic pair (P)



DOF = 1

When the two elements of a pair are connected in such a way that one can only slide relative to the other, has a completely constrained motion.

E.g: The motion of a square bar in a square hole

E.g.: The piston and cylinder, Cross-head and guides of a reciprocating steam engine, ram and its guides in shaper, tail stock on the lathe bed

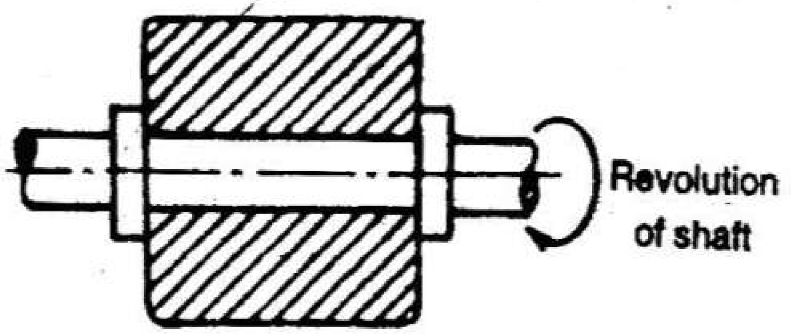
2. Turning Pair / Revolute Pair (R)

When the two elements of a pair are connected in such a way that one can only turn or revolve about a fixed axis of another link.

Examples Lathe spindle supported in head stock.

cycle wheels turning over their axles.

Shaft with collar at both end fitted into a circular hole



DOF = 1

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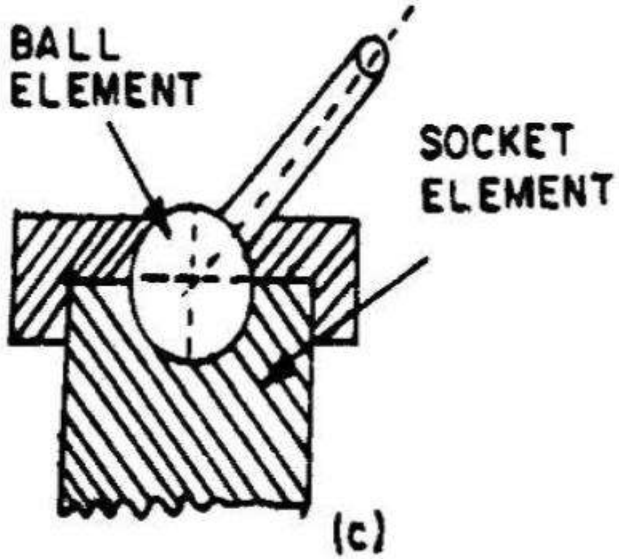
3. Spherical pair (S)

When the two elements of a pair are connected in such a way that one element turns or swivels about the other fixed element.

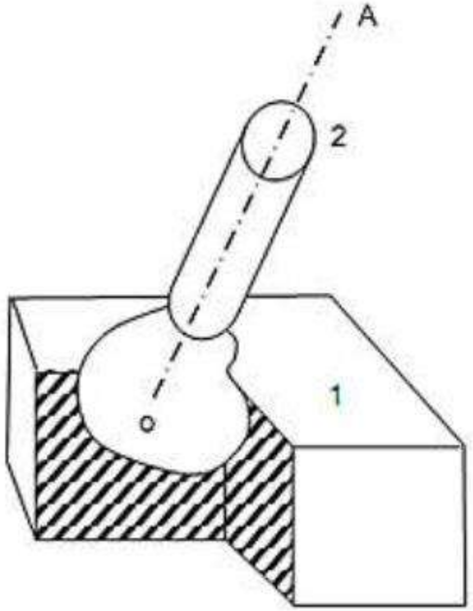
Eg: The ball and socket joint.

Attachment of a car mirror.

Pen stand.



DOF = 3

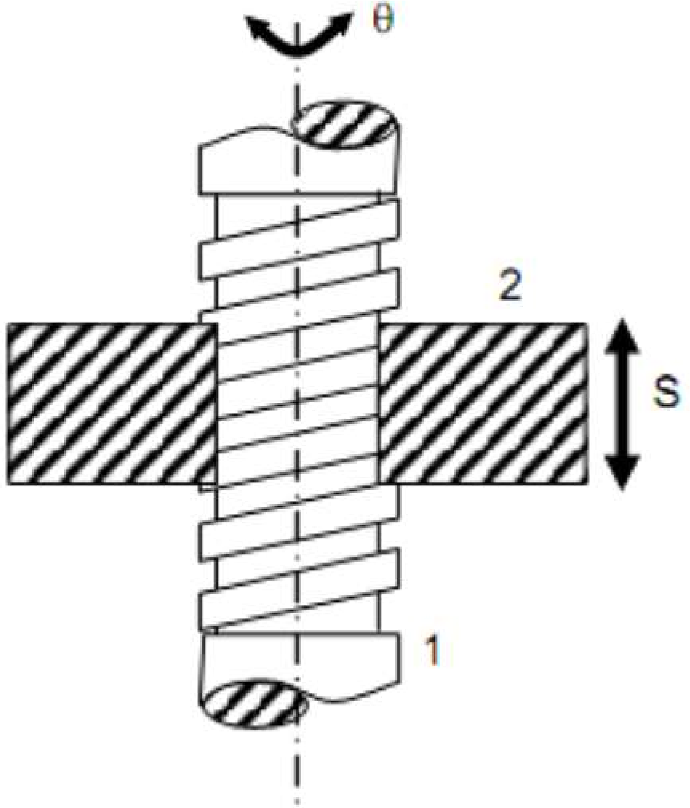


4. Screw pair

When the two elements of a pair are connected in such a way that one element can turn about the other by screw threads.

Examples:

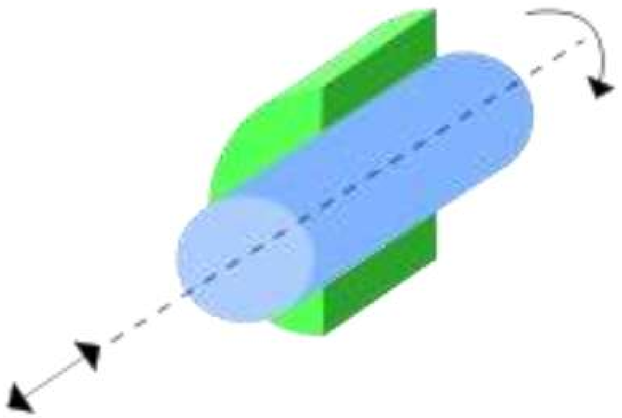
- The lead screw of a lathe with nut
- Bolt with a nut

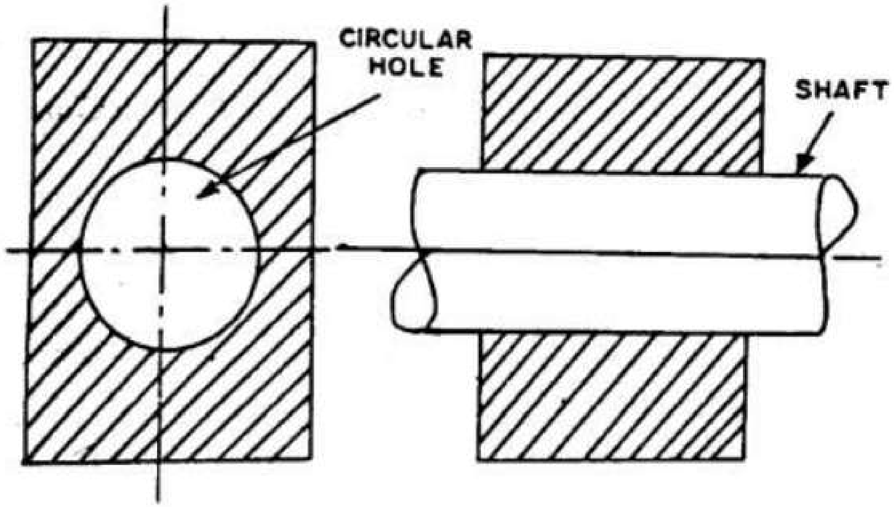


DOF = 1

5. Cylindrical pair

If the relative motion between the pairing elements is the combination of turning and sliding, then it is called as cylindrical pair.



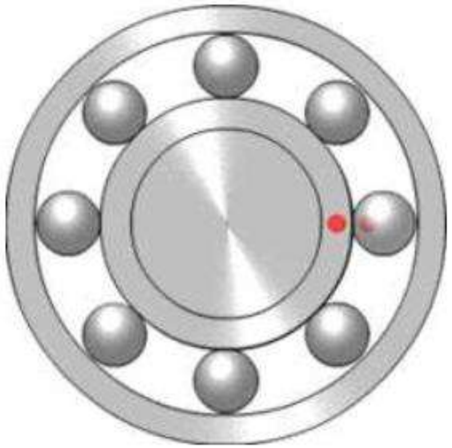


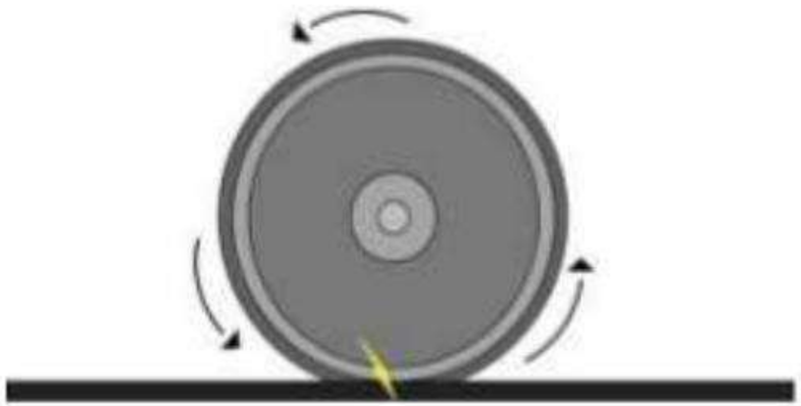
DOF = 2

6. Rolling Pair

When the two elements of a pair are connected in such a way that one rolls over another fixed link.

E.g.: Ball and roller bearings, railway wheel rolling over a fixed rail





DOF = 1

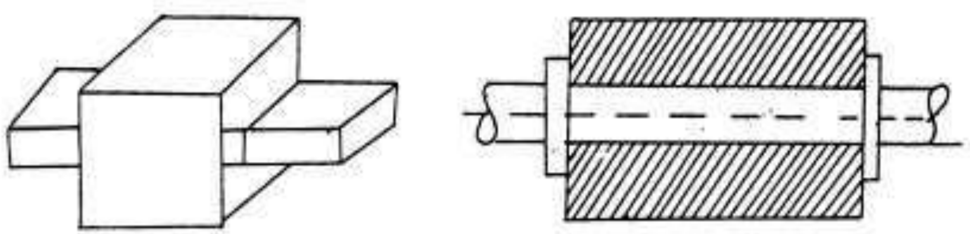
Belt and pulley

2. According to the type/nature of contact between the elements/links.

1. Lower pair

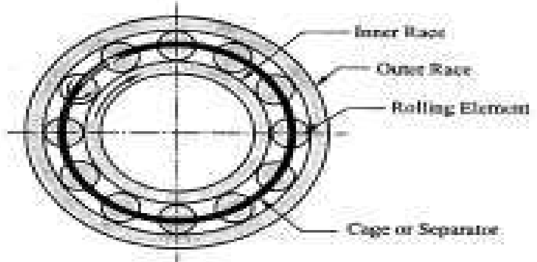
When the two elements of a pair have a **surface contact,** and the surface of one element slides or rolls over the surface of the other.

E.g. sliding pairs, turning pairs and screw pairs form lower pairs.



2. Higher pair

When the two elements of a pair have a **line or point contact,** and the motion between the two elements is partly turning and partly sliding.



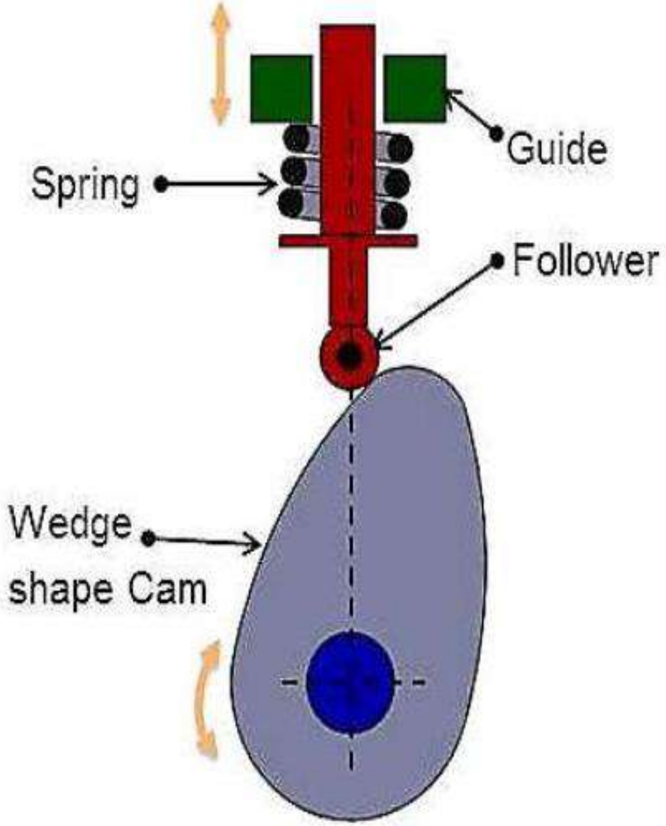
E.g.: toothed gearing, belt and rope drives, ball and roller bearings and cam and follower.

3. According to the Mechanical Arrangement

1. Self closed pair

When the two elements of a pair are **connected together mechanically** in such a way that only required relative motion occurs.

E.g. Lower pairs are self closed pair.



2. Force - closed pair

When the two elements of a pair are **not connected mechanically** but **are kept in contact by the action of external forces**, the pair is said to be a force-closed pair.

E.g.: Cam and follower

Kinematic Pairs

Lower pair

- Sliding pairs
- Turning pairs
- Screw pairs

Higher pair

- Rolling pair

Self closed pair

Force - closed pair

- Based on the possible motions (Few Important Types only)

Name of Pair	Letter Symbol	D.O.F
1. Revolute / Turning Pair	R	1
2. Prismatic / Sliding Pair	P	1
3. Helical / Screw Pair	H	1
4. Cylindrical Pair	C	2
5. Spherical / Globular Pair	S (or) G	3
6. Flat / Planar Pair	E	3

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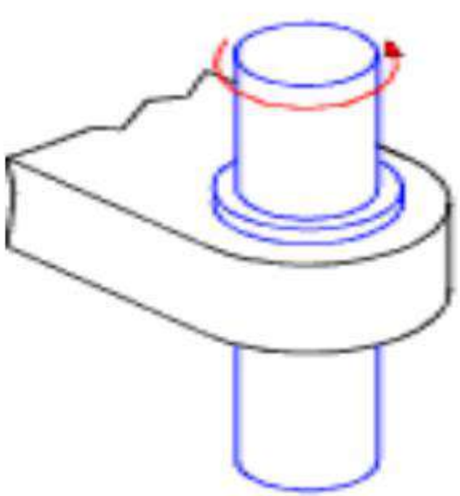
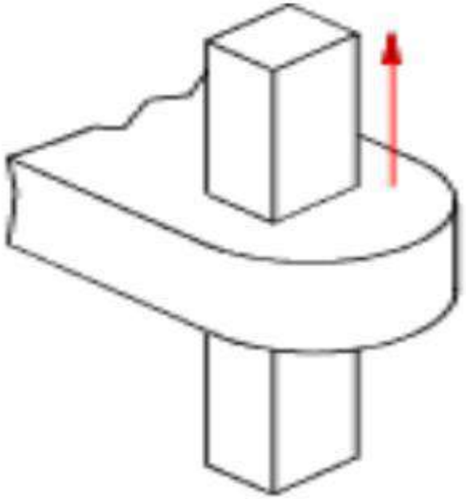
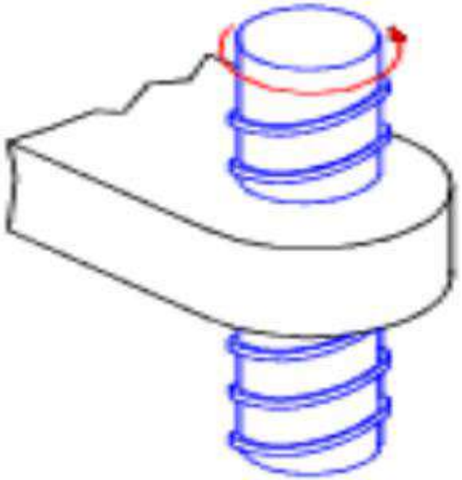
Degrees of Freedom & Links Pairs



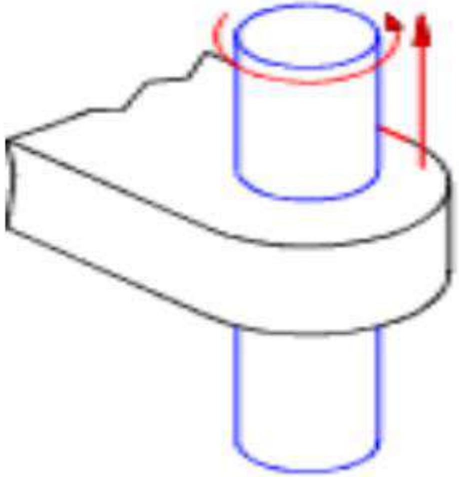
Turning Pair...1-DOF



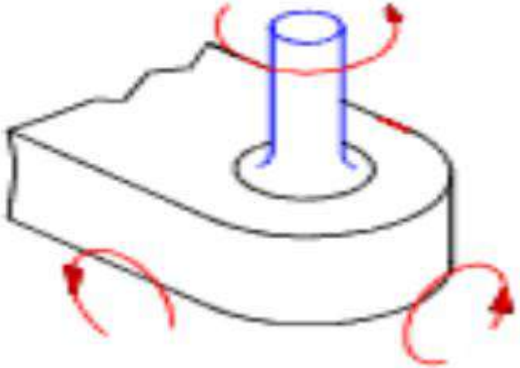
Prismatic (Sliding) Pair...1-DOF



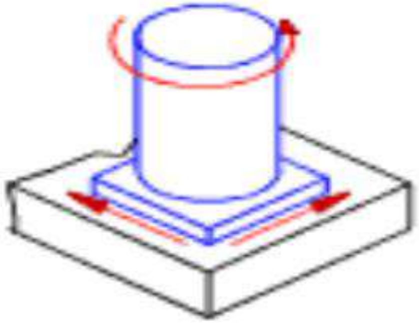
Screw Pair ...1-DOF



Cylinderical Pair ...2-DOF



Spherical (Globular) Pair...3-DOF



Flat Pair ...3-DOF

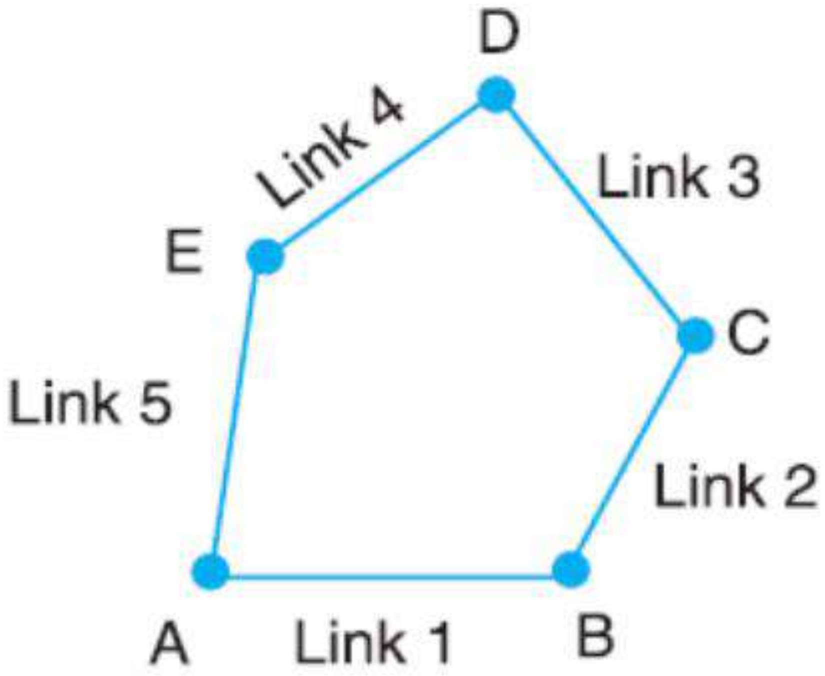
- **Kinematic Chain**
  - When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion (i.e. completely or successfully constrained motion), it is called a kinematic chain.

OR

Assembly of links (Kinematic link/ element) and kinematic pairs to transmit required/ specified output motion(s) for given input motion(s)

OR

kinematic chain can be defined as a combination of kinematic pairs, joint in such a way that relative motion between them is completely or successfully constrained

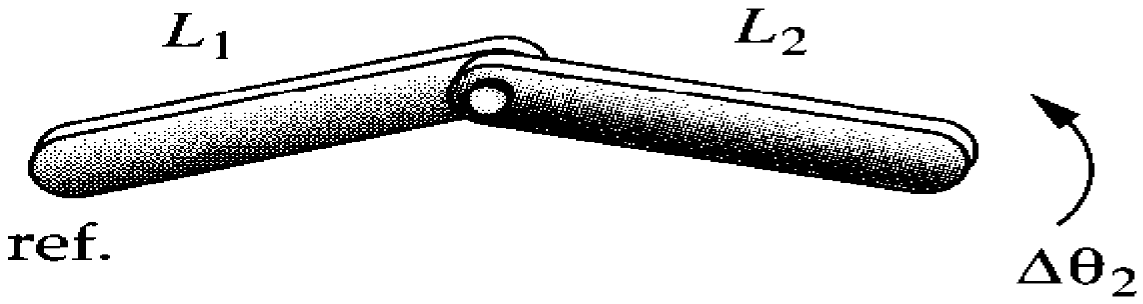


Types of joints in a chain

Joint can be defines as the point where two or more links meets

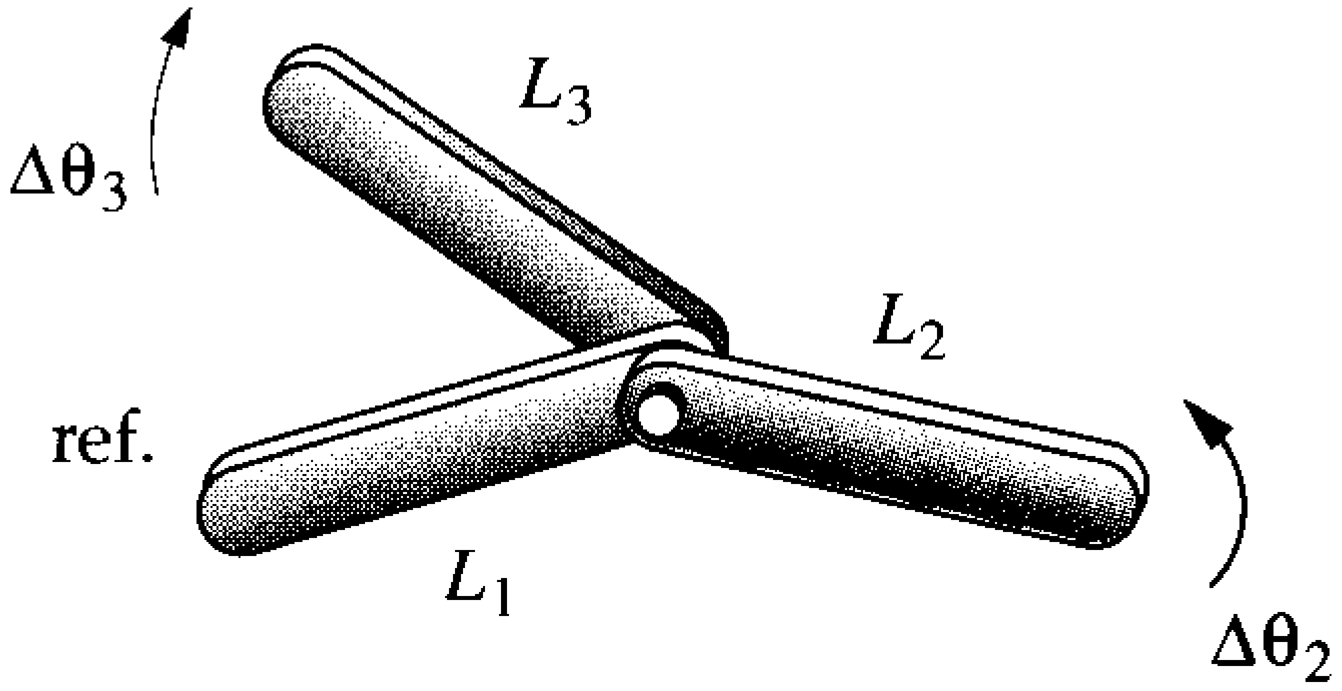
- The following types of joints are usually found in a chain :
  Binary joint.
  Ternary Joint
  Quaternary Joint

- **Binary joint:**
  - When two links are joined at the same connection, the joint is known as binary joint



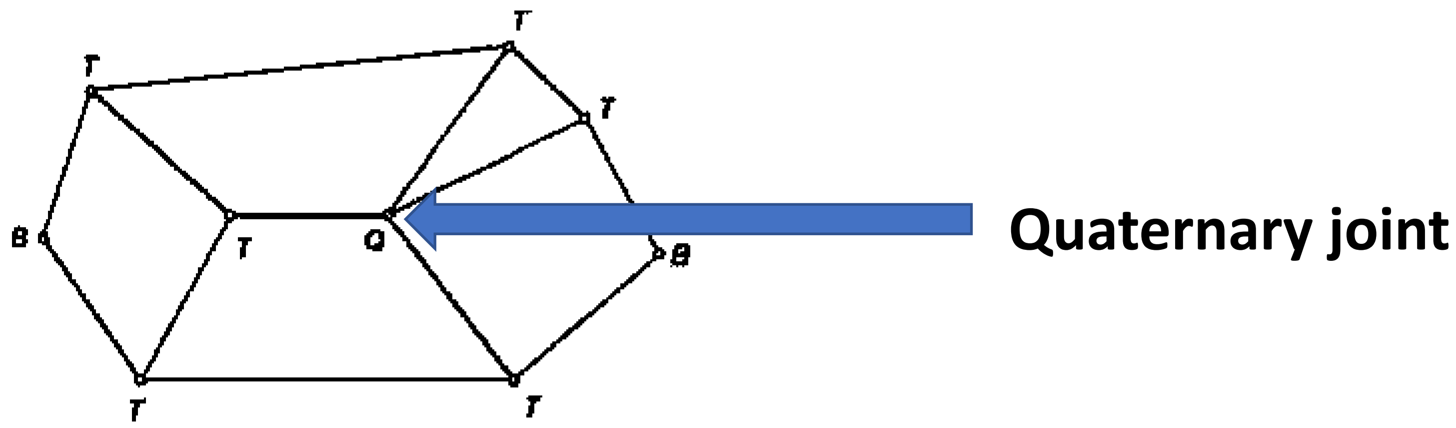
First order pin joint - one *DOF*
(two links joined)

- **Ternary joint:**
  - When three links are joined at the same connection, the joint is known as ternary joint.
  - Ternary joint is equivalent to (l-1)two binary joints as one of the three links joined carry the pin for the other two links.



Second order pin joint - two *DOF*
(three links joined)

- **Quaternary joint:**
  - When four links are joined at the same connection , the joint is called a quaternary joint.
  - It is equivalent to three binary joints.
  - In general, when l number of links are joined at the same connection, the joint is equivalent to (l - 1) binary joints.



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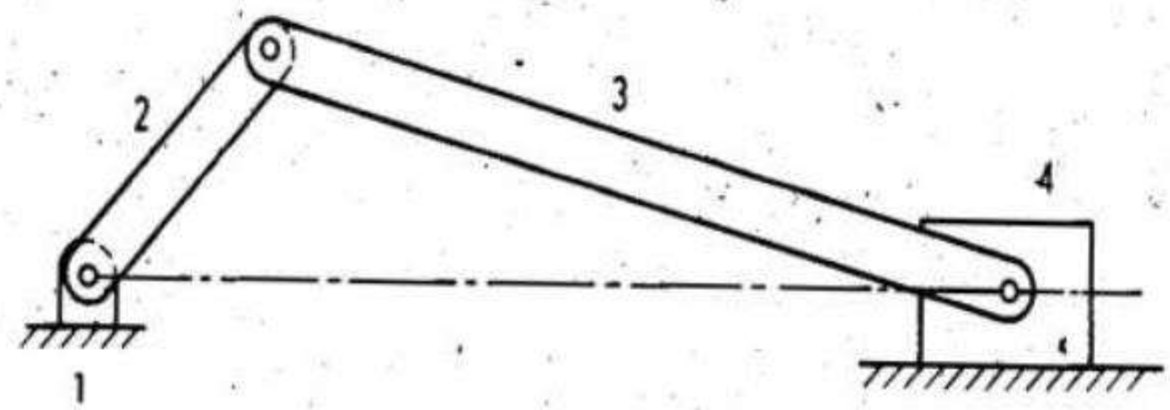
Mechanism

- **Mechanism**: A linkage is obtained if one of the links of a kinematic chain is fixed to the ground . If motion of any of the movable links results in definite motions of the others, if linkage is known as a Mechanism
- When one of the links of a kinematic chain is fixed, the chain is known as mechanism. It may be used for transmitting or transforming motion

  E.g .Printing machine, windshield wiper, robot arms
- A mechanism with four links is known as simple mechanism, and

  the mechanism with more than four links is known as compound mechanism.
- When a mechanism is required to transmit power or to do some particular type of work, it then becomes a machine.



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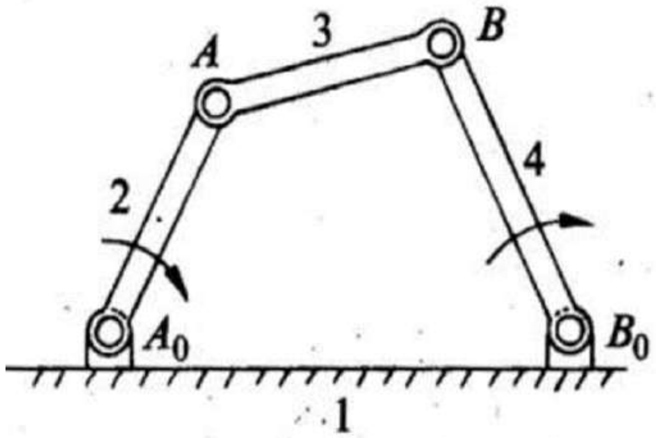
- **Mechanisms and Simple Machines**
  - **Machine**: an assemblage of parts that transmit forces, motion and energy in a predetermined manner.
  - The term **mechanism** is applied to the combination of geometrical bodies which constitute a machine or part of a machine.

- A **mechanism** may therefore be defined as a combination of rigid or resistant bodies, formed and connected so that they move with definite relative motions with respect to one another .



- The similarity between machines and mechanisms is that

– they are both combinations of rigid bodies

– the relative motion among the rigid bodies are definite.

- The difference between machine and mechanism is that machines transform energy to do work, while mechanisms do not necessarily perform this function.
- All machines are mechanisms. But all mechanisms are not machines

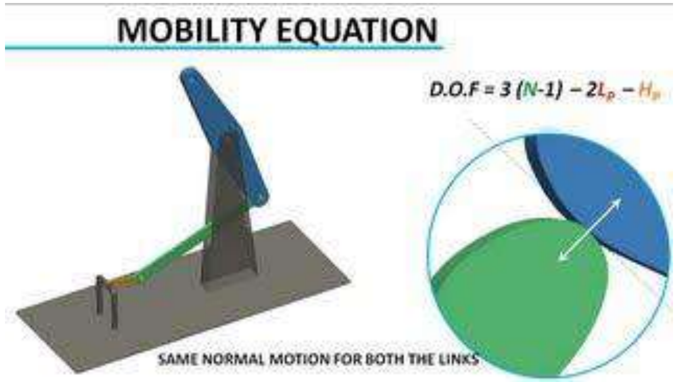
Degrees of Freedom/Mobility of a Mechanism:

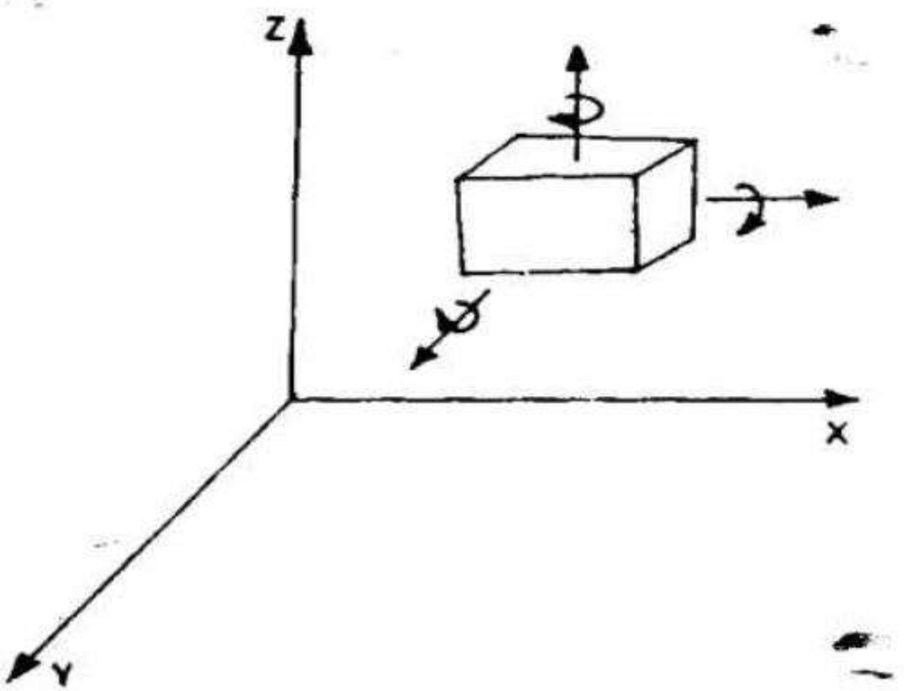
It is the number of inputs (number of independent coordinates), required to describe /Specify the configuration or position of all the links of the mechanism, with respect to the fixed link at any given instant.

In a kinematic pair, depending on the constraints imposed on the motion, the links may loose some of the six degree of freedom

https://youtu.be/vOFM8eG8kVc

Preview YouTube video Understanding Degrees of Freedom





Degrees of Freedom /Mobility of a Mechanism:

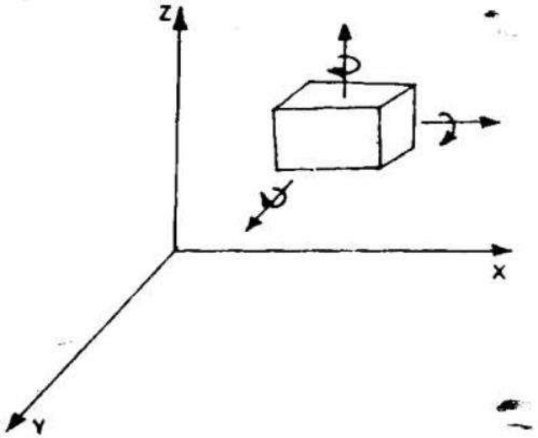
An unconstrained rigid body moving in space can describe the following independent motions (Fig. 1.10):

1. Translational motions along any three mutually perpendicular axes x, y and z
2. Rotational motions about these axes

Thus, a rigid body possesses six degrees of freedom. The connection of a link with another imposes certain constraints on their relative motion. The number of restraints can never be zero (joint is disconnected) or six (joint becomes solid).

Degrees of freedom of a pair is defined as the number of independent relative motions, both translational and rotational, a pair can have.

Degrees of freedom = 6 – Number of restraints



Degrees of Freedom/Mobility of a Mechanism:

Expressing the number of degrees of freedom of a linkage in terms of the number of links and the number of pair connections of different types is known as *number synthesis*. *Degrees of freedom* of a mechanism in space can be determined as follows:

Let

N = total number of links in a mechanism

F = degrees of freedom

P_1 = number of pairs having one degree of freedom

P_2 = number of pairs having two degrees of freedom, and so on

In a mechanism, one link is fixed.

Therefore,

Number of movable links = $N - 1$

Number of degrees of freedom of $(N - 1)$ movable links = $6(N - 1)$

Degrees of Freedom/Mobility of a Mechanism:

Each pair having one degree of freedom imposes 5 restraints on the mechanism, reducing its degrees of freedom by $5P_1$.

Each pair having two degrees of freedom will impose 4 restraints, reducing the degrees of freedom of the mechanism by $4P_2$.

Similarly, other pairs having 3, 4 and 5 degrees of freedom reduce the degrees of freedom of the mechanism. Thus,

$$F = 6(N - 1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - P_5 \qquad (1.1)$$

The above criterion is hardly necessary to find the degrees of freedom, as space mechanisms, especially of the zero order are not practical. Most of the mechanisms are two-dimensional such as a four-link or a slider-crank mechanism in which displacement is possible along two axes (one restraint) and rotation about only one axis (two restraints). Thus, there are three general restraints.

- **Kutzbach Criterion to Plane Mechanisms :** For plane Mechanisms(2D) , Number of degrees of freedom of a mechanism is given by

$$n = 3(l - 1) - 2j - h$$

- Where
  n = total degrees of freedom in the mechanism(F)
  l = number of links (including the frame) (N)
  j = Number of equivalent binary joints (P1)
  h = number of higher pairs (two degrees of freedom) (P2)

- If, n=1 ,it gives Kinematic chain (i.e mechanism can be driven by a single input motion)
- n=0 , it gives a Frame/ structure
- n= -1,-2,-3……. it gives super structure
- n= 2,3,4 ….. Unconstrained chain (i.e n=2 two separate input motions are necessary to produce constrained motion for the mechanism)

DOF of a mechanism is equal to the No. of inputs required to get a constrained output

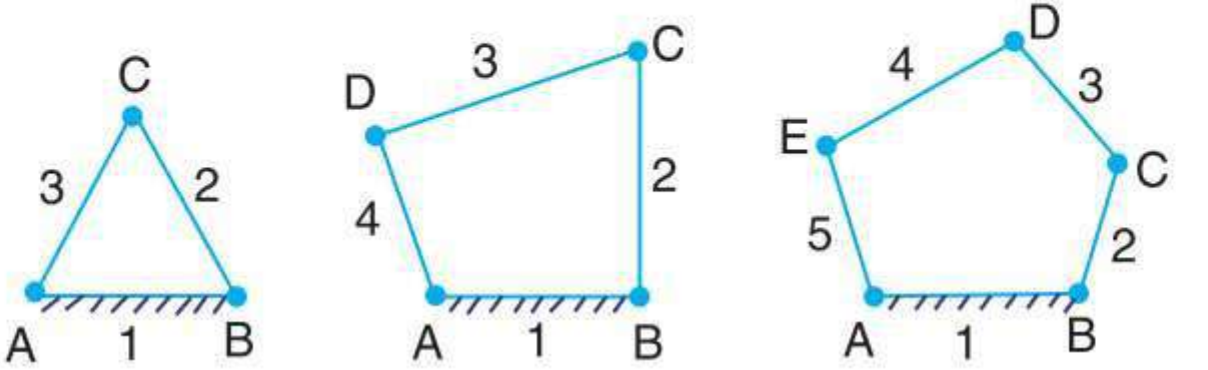
• Grubler's Criterion for Plane Mechanisms

The Grubler's criterion applies to mechanisms with only single degree of freedom joints where the overall movability of the mechanism is unity. Substituting n = 1 and h = 0 in Kutzbach equation, we have

$$n = 3(l - 1) - 2j - h$$

$$1 = 3(l - 1) - 2j \quad \text{or} \quad 3l - 2j - 4 = 0$$

- This equation is known as the Grubler's criterion for plane mechanisms with constrained motion.
  A little consideration will show that a plane mechanism with a movability of 1 and only single degree of freedom joints can not have odd number of links.
- The simplest possible machanisms of this type are a four bar mechanism and a slider-crank mechanism in which l = 4 and j = 4.



(*a*) Three-bar mechanism. (*b*) Four bar mechanism. (*c*) Five bar mechanism

1. The mechanism, as shown in Fig. *(a)*, has three links and three binary joints,

   i.e. $l = 3$ and $j = 3$.

   $n = 3(3-1) - 2 \times 3 = 0$ Frame/ structure

2. The mechanism, as shown in Fig. *(b)*, has four links and four binary joints,

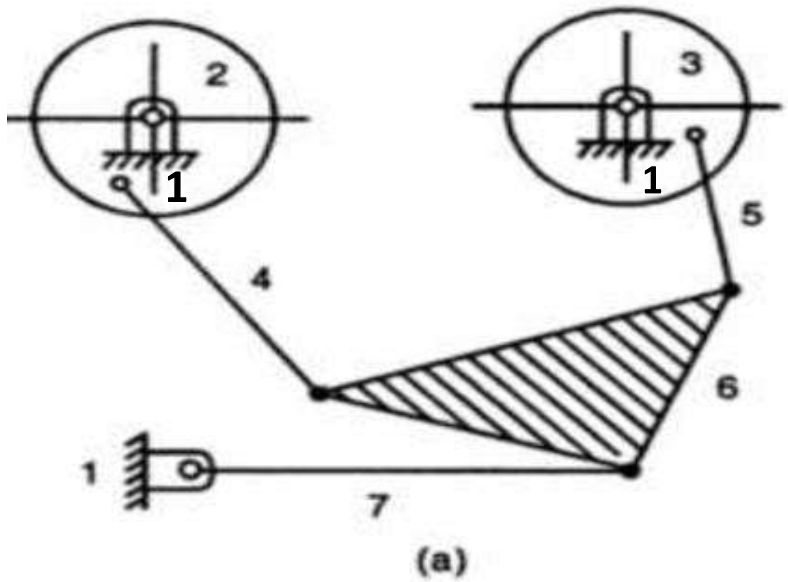
   i.e. $l = 4$ and $j = 4$.

   $n = 3(4-1) - 2 \times 4 = 1$ Kinematic chain (Mechanism)

3. The mechanism, as shown in Fig. (c), has five links and five binary joints,

   i.e. $l = 5$, and $j = 5$.

   $n = 3(5-1) - 2 \times 5 = 2$ Unconstrained chain (two separate input motions are necessary to produce constrained motion for the mechanism)



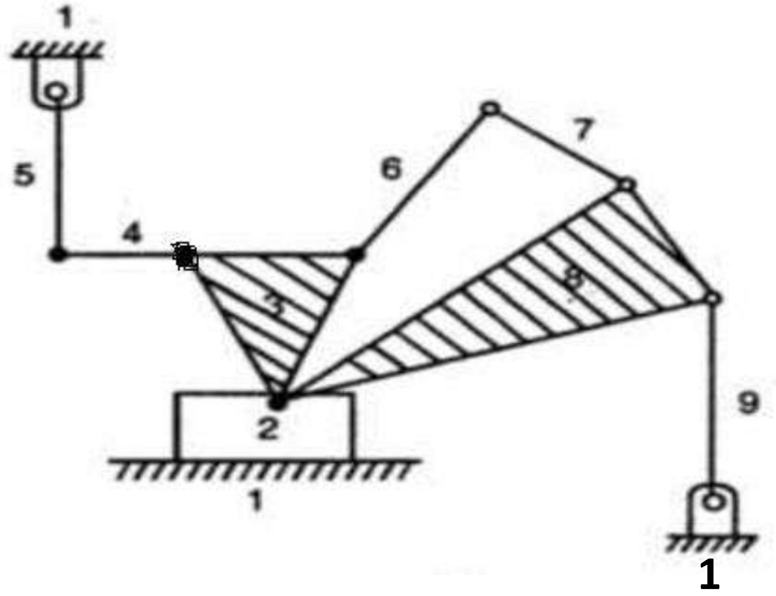
Number of links, $l = 7$

Number of Equivalent binary joints, $J_b = 8$

Number of Higher pairs, h = 0

$$dof = 3(l - 1) - 2J_b - h$$
$$= 3(7-1) - 2x8 - 0$$
$$= 18 - 16$$
$$= 2$$

Unconstrained chain (**two** separate input motions are necessary to produce constrained motion for the mechanism)



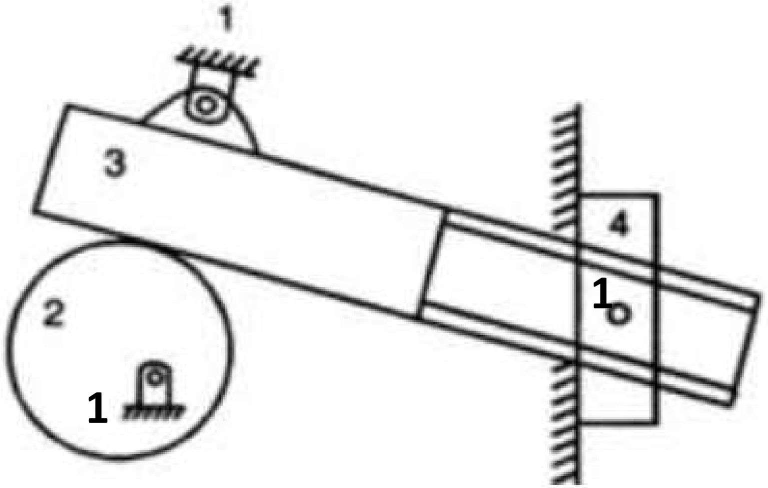
Number of links, $l = 9$

Number of Equivalent binary joints, $J_b = 11$

Number of Higher pairs, h = 0

$$dof = 3(l - 1) - 2J_b - h$$
$$= 3(9-1) - 2x11 - 0$$
$$= 24 - 22$$
$$= 2$$

Unconstrained chain (**two** separate input motions are necessary to produce constrained motion for the mechanism)



Number of links, $l = 4$

Number of Equivalent binary joints, $J_b = 4$

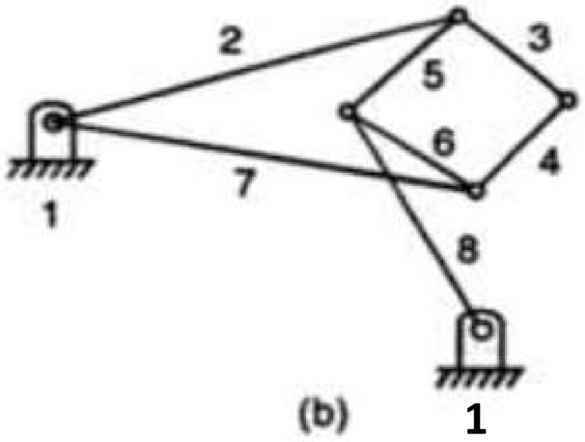
Number of Higher pairs, $h = 1$

$\text{dof} = 3(l - 1) - 2J_b - h$

$= 3(4-1) - 2 \times 4 - 1$

$= 9 - 8 - 1$

$= 0$ Frame/ structure



Number of links, $l = 8$

Number of Equivalent binary joints, $J_b = 10$

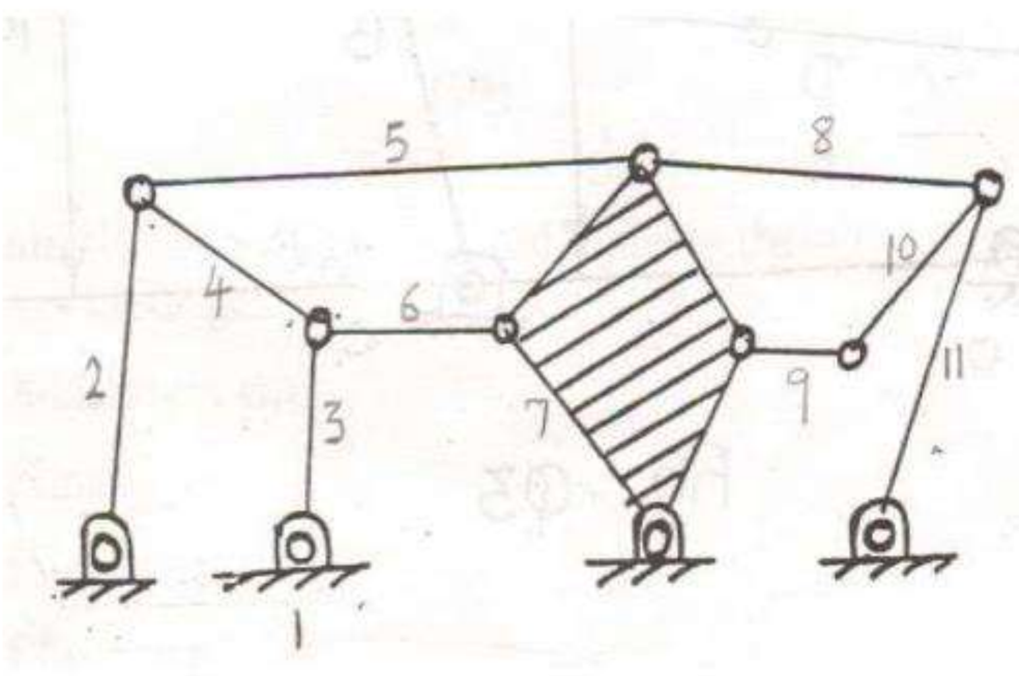
Number of Higher pairs, $h = 0$

$\text{dof} = 3(l - 1) - 2J_b - h$

$= 3(8-1) - 2 \times 10 - 0$

$= 21 - 20 - 0$

$= 1$ Kinematic chain (mechanism can be driven by a single input motion)



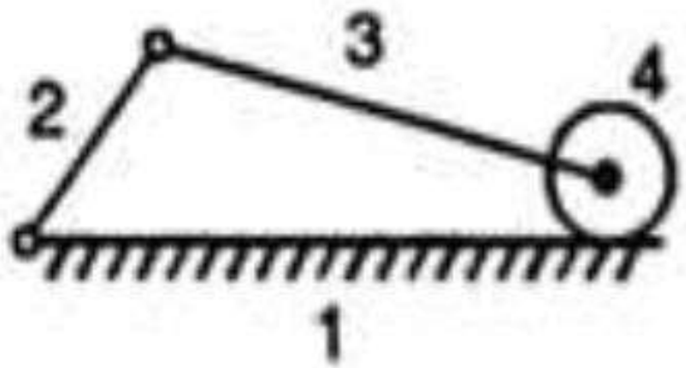
Number of links, $l = 11$

Number of Equivalent binary joints, $J_b = 15$

Number of Higher pairs, h = 0

$$\begin{aligned} dof &= 3(l-1) - 2J_b - h \\ &= 3(11-1) - 2x15 - 0 \\ &= 30 - 30 - 0 \\ &= 0 \end{aligned}$$

Frame/ structure



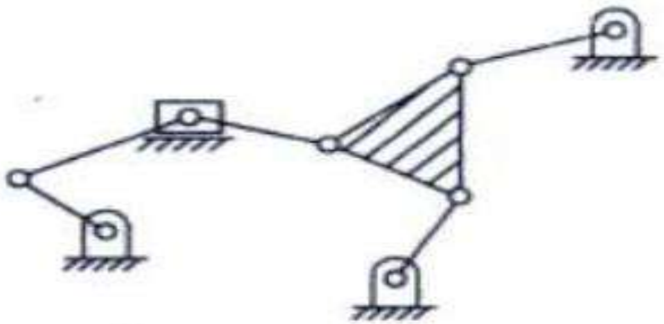
Number of links, $l = 4$

Number of Equivalent binary joints, $J_b = 3$

Number of Higher pairs, h = 1

$$\begin{aligned} dof &= 3(l-1) - 2J_b - h \\ &= 3(4-1) - 2x3 - 1 \\ &= 9 - 6 - 1 \\ &= 2 \end{aligned}$$

Unconstrained chain (two separate input motions are necessary to produce constrained motion for the mechanism)



(a)

Solution

(a) The mechanism has a sliding pair. Therefore, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 8 (Fig. 1.22)

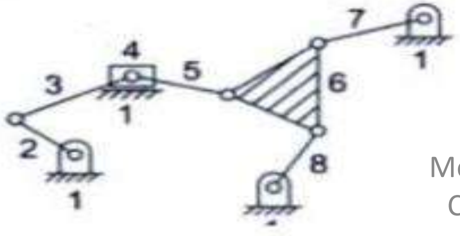
Number of pairs with 1 degree of freedom = 10

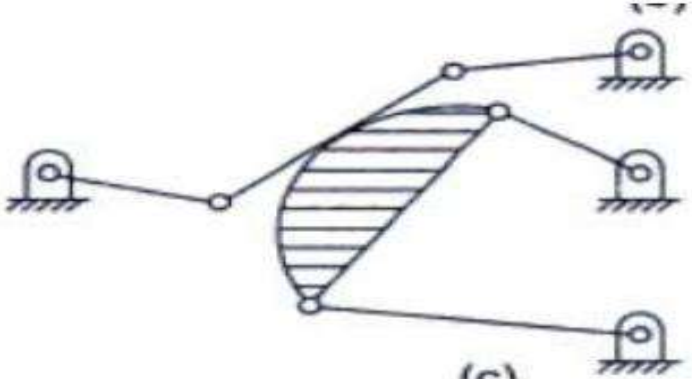
(At the slider, one sliding pair and two turning pairs)

$F = 3(N-1) - 2P_1 - P_2$

$= 3(8-1) - 2 \times 10 - 0 = 1$

Thus, it is a mechanism with a single degree of freedom.





(c)

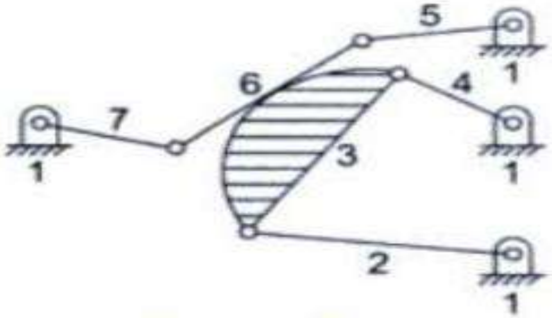


Fig. 1.23

(c) The mechanism has a cam pair. Therefore, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 7 (Fig. 1.23)

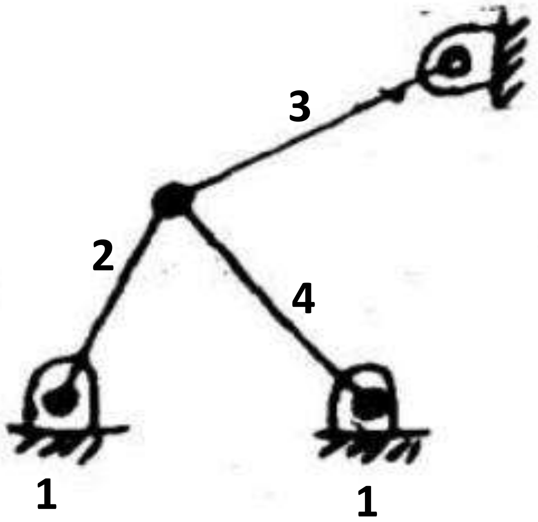
Number of pairs with 1 degree of freedom = 8

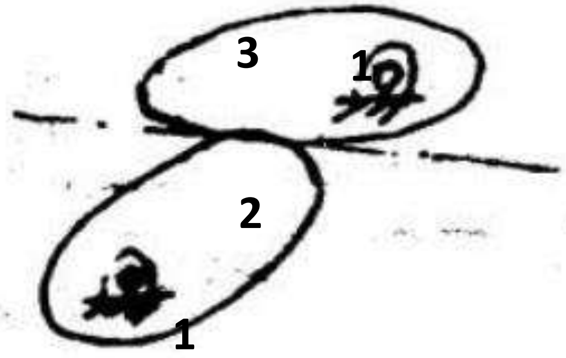
Number of pairs with 2 degrees of freedom = 1

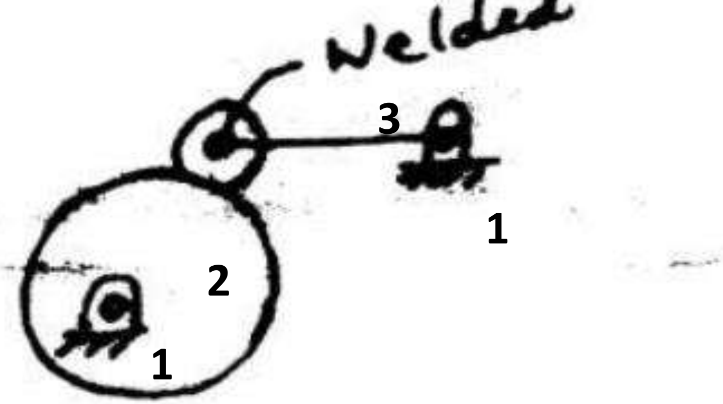
$F = 3(N-1) - 2P_1 - P_2$

$= 3(7-1) - 2 \times 8 - 1 = 1$

Thus, it is a mechanism with one degree of freedom.







Unconstrained chain (4 separate input motions

(a)

$F = 3(n-1)-2l-h$

Here, n = 4, l = 5 and h = 0.

$F = 3(4-1)-2(5) = -1$

I.e., it is a structure

super structure

(b)

$F = 3(n-1)-2l-h$

Here, n = 3, l = 2 and h = 1.

$F = 3(3-1)-2(2)-1 = 1$

(c)

$F = 3(n-1)-2l-h$

Here, n = 3, l = 2 and h = 1.

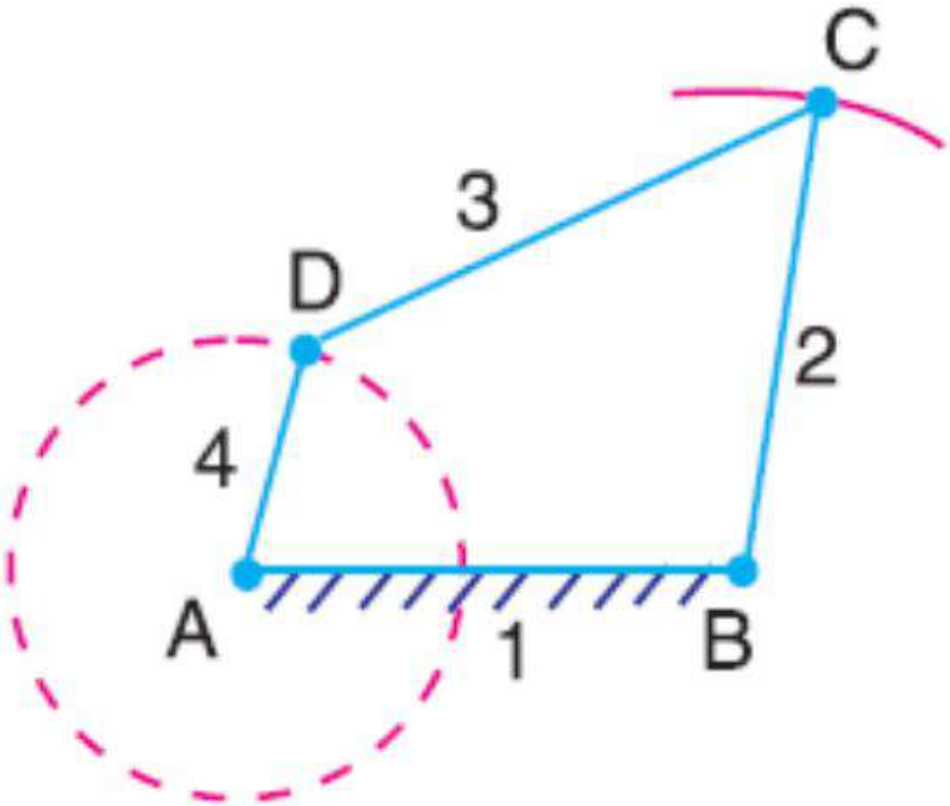
$F = 3(3-1)-2(2)-1 = 1$

Kinematic chain(Mechanism)

Inversion of Mechanism

- A Mechanism is one in which one of the links of a kinematic chain is fixed .
- Different mechanisms can be obtained by fixing different links of the same kinematic chain These are called as inversions of the mechanism.

- By changing the fixed link, the number of mechanisms which can be obtained is equal to the number of links.
- **Except the original mechanism, all other mechanisms will be known as inversions of original mechanism.**
- The inversion of a mechanism does not change the motion of its links relative to each other.



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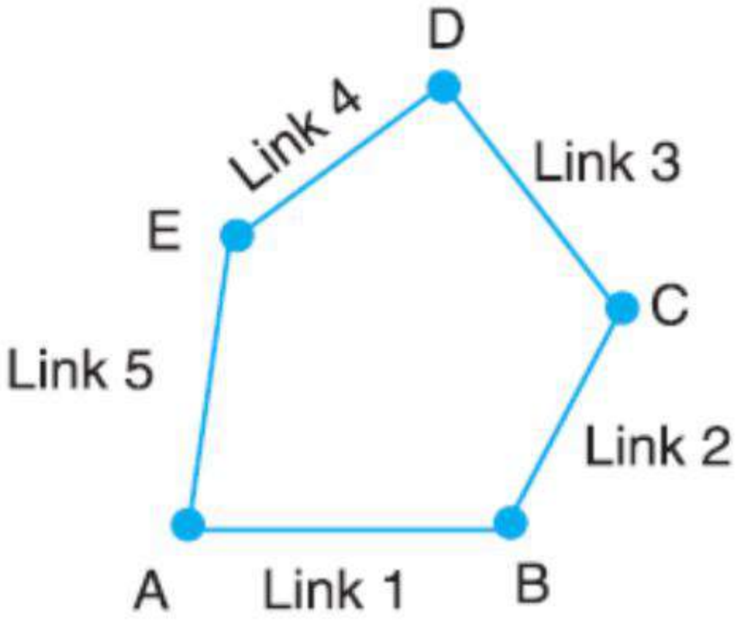
Kinematic Chain

When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion (*i.e. completely or* successfully constrained motion).



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- Types of Kinematic Chains
  - The most important kinematic chains are those which consist of four lower pairs, each pair being a sliding pair or a turning pair.

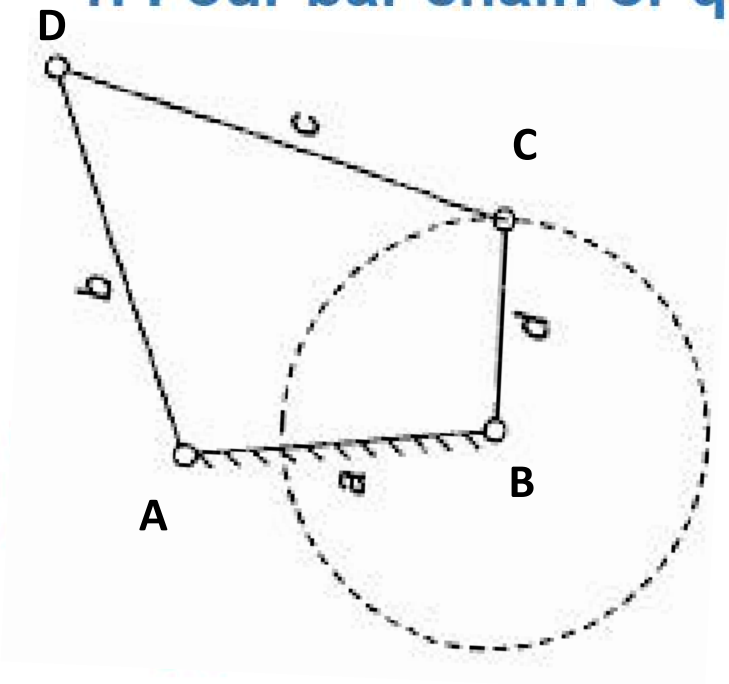
- The following three types of kinematic chains with four lower pairs are important from the subject point of view :

1. Four bar chain or quadric cyclic chain,: it has 4links and all 4 have turning (revolute) pairs

2. Single slider crank chain, : it has 1 sliding pair and 3 turning pair

3. Double slider crank chain : it has 2 sliding pair and 2 turning pair

1. Four bar chain or quadric chain



- Four bar chain (mechanism) is the simplest and the basic kinematic chain and consists of four rigid links
- Each of them forms a *turning pair at A, B, C and D.*

The link that makes a complete revolution is called a **crank**

- The four links may be of different lengths.
- According to **Grashof 's law** for a four bar mechanism, "the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths" if there is to be continuous relative motion between the two links.

i.e $d+b < a+c$

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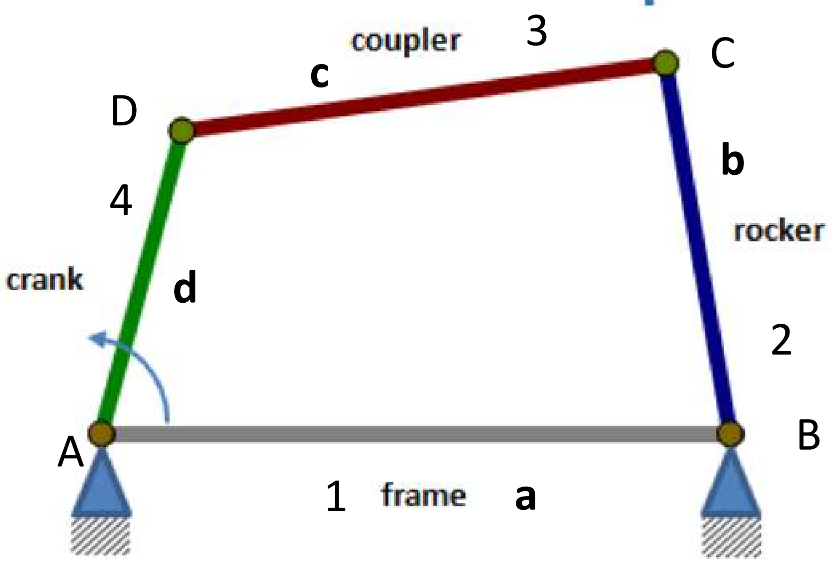
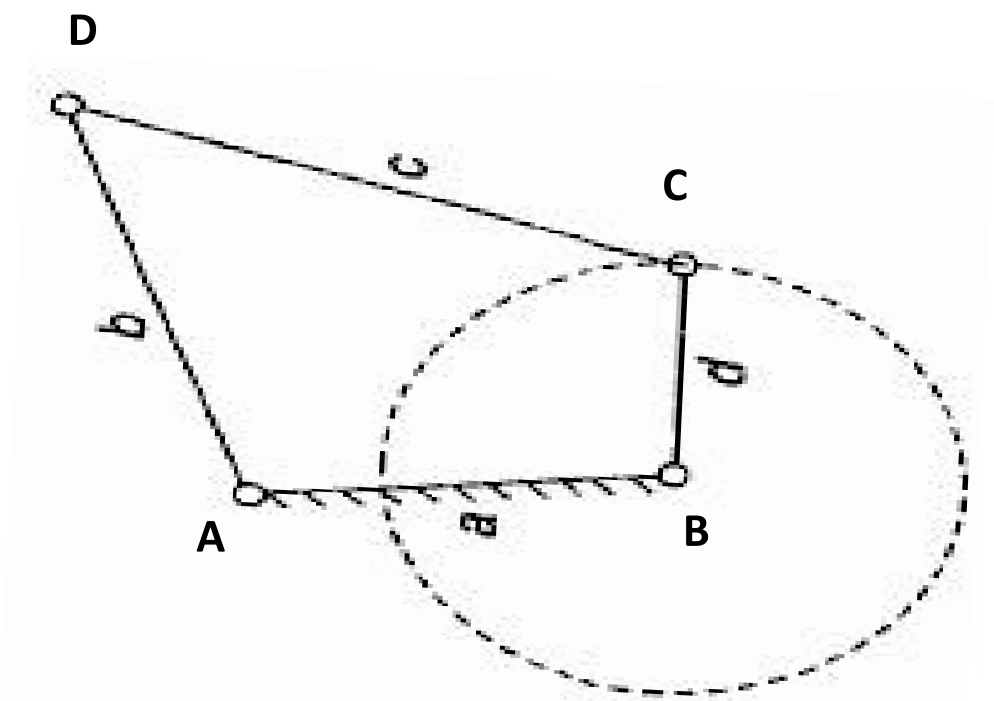
1. Four bar chain or quadric chain





- The shortest link, will make a complete revolution relative to the other three links **crank or driver.** **In Fig.**, **AD (link 4) is a crank**.

- link BC (link 2) which makes a partial rotation or oscillates is known as **lever or rocker or follower**

- link CD (link 3) which connects the crank and lever is called **connecting rod or coupler.**

- The fixed link AB (link 1) is known as **frame of the mechanism.**

The mechanism transforms rotary motion into oscillating motion.

Where d is the shortest link, b is the longest link, c and a are the other links

A linkage in which the sum of the lengths of the longest and the shortest links is less than the sum of the lengths of the other two links, is known as a Class-I , four bar linkage

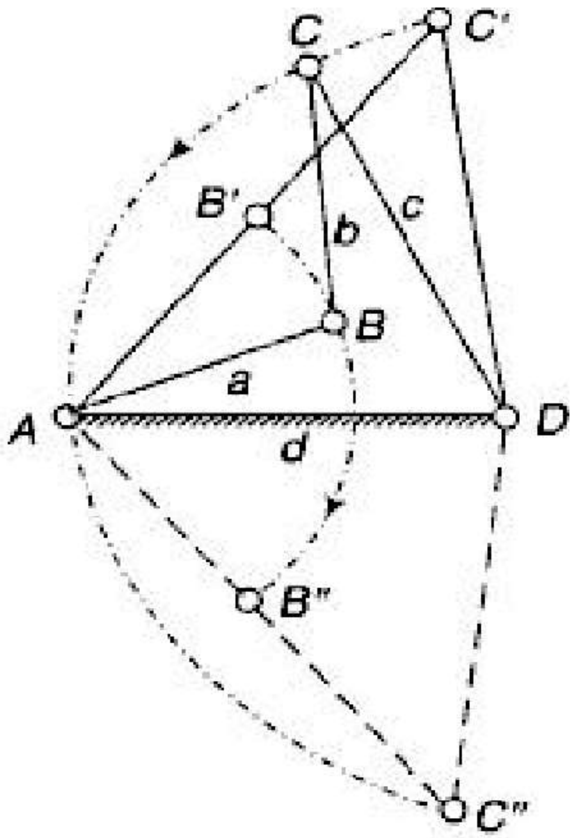
i.e $d+b< c+a$

When the sum of the lengths of the longest and the shortest links is more than the sum of the lengths of the other two links, the linkage is known as a Class-II , four bar linkage.

i.e $d+b > c+a$

In a Class-II , four bar linkage, fixing of any of the links always results in a rocker – rocker mechanism .

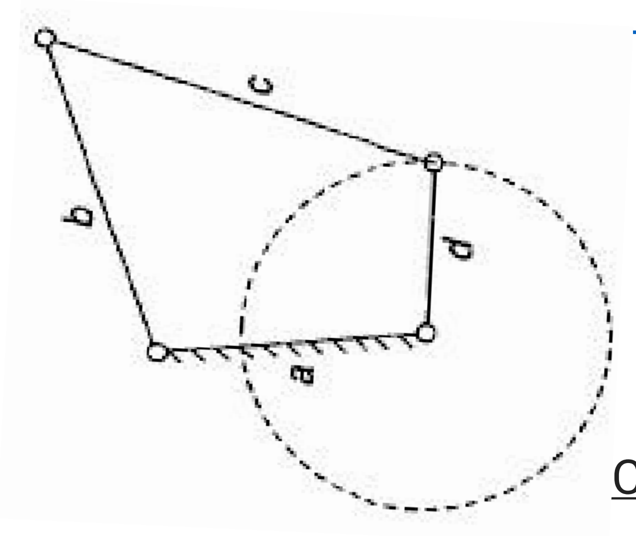
In other wards, the mechanism and its inversions give the same type of motion (of a double-rocker mechanism)



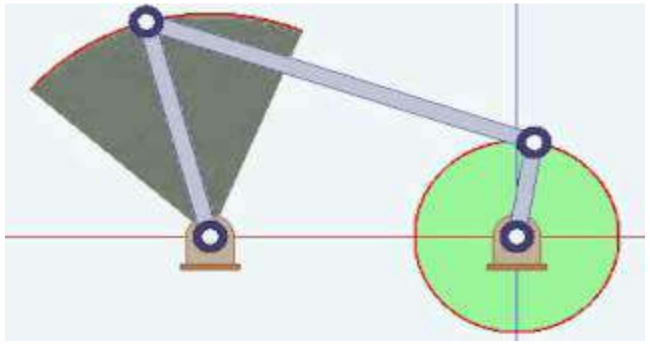
Class-I , four bar linkage:

The above observations are summarised in Grashof’s law states that a four- bar mechanism has at least one revolving link if the sum of the lengths of the longest and the shortest links is less than the sum of the lengths of the other two links

i.e $d+b< c+a$



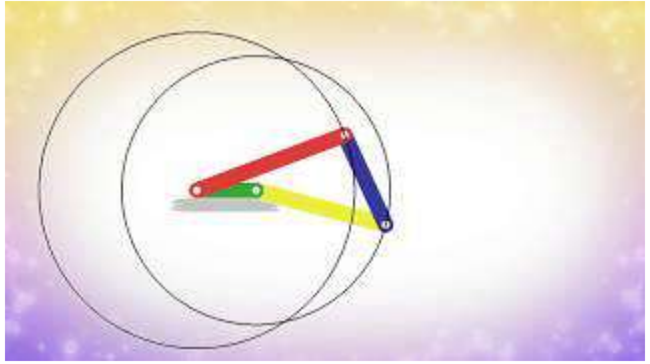
https://youtu.be/4tIo3AQQiU8



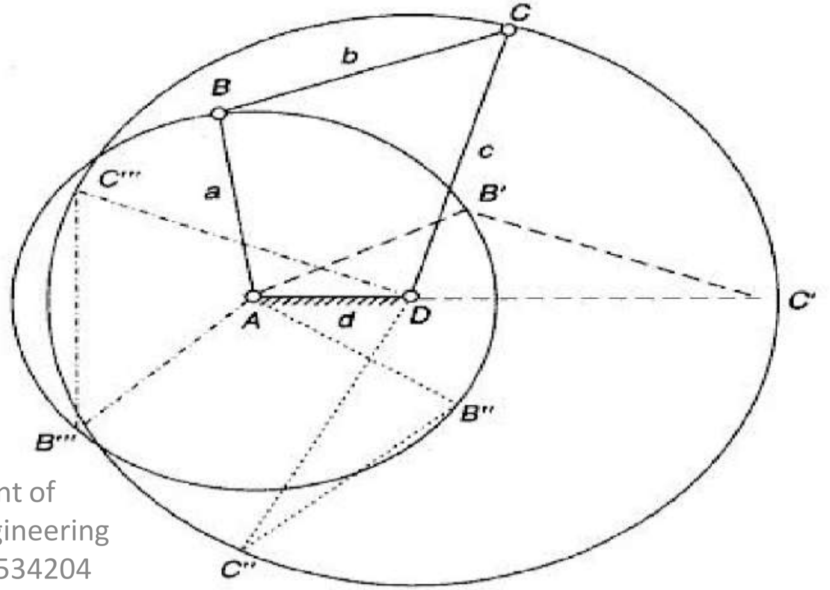
Crank-Rocker Four-Bar Linkage

Further ,if the shortest link is fixed, the chain will act as a double-crank mechanism in which links adjacent to the fixed link will have complete revolution.

https://youtu.be/xFg8WQRfHHo



Double Crank Mechanism



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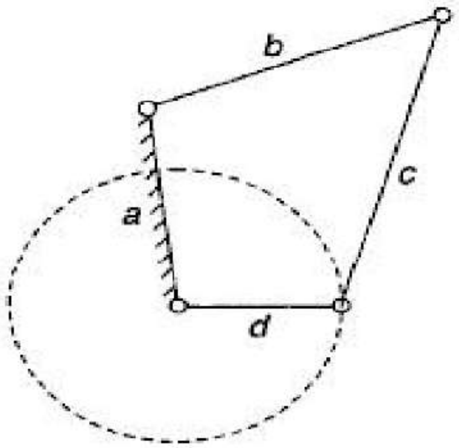
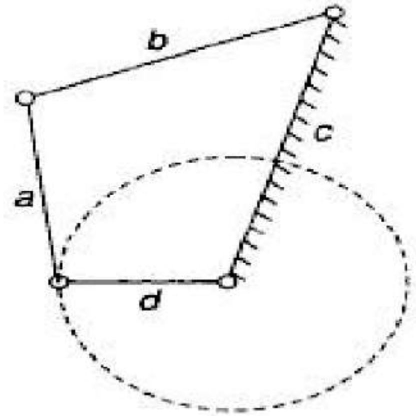
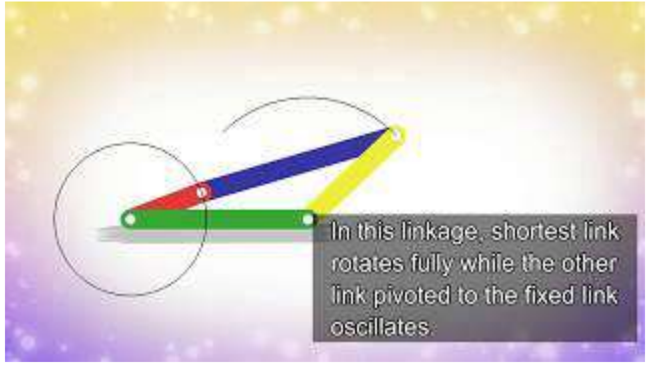
Inversions of four bar Mechanism

If the link adjacent to the shortest link is fixed, the chain will act as crank- rocker mechanism in which the shortest link will revolve and the adjacent to the fixed link will oscillate

https://youtu.be/fCcWGXL-2Z8







Crank Rocker Mechanism

If the link opposite to the shortest link is fixed , the chain will act as double- rocker mechanism in which links adjacent to the fixed link will oscillate.

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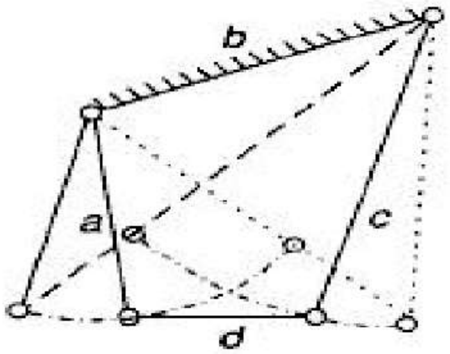

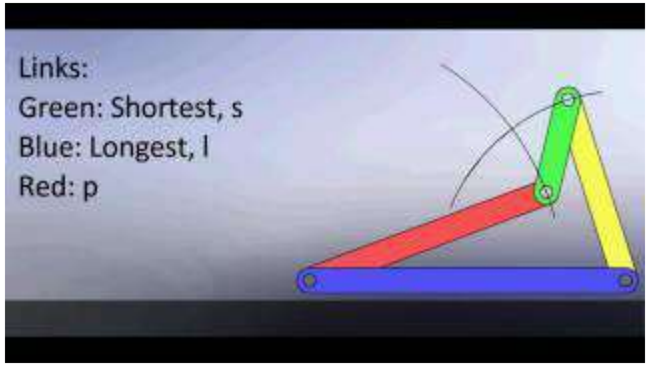

Double rocker mechanism

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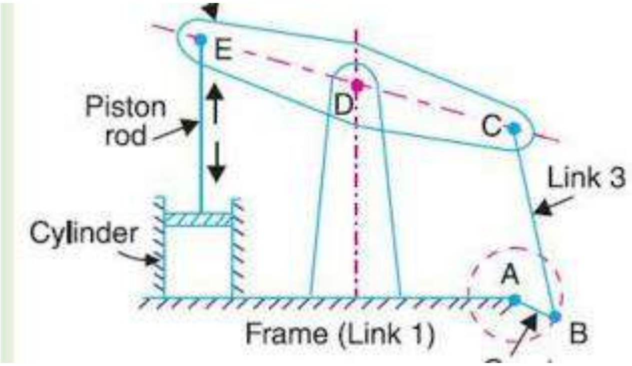
Inversions of Four Bar Chain

Coupling rod of a locomotive (Double crank mechanism)

Beam engine (crank and lever mechanism)

Watt's indicator mechanism (Double lever mechanism).

https://youtu.be/ia-295RVEP0



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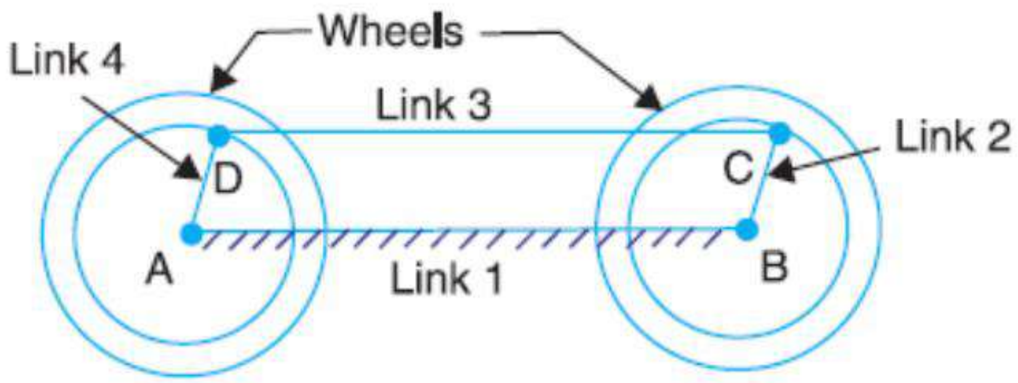
inversion of mechanism, four bar chain and single slider crank chain

Inversions of Four Bar Chain

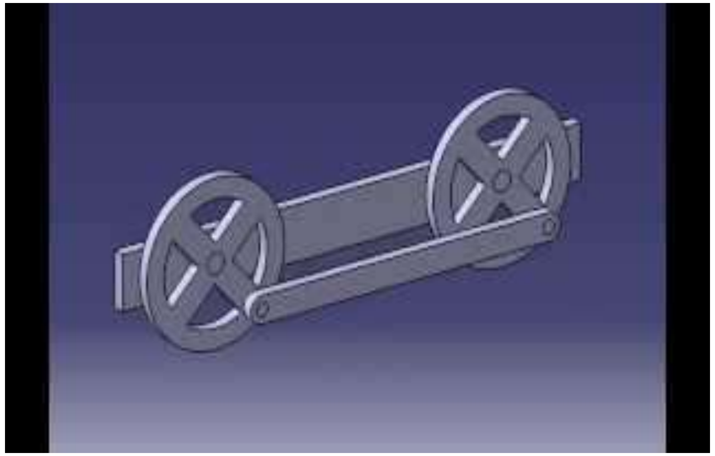
1. *Coupling rod of a locomotive (Double crank mechanism)*

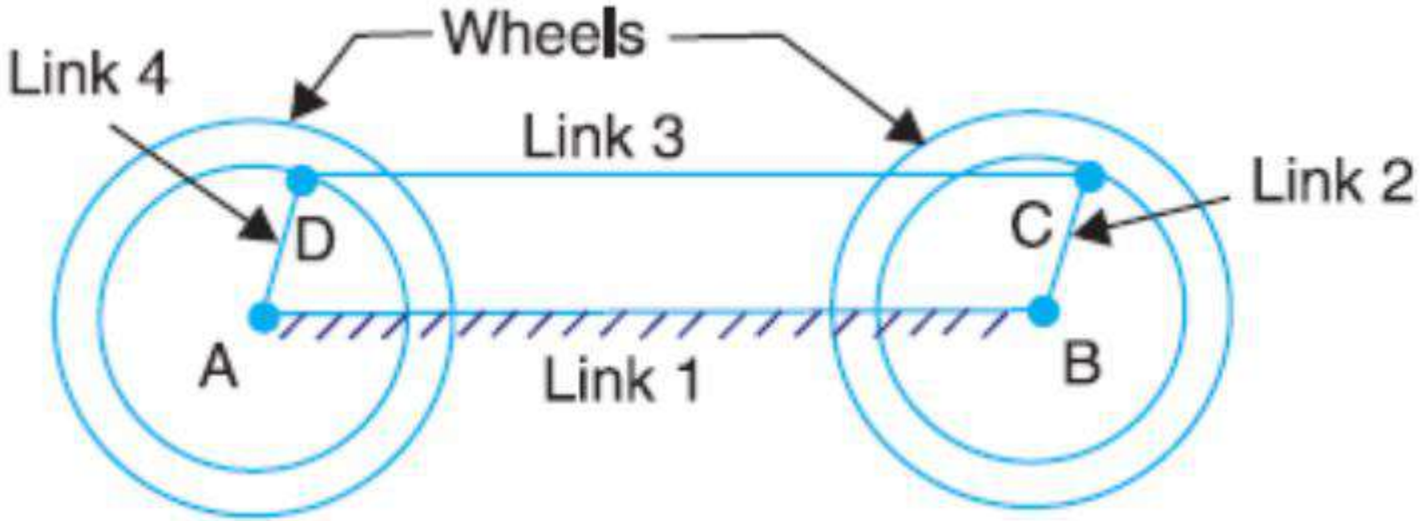


- In this mechanism, the links AD and BC (having equal length) act as cranks and are connected to the respective wheels.
- The link CD acts as a coupling rod.
- The link AB is fixed in order to maintain a constant centre to centre distance between them.

Purpose: Transmitting rotary motion from one wheel to the other wheel.

https://youtu.be/OjPyo0yikuA





COUPLING ROD OF LOCOMOTIVE MECHANISM

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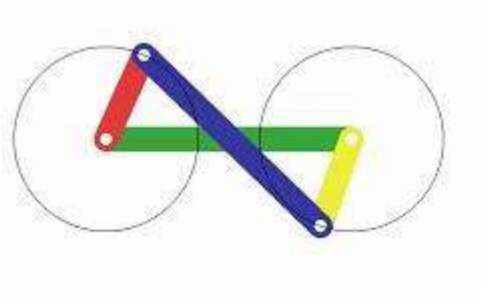
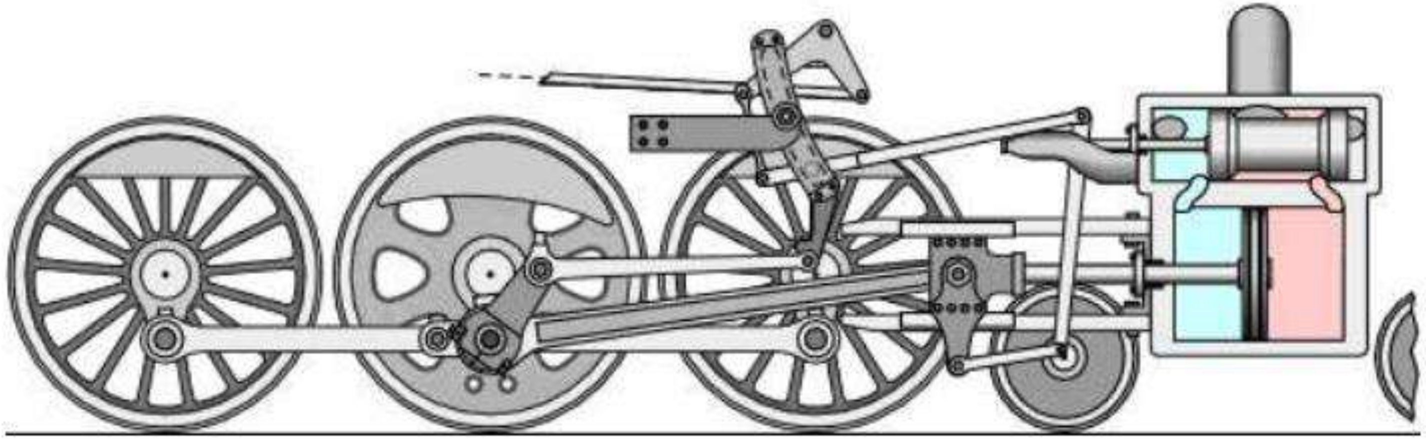


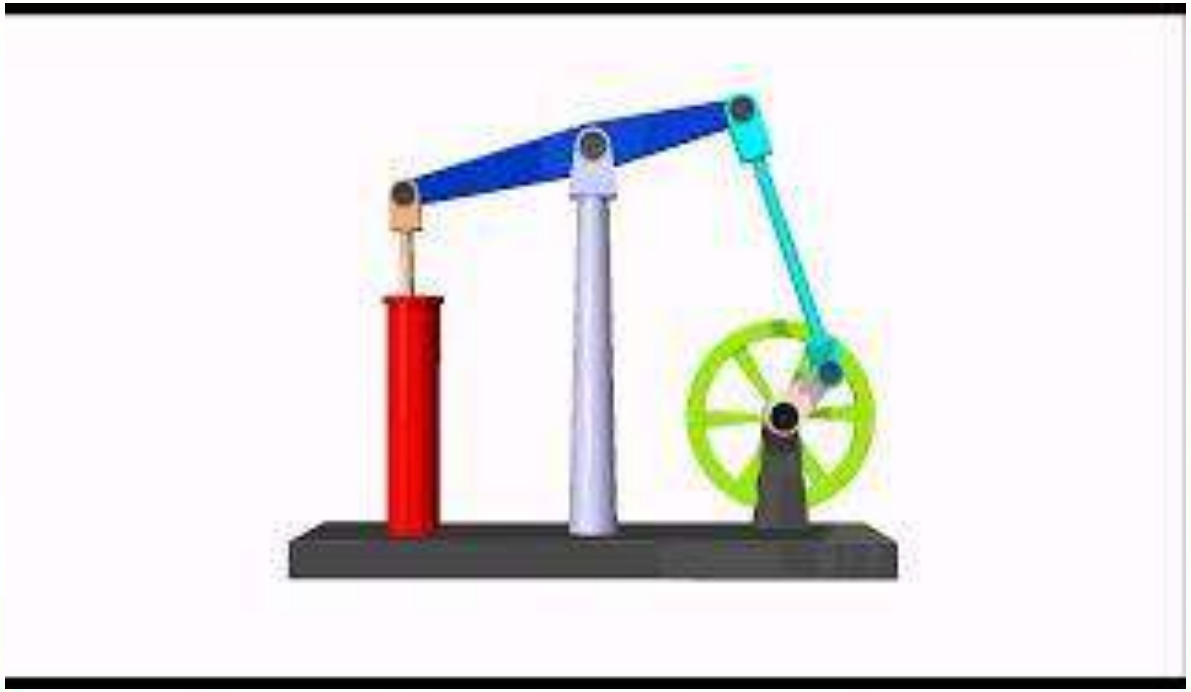


Antiparallelogram linkage

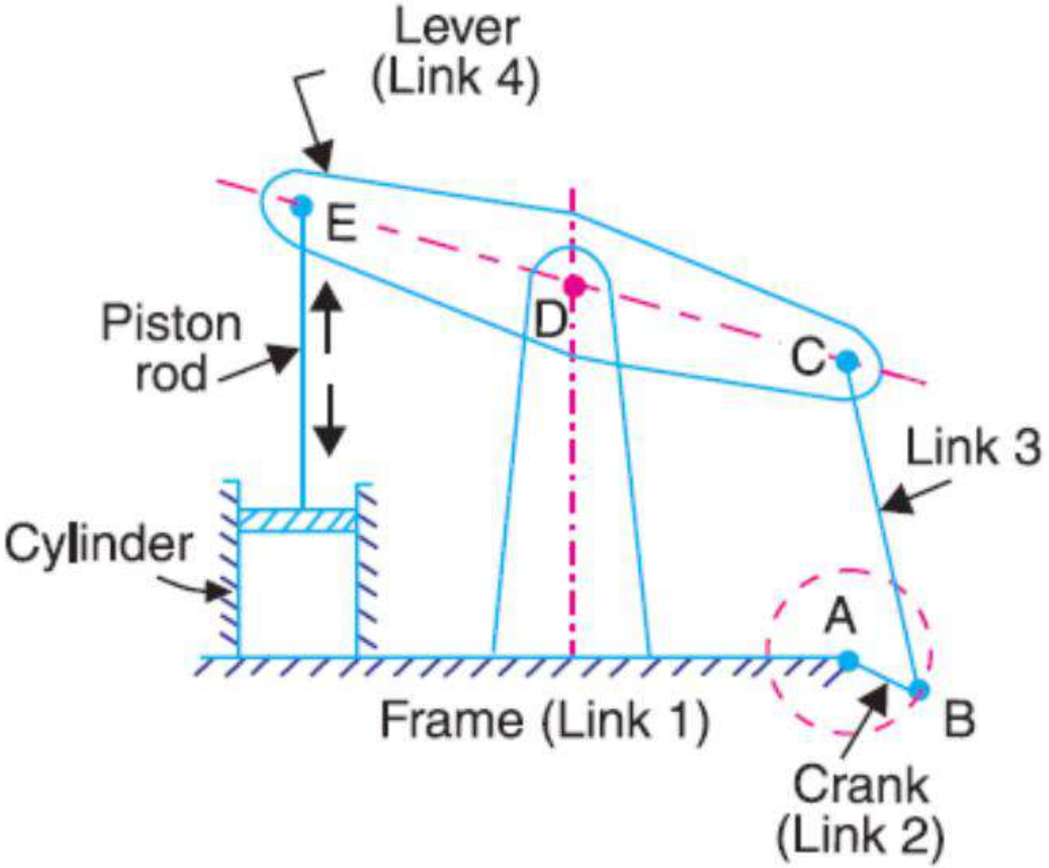
Beam engine (crank and lever mechanism)

- When the crank AB rotates about the fixed point A.
- The lever oscillates about another fixed point D.
- The end E of lever is connected to a piston rod which reciprocates in a cylinder.`

https://youtu.be/QEeysl4wpn4



[Beam Engine Mechanism animation]()



Purpose of this mechanism is to convert rotary motion into reciprocating motion.

3. Watt's indicator mechanism (Double lever mechanism).

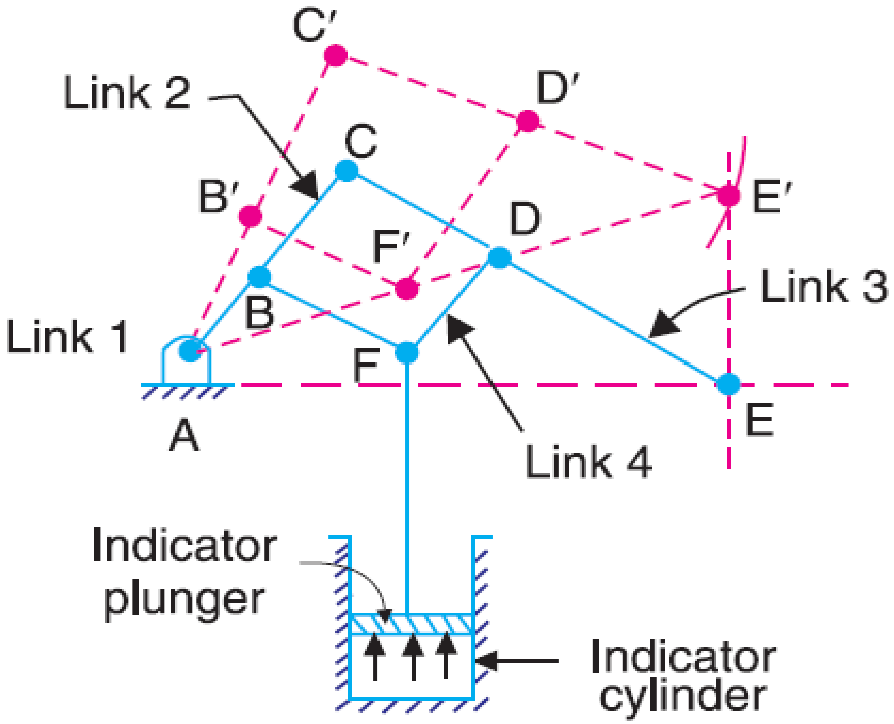
- A Watt's indicator mechanism (also known as Watt's straight line mechanism or double lever mechanism) which consists of four links, is shown in Fig.

- The four links are : fixed link at A, link AC, link CE and link BFD. It may be noted that BF and FD form one link because these two parts have no relative motion between them. The links CE and BFD act as levers.

- The displacement of the link BFD is directly proportional to the pressure of gas or steam which acts on the indicator plunger.

- On any small displacement of the mechanism, the tracing point E at the end of the link CE traces out approximately a straight line.

- The initial position of the mechanism is shown in Fig. by full lines whereas the dotted lines show the position of the mechanism when the gas or steam pressure acts on the indicator plunger.



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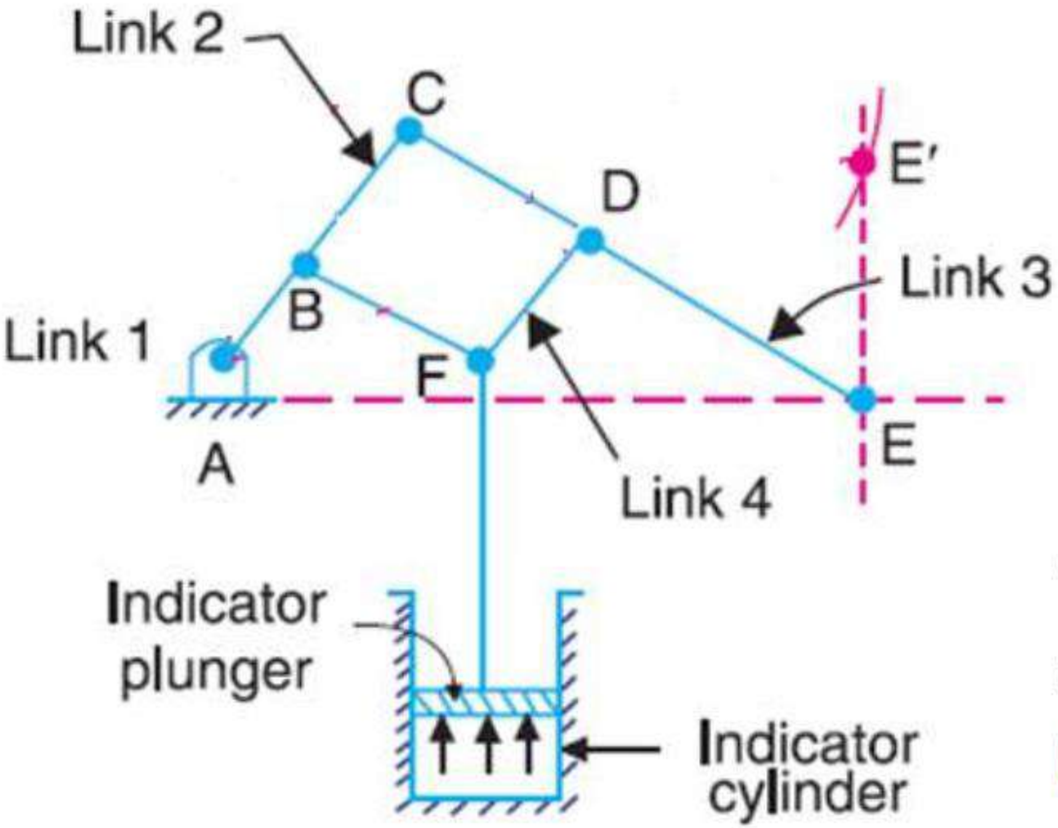
3. *Watt's indicator mechanism (Double lever mechanism)*

Watt's straight line mechanism

The four links are : fixed link at A, link AC, link CE and link BFD. Links CE and BFD act as levers.

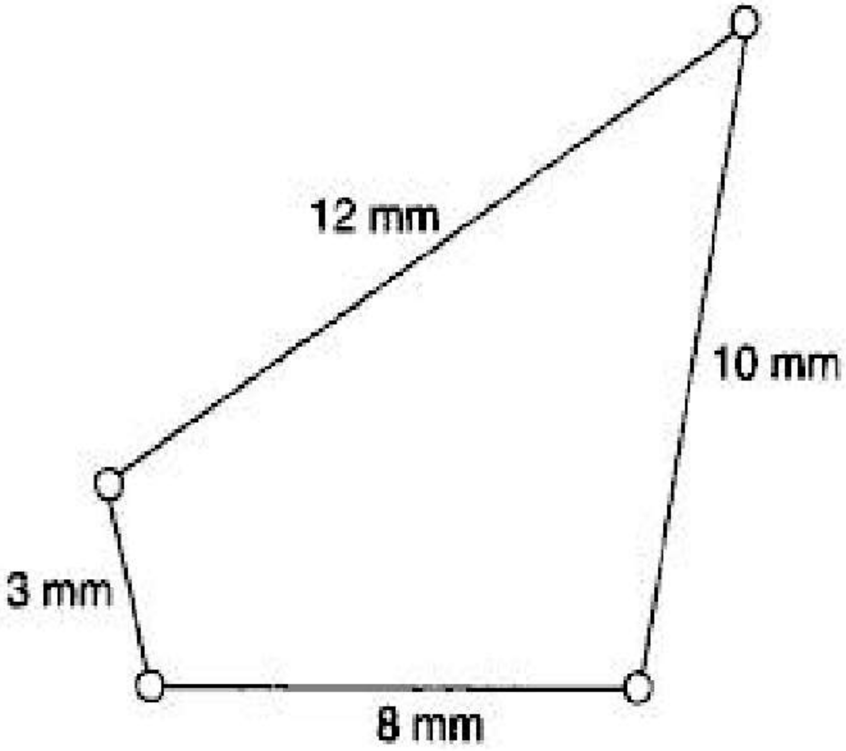


https://youtu.be/xsh5H654llg

Watt indicator mechanism animation

On displacement of the mechanism, the tracing point ***E** at the end of the* link CE traces out approximately a straight line.

Find all the inversion of the chain given in Fig. 1.39.

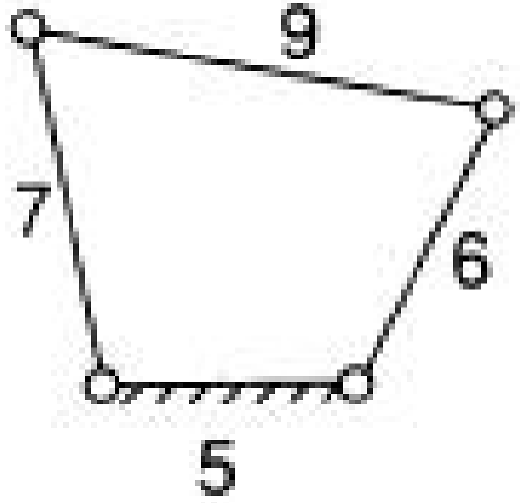


Length of the longest link = 12 mm
Length of the shortest link = 3 mm
Length of other links = 10 mm and 8 mm
Since $12 + 3 < 10 + 8$, it belongs to the class-I mechanism and according to Grashoff's law, three distinct inversions are possible.

Shortest link fixed, i.e., when the link with 3-mm length is fixed, the chain will act as double-crank mechanism in which links with lengths of 12 mm and 8 mm will have complete revolutions.

Link opposite to the shortest link fixed, i.e., when the link with 10-mm length is fixed, the chain will act as double-rocker mechanism in which links with lengths of 12 mm and 8 mm will oscillate.

Link adjacent to the shortest link fixed, i.e., when any of the links adjacent to the shortest link, i.e., link with a length of 12-mm or 8 mm is fixed, the chain will act as crank-rocker mechanism in which the shortest link of 3-mm length will revolve and the link with 10-mm length will oscillate.

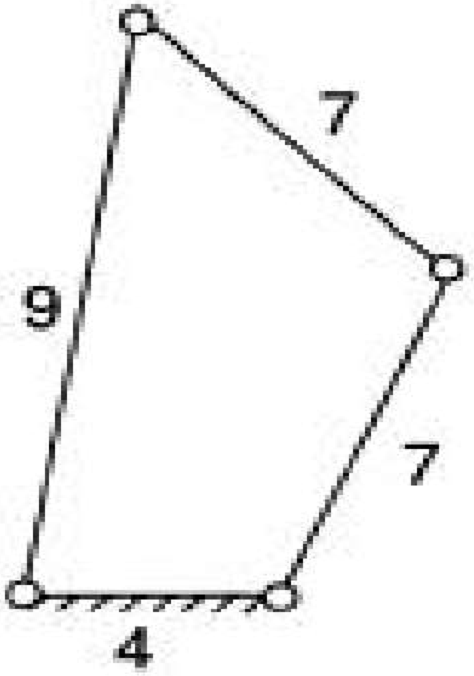


Length of the longest link = 9

Length of the shortest link = 5

Length of other links = 7 and 6

Since 9 + 5 > 7 + 6, it does not belong to the class-I mechanism. Therefore, it is a double-rocker mechanism.



Length of the longest link = 9

Length of the shortest link = 4

Length of other links = 7 and 7

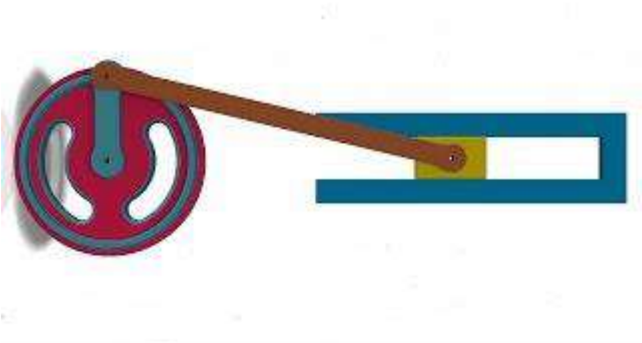
Since 9 + 4 < 7 + 7, it belongs to the class-I mechanism. In this case as the shortest link is fixed, it is a double-crank mechanism.

Single Slider Crank Chain

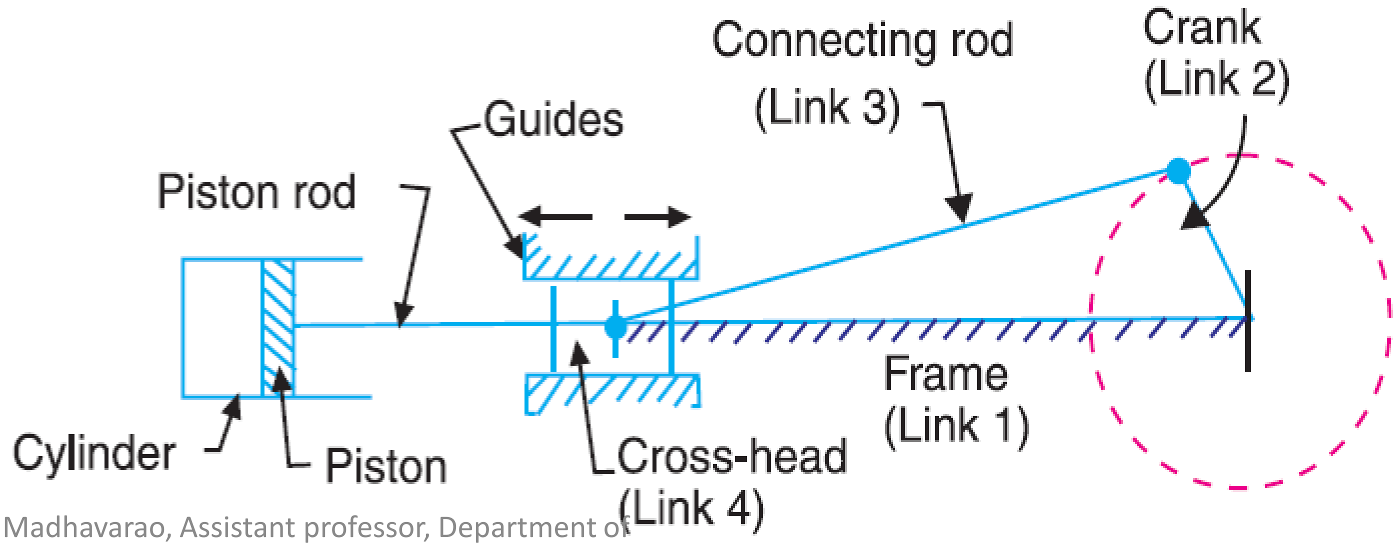
- A single slider crank chain is a modification of the basic four bar chain. It consist of one sliding pair and three turning pairs. It is, usually, found in reciprocating steam engine mechanism.
- This type of mechanism converts rotary motion into reciprocating motion and vice versa.

- In a single slider crank chain, as shown in Fig., the links 1 and 2, links 2 and 3, and links 3 and 4 form three turning pairs while the links 4 and 1 form a sliding pair.
- The link 1 corresponds to the frame of the engine, which is fixed. The link 2 corresponds to the crank ; link 3 corresponds to the connecting rod and link 4 corresponds to crosshead.
- As the crank rotates, the cross-head reciprocates in the guides and thus the piston reciprocates in the cylinder

https://youtu.be/ZO8QEG4x0wY



Slider Crank Mechanism Works



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Inversions of Single Slider Crank Chain

- **First inversion:** This inversion is obtained when link 1 (ground body) is fixed. Application- Reciprocating engine, Reciprocating compressor etc...
- **Second inversion:** This inversion is obtained when link 2 (crank) is fixed. Application- Whitworth quick return mechanism, Rotary engine, etc...
- **Third inversion:** This inversion is obtained when link 3 (connecting rod) is fixed. Application- Slotted crank mechanism, Oscillatory engine etc..,
- **Fourth inversion:** This inversion is obtained when link 4 (slider) is fixed. Application- Hand pump, pendulum pump or Bull engine, etc...

Inversions of Single Slider Crank Chain

- Four inversions of a single slider crank chain are possible. These inversions are found in the following mechanisms.

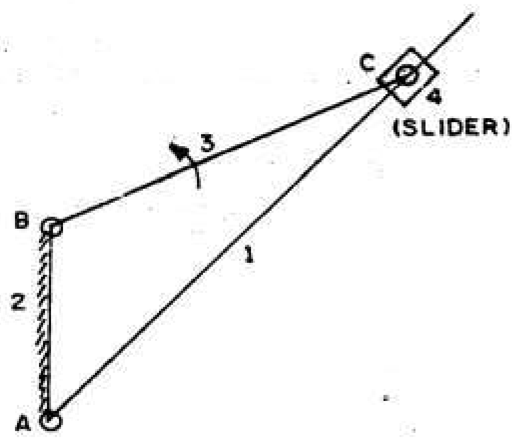
1. Pendulum pump or Bull engine.(slider is fixed**)**

2. Oscillating cylinder engine (connecting rod is fixed **)**

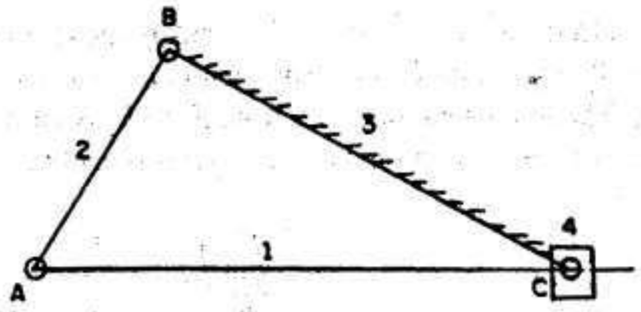
3. Rotary internal combustion engine(crank is fixed)

4. Crank and slotted lever quick return motion mechanism. (connecting rod is fixed **)**

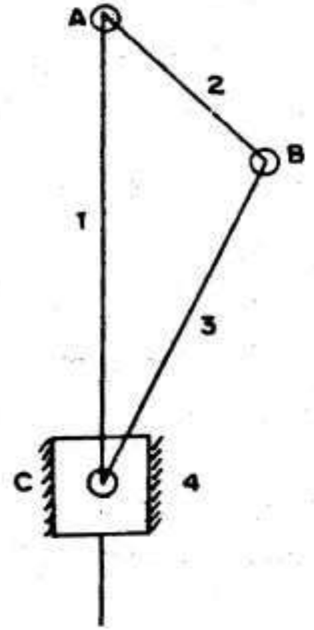
5. Whitworth quick return motion mechanism. (crank is fixed)



(a) crank fixed



(b) connecting rod fixed



(c) slider fixed

Inversions of Single Slider Crank Chain

Pendulum pump or Bull engine.

Oscillating cylinder engine.

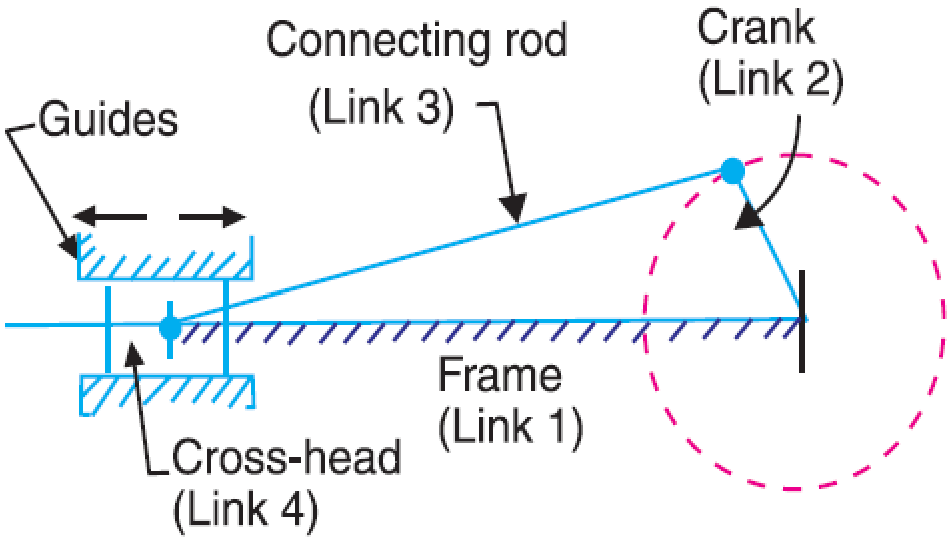
Rotary internal combustion or Gnome engine.

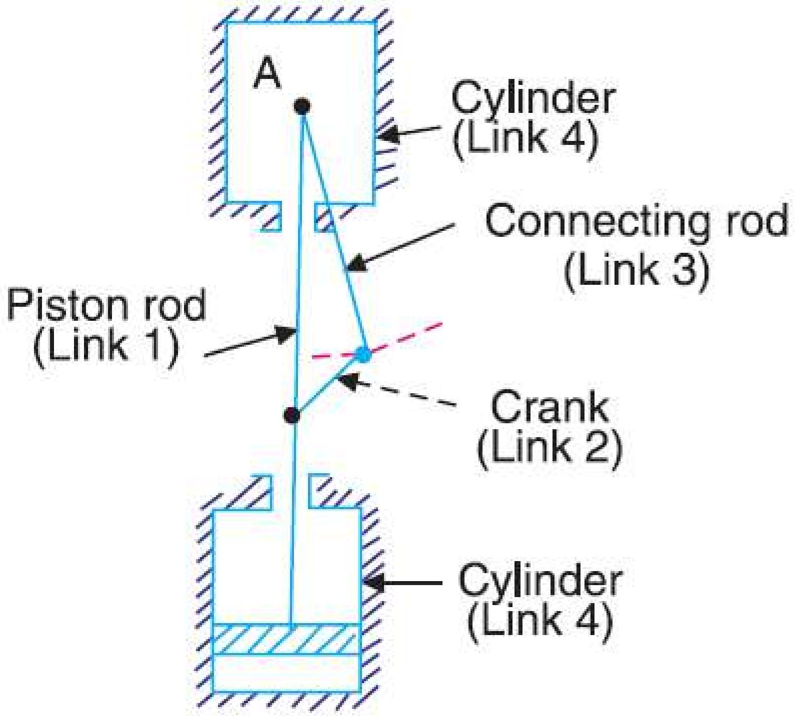
Crank and slotted lever quick return motion mechanism.

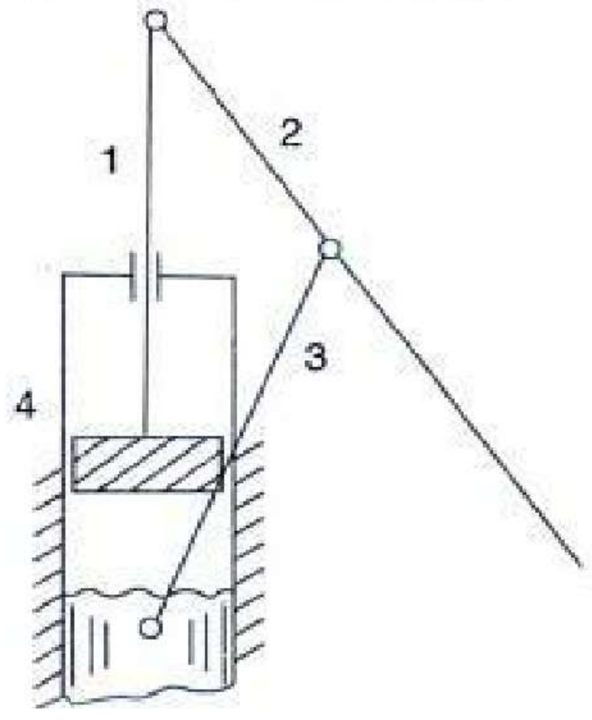
Whitworth quick return motion mechanism.

Pendulum pump or Bull engine.(Fourth Inversion)

- In this mechanism, the inversion is obtained by fixing the cylinder or link 4 (i.e. sliding pair), as shown in Fig.
- In this case, when the crank (link 2) rotates, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at A and the piston attached to the piston rod (link 1) reciprocates.
- The duplex pump which is used to supply feed water to boilers have two pistons attached to link 1, as shown in Fig



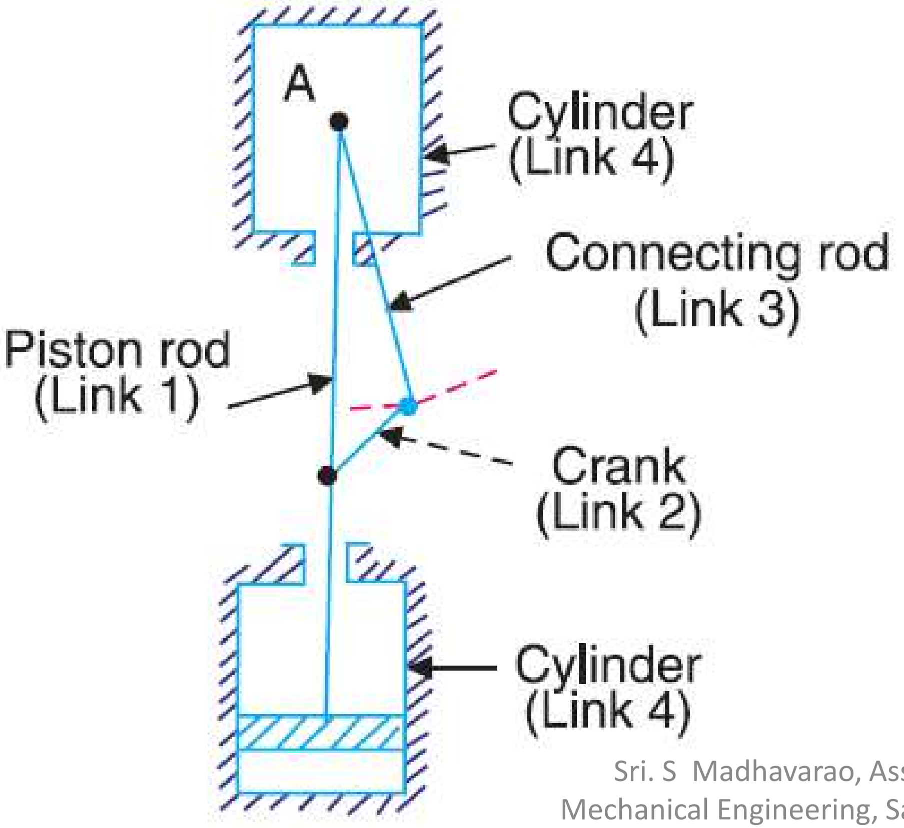




HAND PUMP

Pendulum pump or Bull engine.(Fourth Inversion)

When the crank (link 2) is given a rotary motion, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at *A*. *The piston* attached to the piston rod (link 1) reciprocates.



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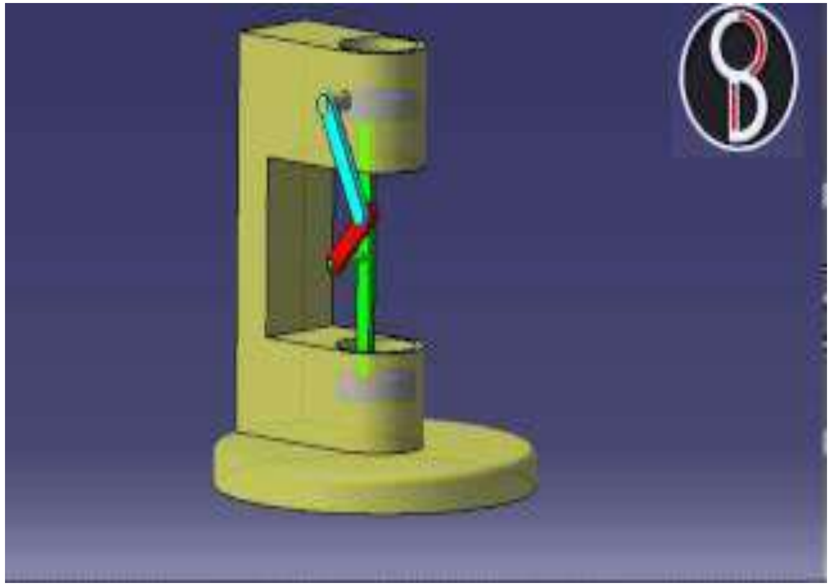

pendulum pump or bull engine animation

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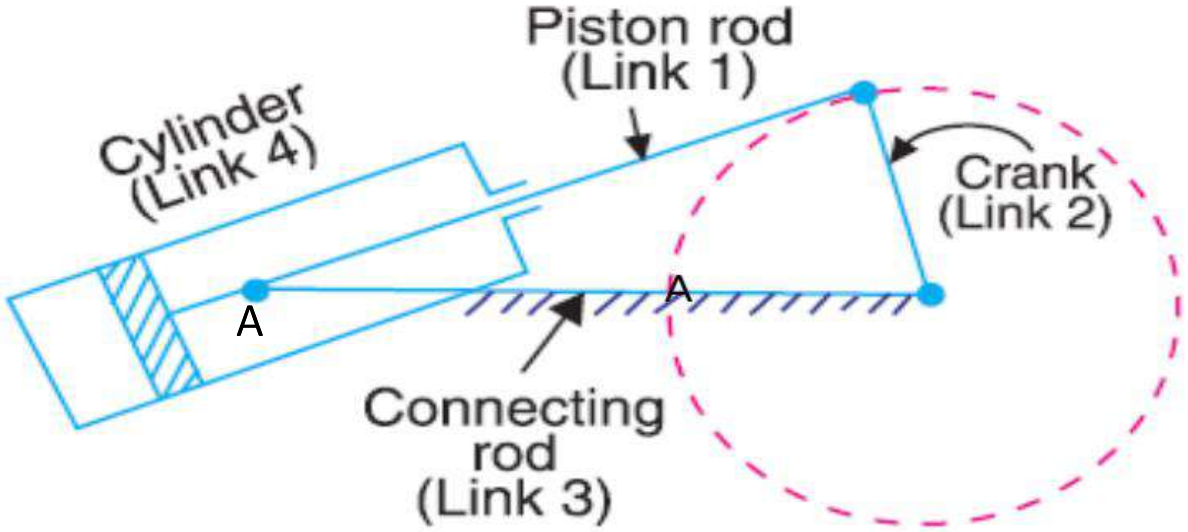
Oscillating Cylinder Engine(Third Inversion)



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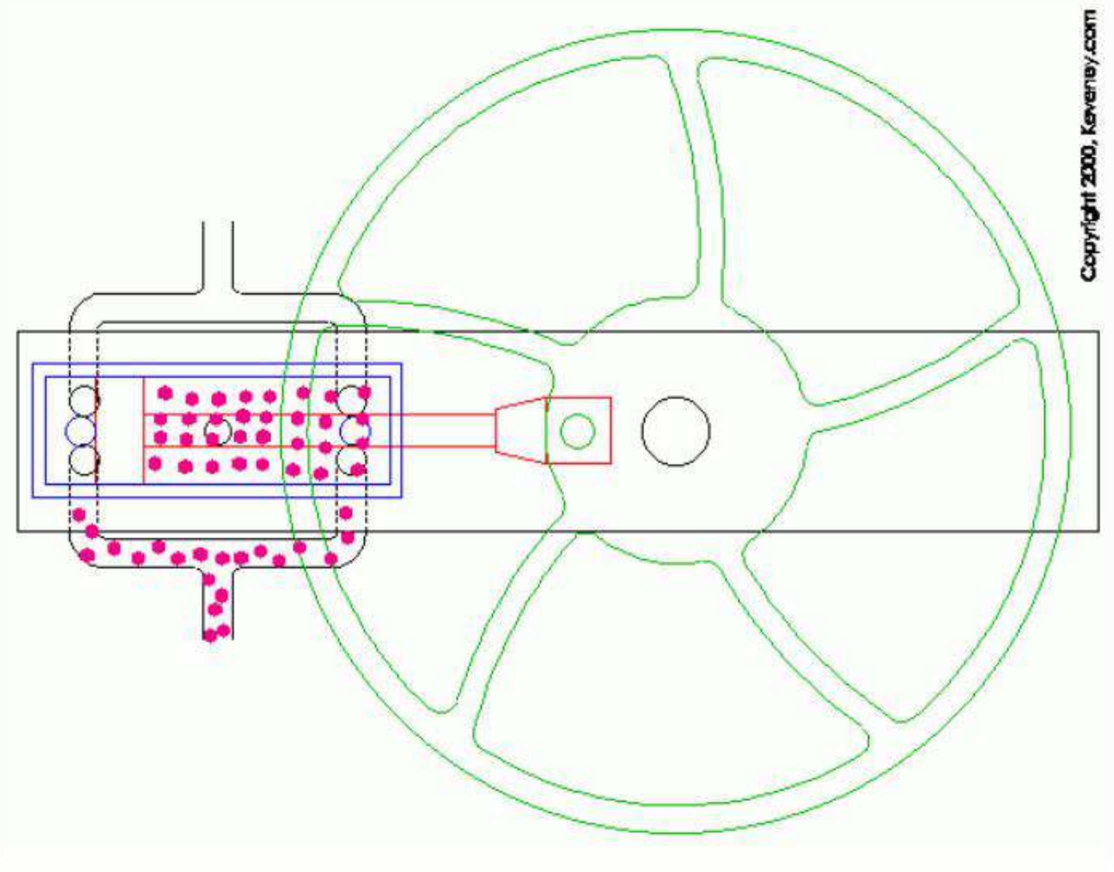
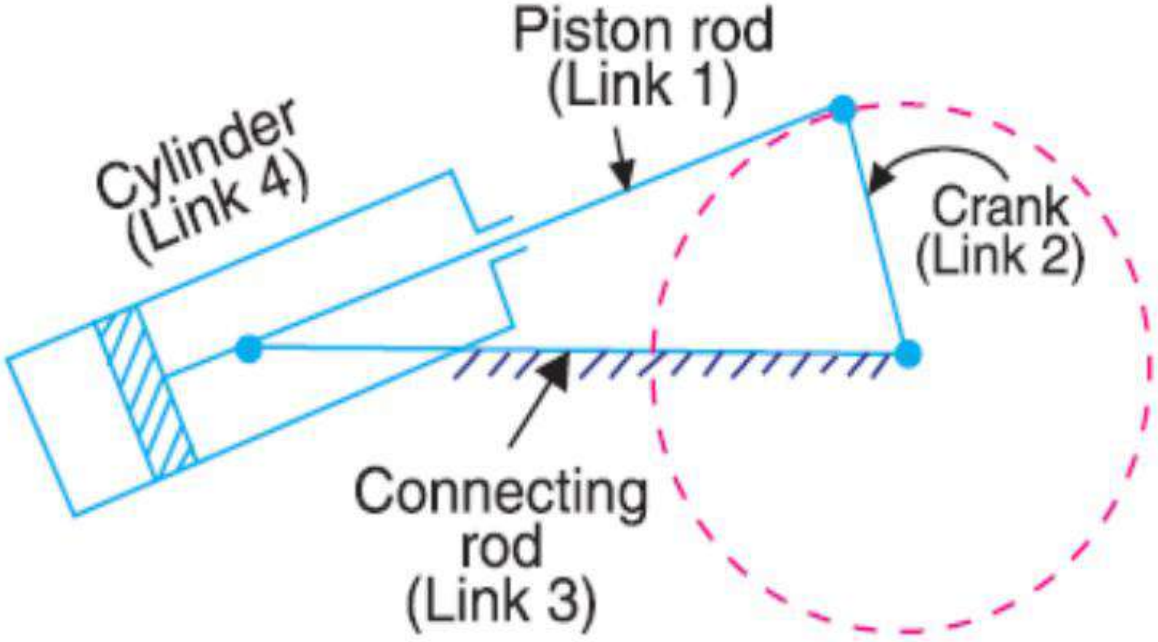
Oscillating Cylinder Engine

Link 3 forming the turning pair is fixed and it corresponds to the connecting rod of a reciprocating steam engine mechanism.

When the crank (link 2) rotates, the piston attached to piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at A

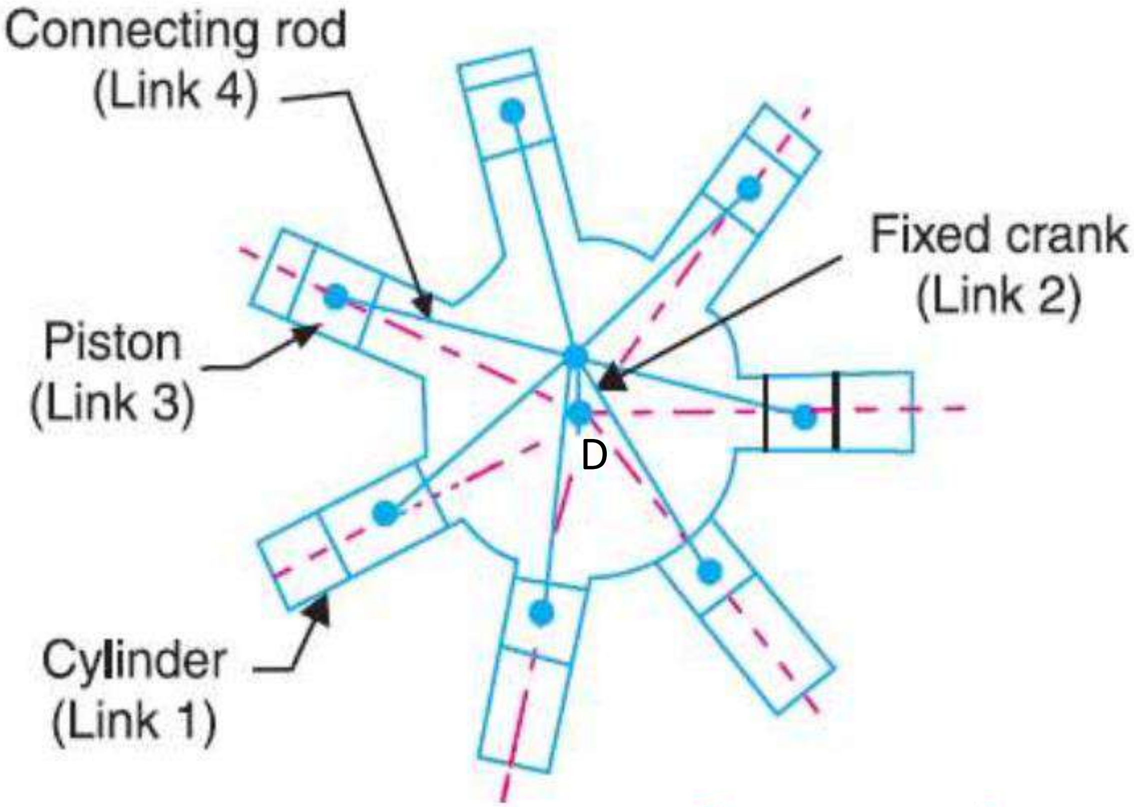
Oscillating cylinder engine.





Rotary internal combustion Engine (Second Inversion)


Rotary engine



Gnome engine

It consists of seven cylinders in one plane all revolve about fixed centre *D*.

Cylinders form link 1, Crank (link 2) is fixed. When the connecting rod (link4) rotates, the piston (link 3) reciprocates inside the cylinder.

Rotary internal combustion Engine (Second Inversion)

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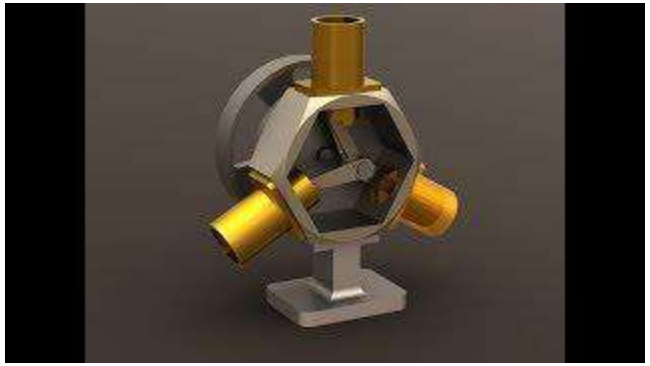
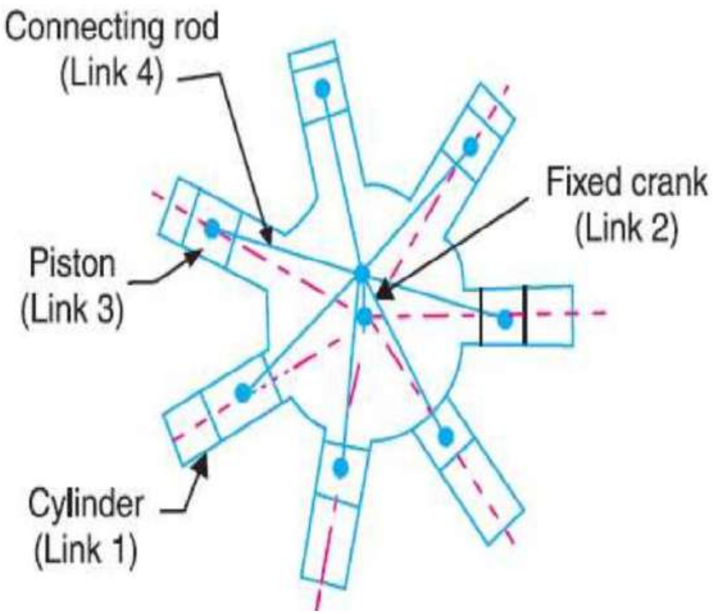
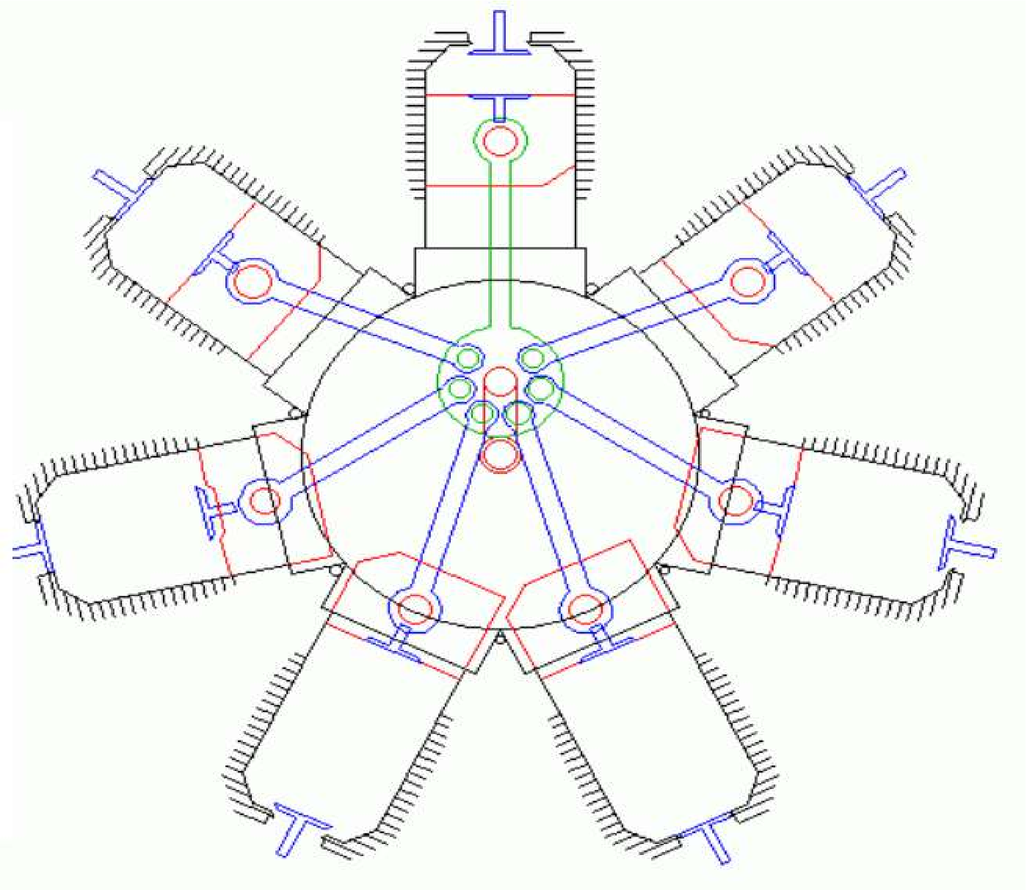


Gnome Engine





Gnome engine

Radial Engine

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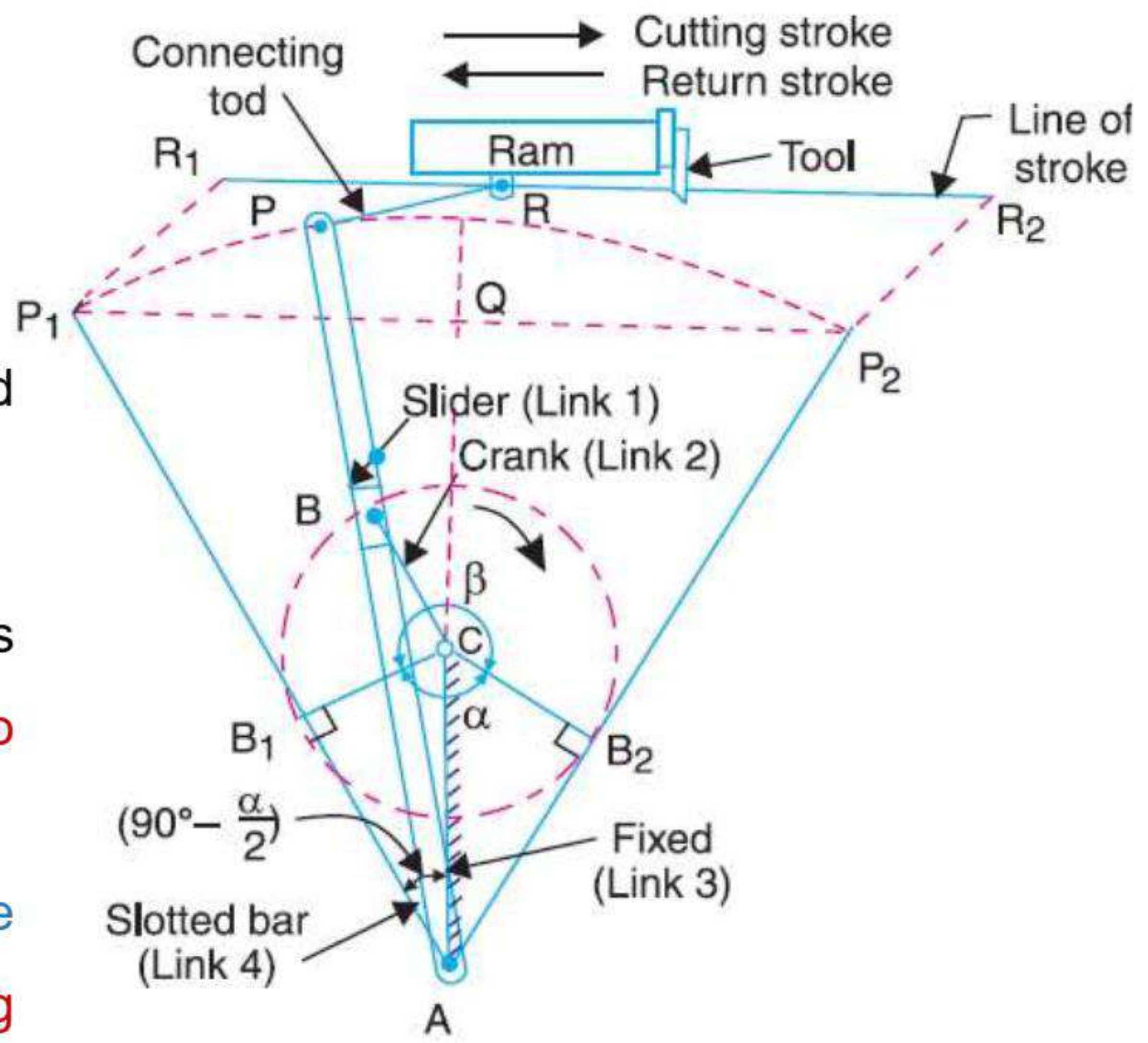
Crank and Slotted Lever quick return motion Mechanism(Third Inversion)

A mechanism used in shaping and slotting machines, where the metal is cut intermittently.

- Link AC (i.e. link 3) is fixed.
- Crank CB revolves with uniform angular speed about the fixed center C.
- Sliding block attached to the crank pin at **B** slides along the slotted bar AP, thus causing AP to oscillate about the pivoted point A.
- Short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke R_1R_2



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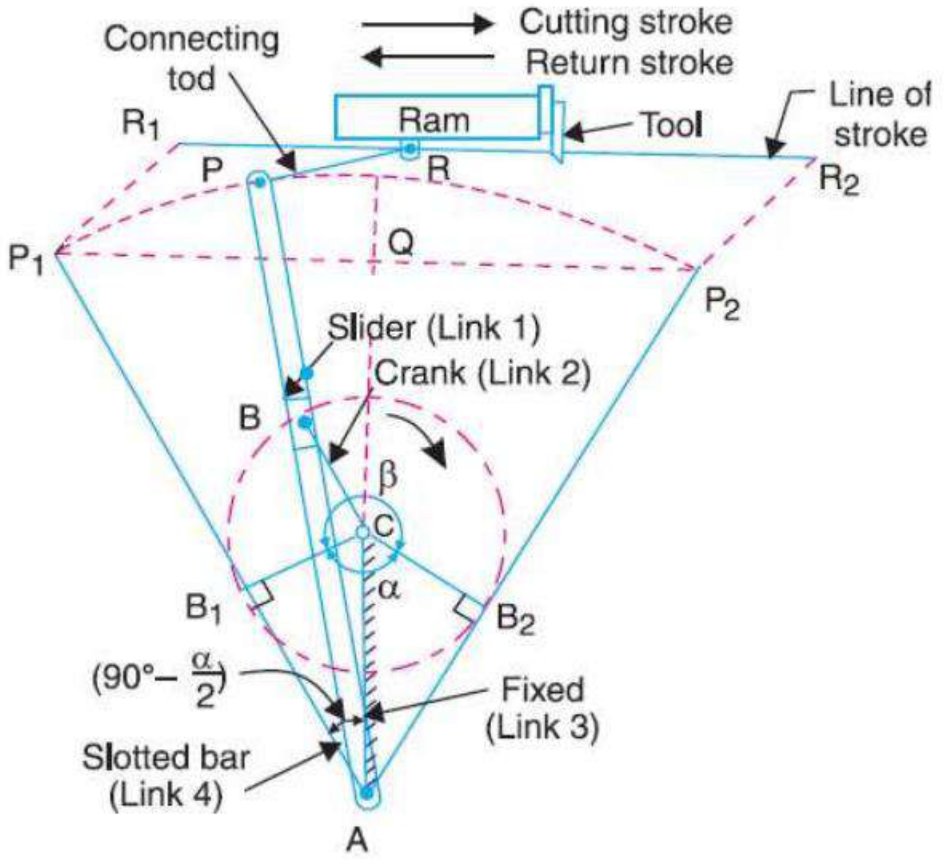
Crank and Slotted Lever quick return motion Mechanism(Third Inversion)

- The forward or cutting stroke occurs when the crank rotates from the position *CB_1 to CB_2 (or* through an angle β) in the clockwise direction.
- The return stroke occurs when the crank rotates from the position *CB_2 to CB_1 (or through angle α)* in the clockwise direction.



- Since the crank has uniform angular speed, therefore

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta} \quad \text{or} \quad \frac{360^\circ - \alpha}{\alpha}$$

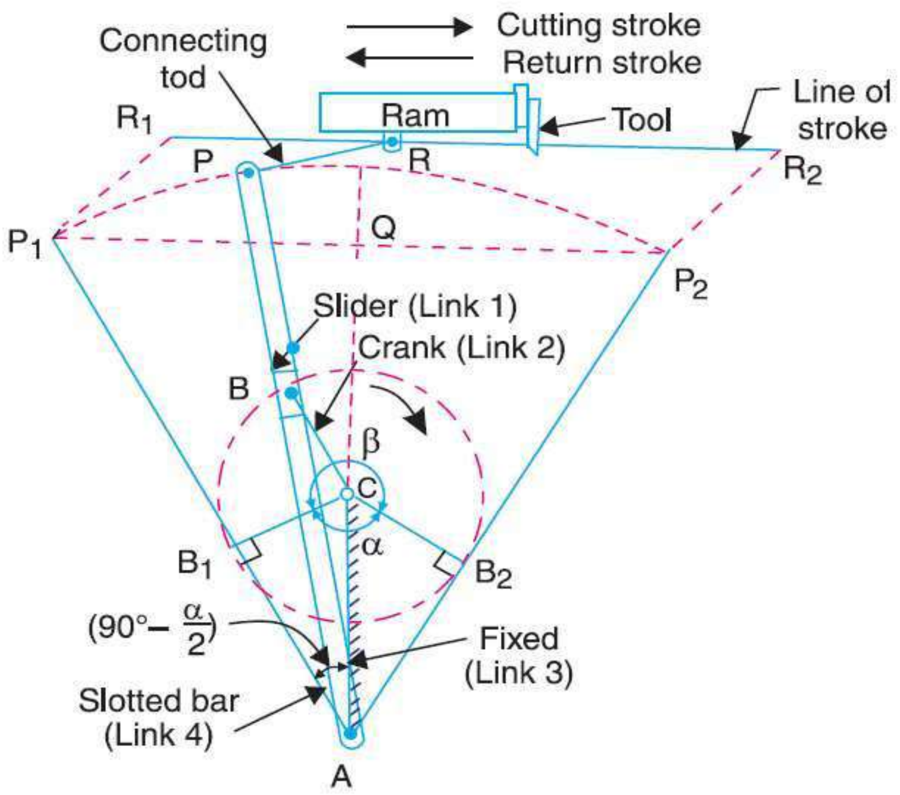
- Since the tool travels a distance of *R*1 *R*2 during cutting and return stroke, therefore travel of the tool or length of stroke

$$= R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 \sin \angle P_1AQ$$

$$= 2AP_1 \sin\left(90^\circ - \frac{\alpha}{2}\right) = 2AP \cos\frac{\alpha}{2} \qquad \ldots\ldots (\because AP_1 = AP)$$

$$= 2AP \times \frac{CB_1}{AC} \qquad \cdots \left(\because \cos\frac{\alpha}{2} = \frac{CB_1}{AC}\right)$$

$$= 2AP \times \frac{CB}{AC} \qquad \ldots (\because CB_1 = CB)$$



Crank and Slotted Lever quick return motion Mechanism(Third Inversion)

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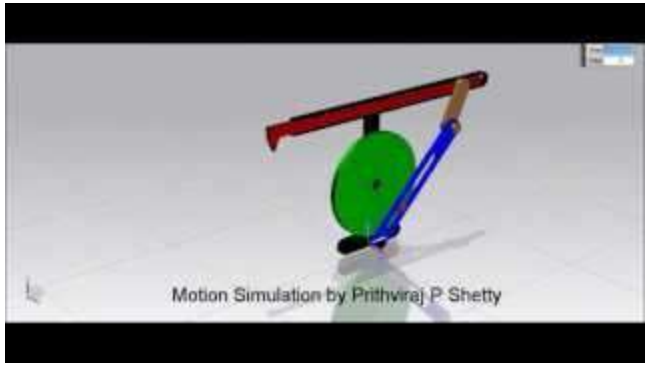
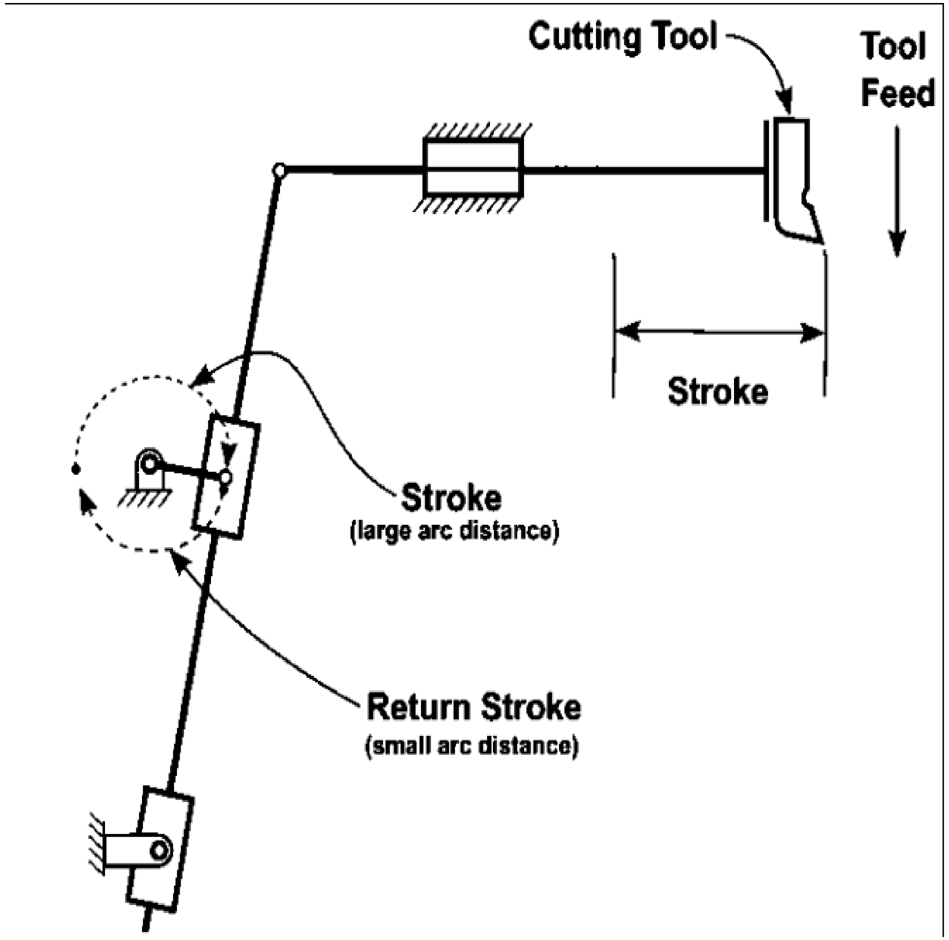
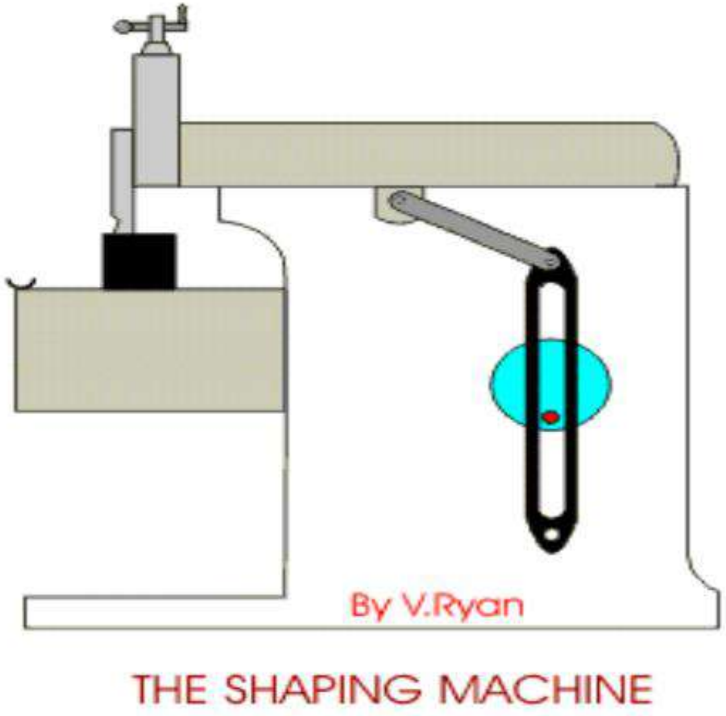
https://youtu.be/s3TiMedJKds







Crank and Slotted Lever Quick Return Motion Mechanism

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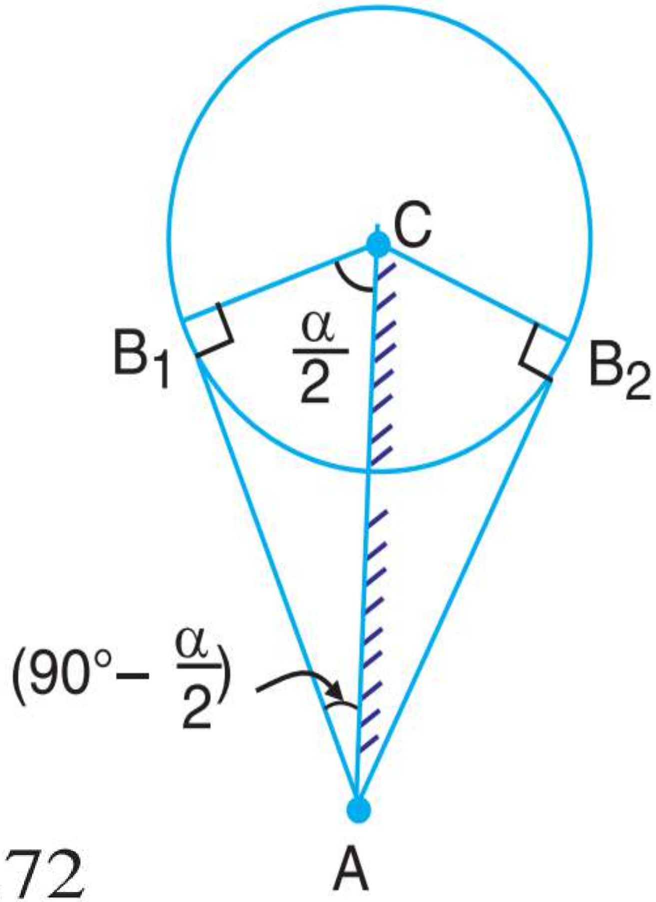
Q. A crank and slotted lever mechanism used in a shaper has a centre distance of 300 mm between the centre of oscillation of the slotted lever and the centre of rotation of the crank. The radius of the crank is 120 mm. Find the ratio of the time of cutting to the time of return stroke.

Sol: Given : AC = 300 mm ; CB_1 = 120 mm

The extreme positions of the crank are shown in Fig



$$\sin \angle CAB_1 = \sin(90° - \alpha/2)$$

$$= \frac{CB_1}{AC} = \frac{120}{300} = 0.4$$

$$\angle CAB_1 = 90° - \alpha/2$$

$$= \sin^{-1} 0.4 = 23.6°$$

We know that $\alpha/2 = 90° - 23.6° = 66.4°$

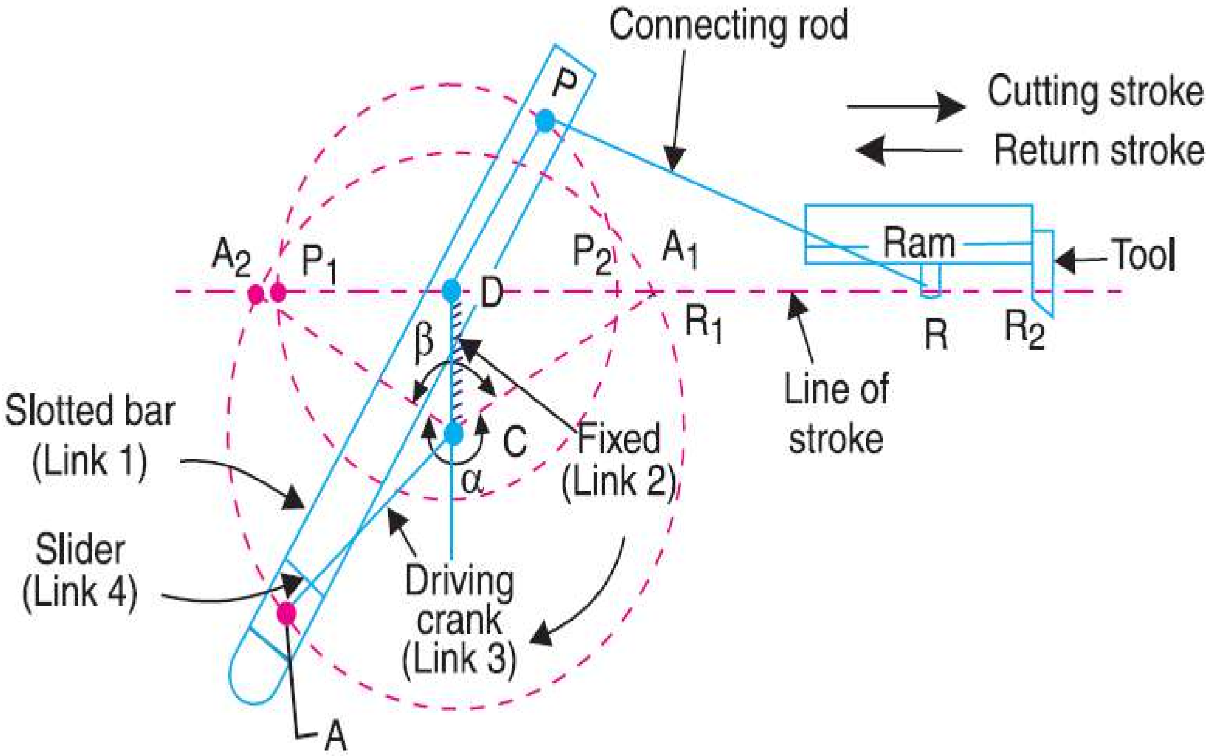
$$\alpha = 2 \times 66.4 = 132.8°$$

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360° - \alpha}{\alpha} = \frac{360° - 132.8°}{132.8°} = 1.72$$

Whitworth quick return motion mechanism(Second Inversion)

This mechanism is mostly used in shaping and slotting machines.

- In this mechanism, the link CD (link 2) forming the turning pair is fixed, as shown in Fig.
- The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank CA (link 3) rotates at a uniform angular speed.
- The slider (link 4) attached to the crank pin at A slides along the slotted bar PA (link 1) which oscillates at a pivoted point D.
- The connecting rod PR carries the ram at R to which a cutting tool is fixed.
- The motion of the tool is constrained Along the line RD produced, i.e. along a Line passing through D and perpendicular to CD.



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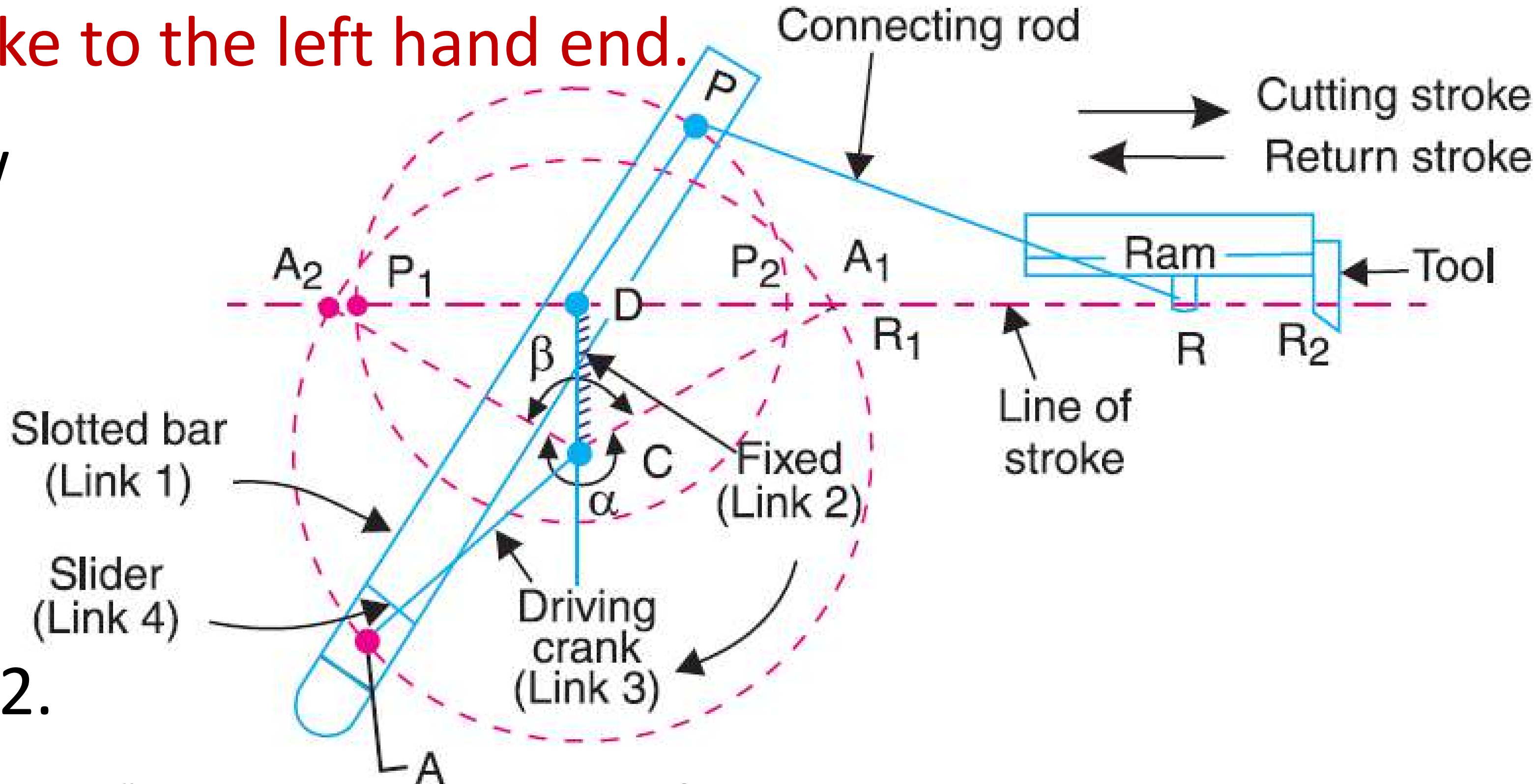
When the driving crank CA moves from the position CA1 to CA2 (or the link DP from the position DP1 to DP2) through an angle in the clockwise direction, the tool moves from the left hand end of its stroke to the right hand end through a distance 2PD.

- Now when the driving crank moves from the position CA2 to CA1 (or the link DP from DP2 to DP1) through an angle in the clockwise direction, the tool moves back from right hand end of its stroke to the left hand end.

A little consideration will show that the time taken during the left to right movement of the ram (i.e. during forward or cutting stroke) will be equal to the time taken by the driving crank to move from CA1 to CA2.



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• Similarly, the time taken during the right to left movement of the ram (or during the idle or return stroke) will be equal to the time taken by the driving crank to move from *CA*2 to *CA*1.

• Since the crank link CA rotates at uniform angular velocity therefore time taken during the cutting stroke (or forward stroke) is more than the time taken during the return stroke.

• In other words, the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke.

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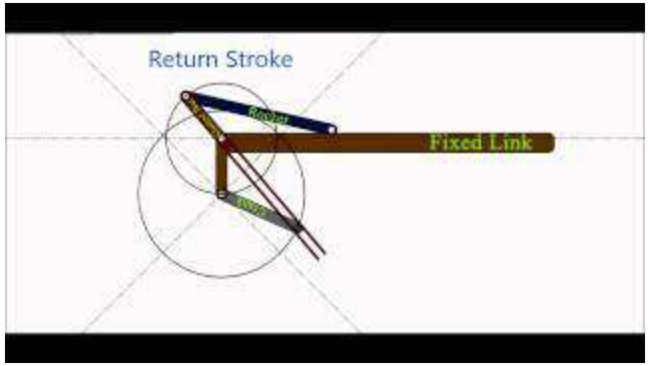
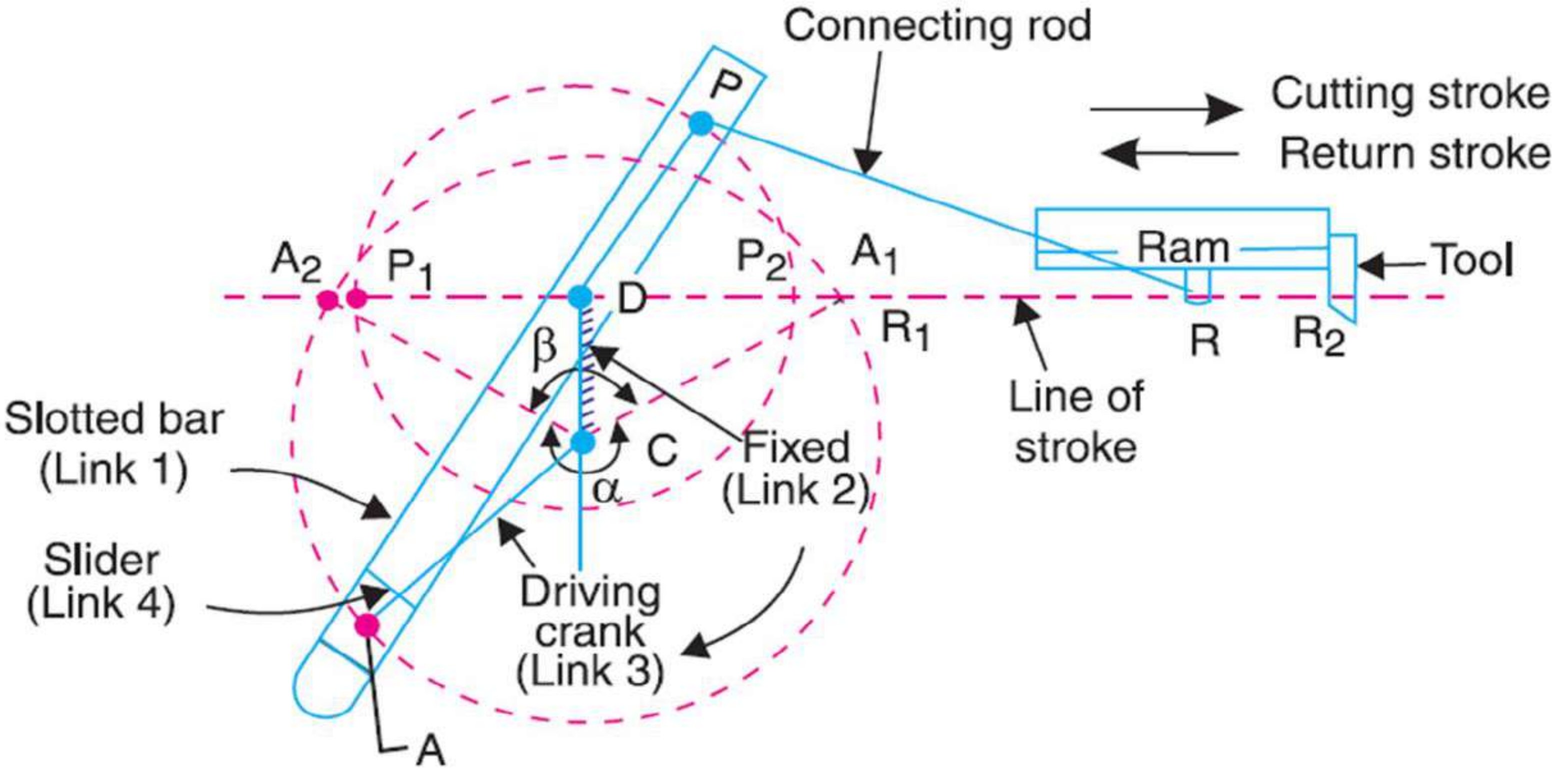


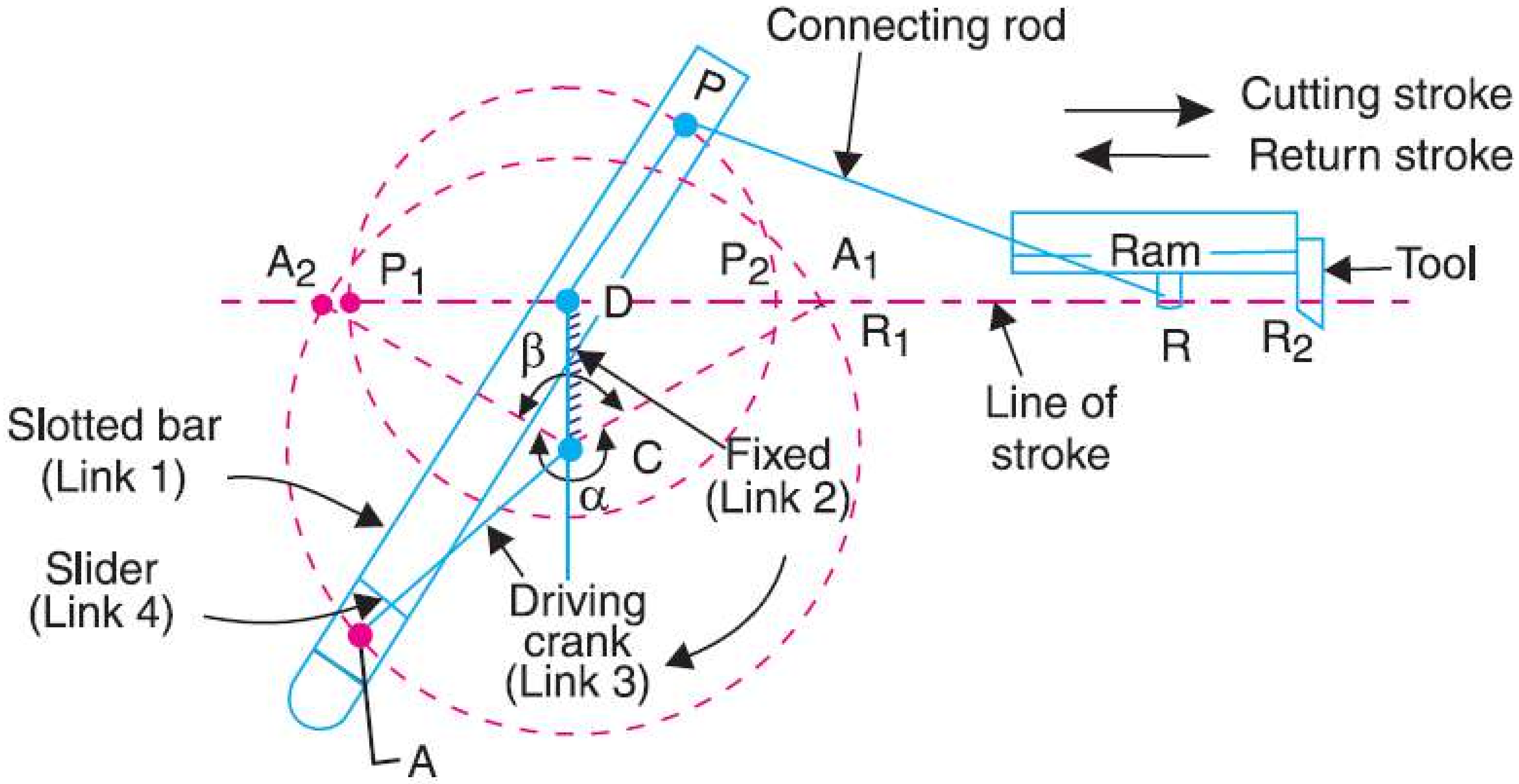
Whitworth Quick-Return Mechanism



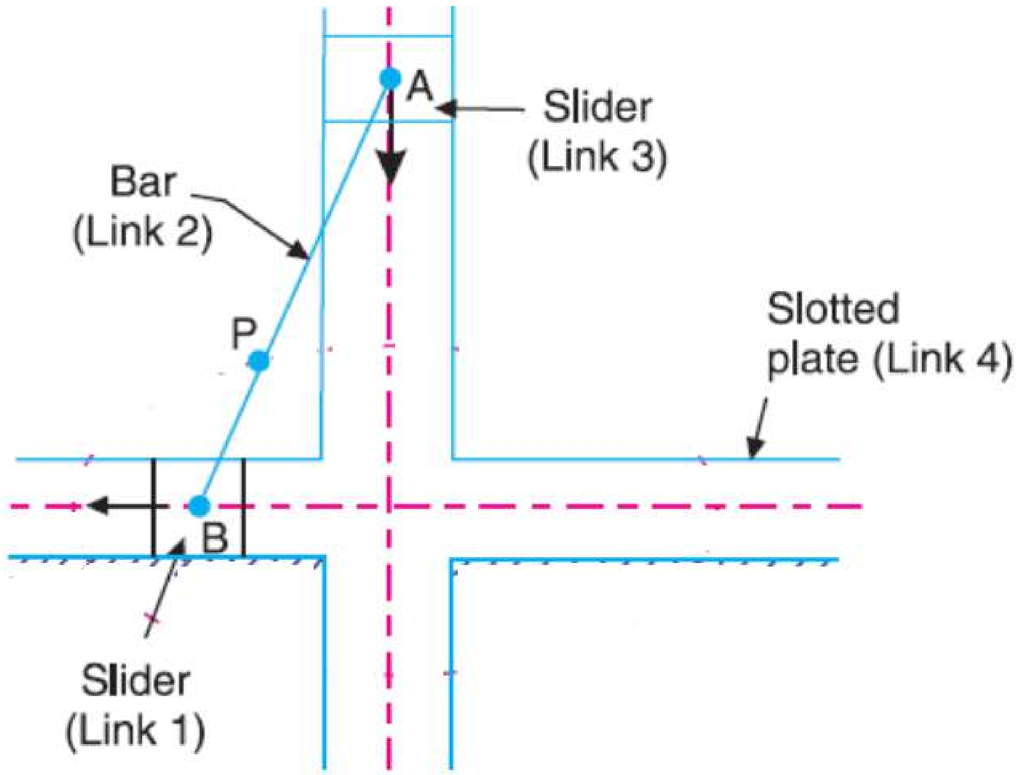
- The ratio between the time taken during the cutting and return strokes is given by

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360° - \alpha} \quad \text{or} \quad \frac{360° - \beta}{\beta}$$

Note. In order to find the length of effective stroke $R_1 R_2$, mark $P_1 R_1 = P_2 R_2 = PR$. The length of effective stroke is also equal to 2 PD.



- **Double-Slider crank Chain:**
  - It consists of four pairs out of which two are turning pairs and two others are sliding pairs.
  - Double-slider crank chain is shown in Fig. 1.28 *(a).*
  - Link 1 and link 4 is sliding pair, link 1 and link 2 is turning pair, link 2 and link 3 is second turning pair, link 3 and link 4 is second sliding pair.
  - Hence there are two turning pairs and two sliding pairs.



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Double Slider Crank Chain

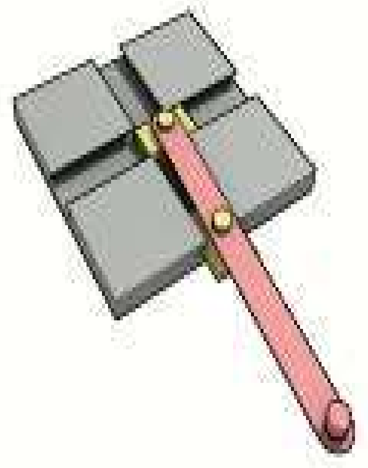
Inversions of Double Slider Crank Chain

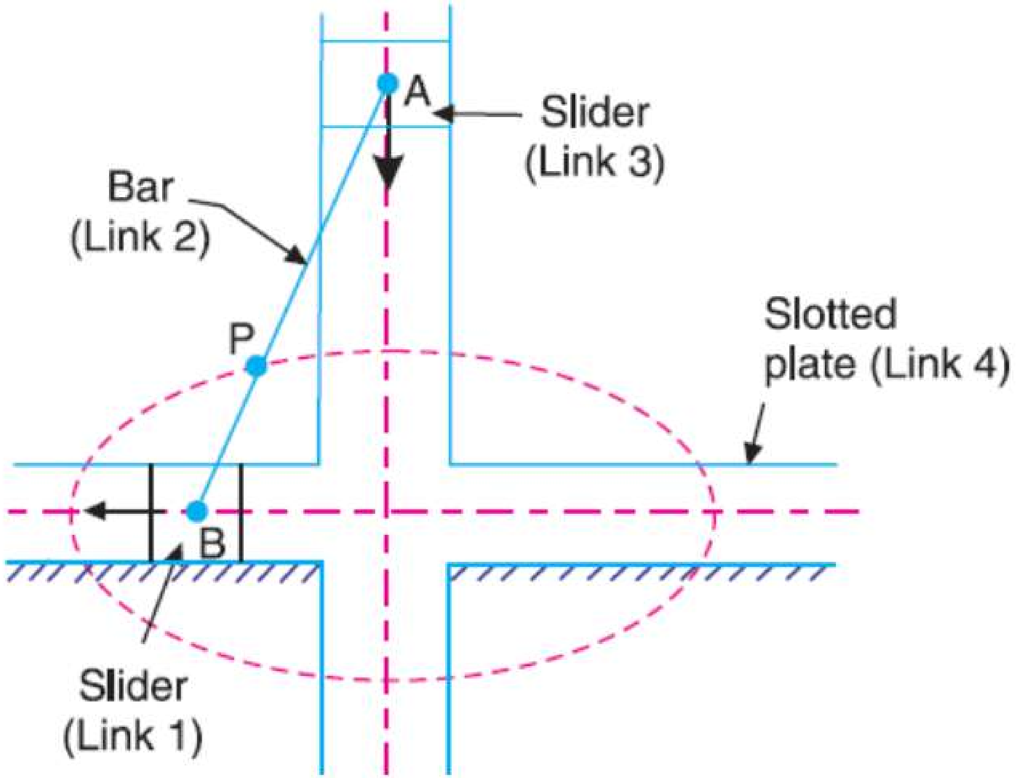
Elliptical trammels

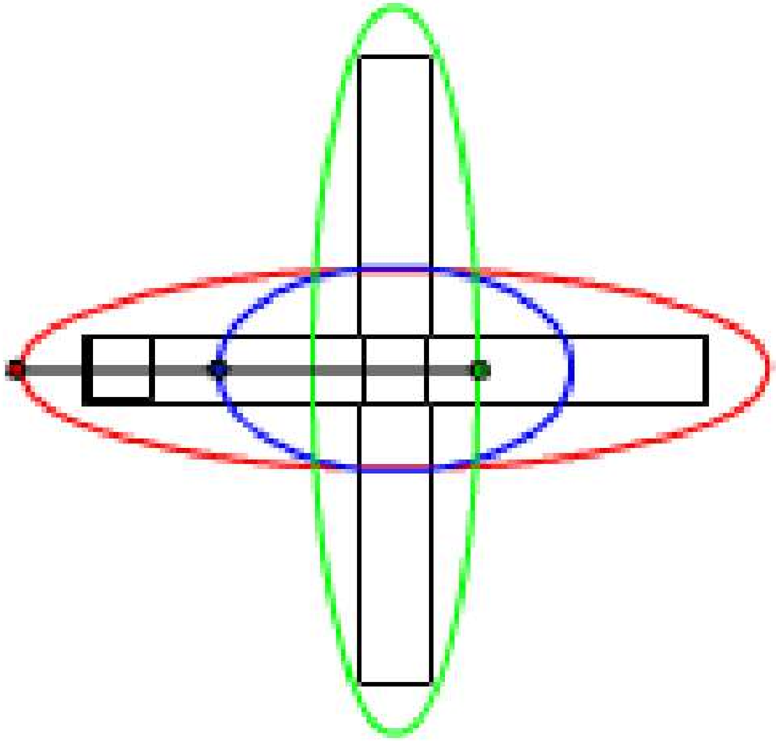
Scotch yoke mechanism

Oldham's coupling

Elliptical trammels





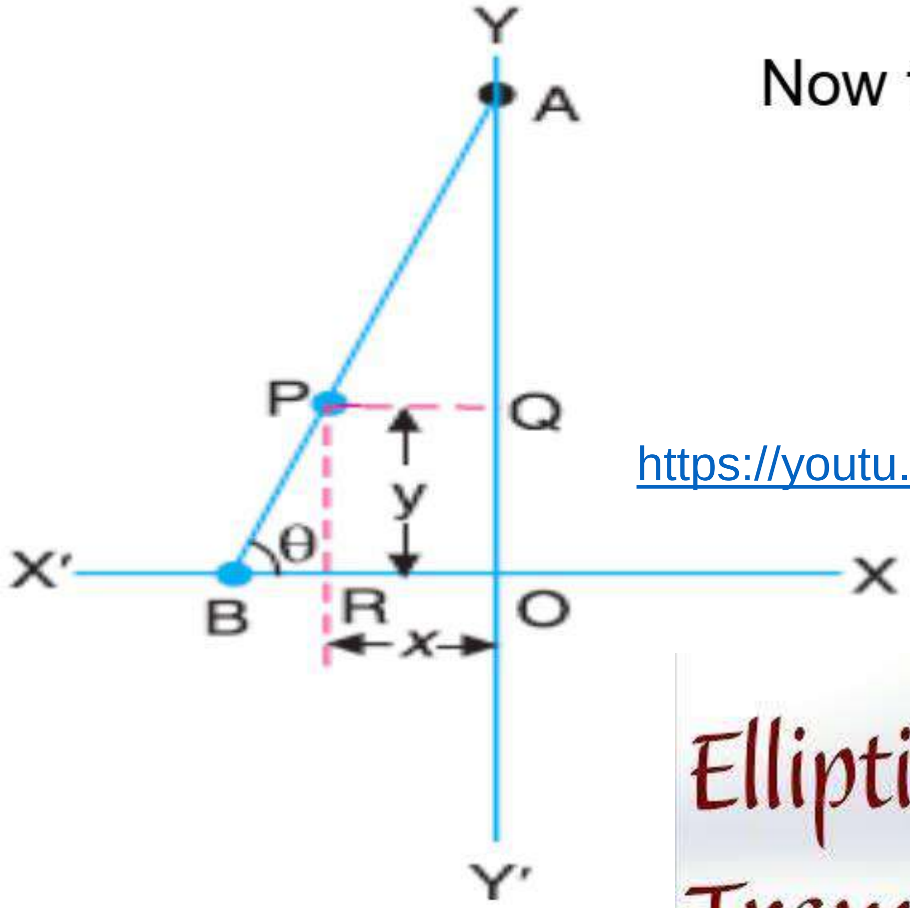


- This inversion is obtained by fixing the slotted plate (link 4).
- fixed plate or link 4 has two straight grooves cut in it, at right angles to each other.
- link 1 and link 3, are known as sliders and form sliding pairs with link 4.
- link AB (link 2) is a bar which forms turning pair with links 1 and 3.
- When the links 1 and 3 slide along their respective grooves, any point on the link 2 such as **P** traces out an ellipse on the surface of link 4

Show that AP and BP are the semi-major axis and semi-minor axis of the ellipse.

OX and OY as horizontal and vertical axes

let the link BA is inclined at an angle θ with the horizontal,



Now the co-ordinates of the point P on the link BA will be

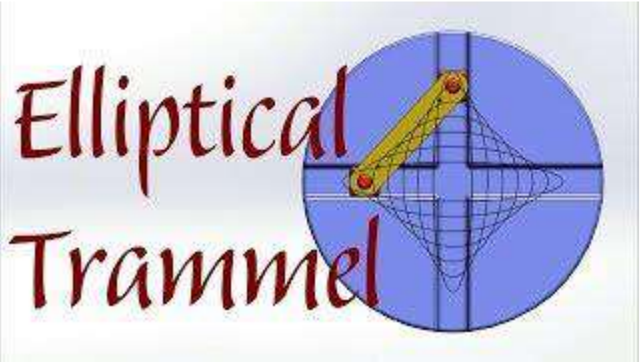
$$x = PQ = AP \cos\theta; \text{ and } y = PR = BP \sin\theta$$

$$\frac{x}{AP} = \cos\theta; \text{ and } \frac{y}{BP} = \sin\theta$$

https://youtu.be/M2Ygp7a5nuE

Squaring and adding,

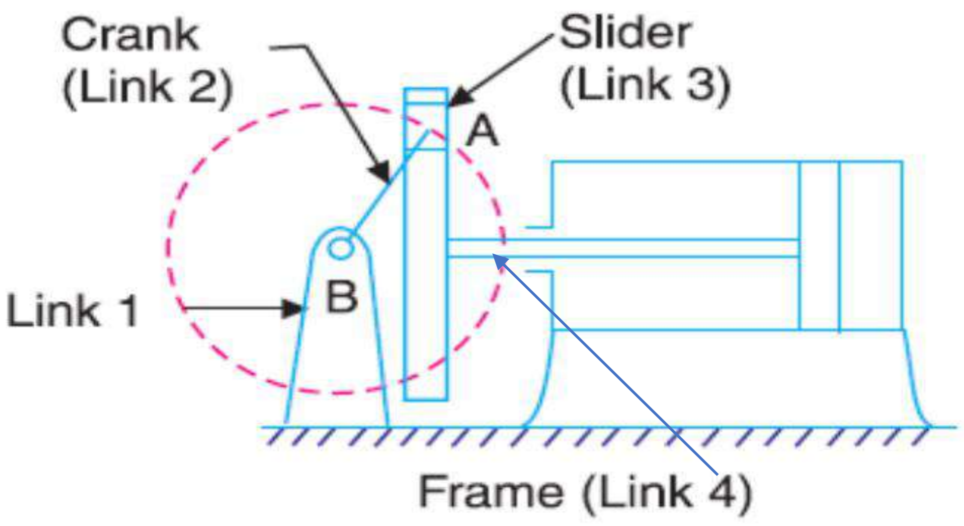
$$\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = \cos^2\theta + \sin^2\theta = 1$$



This is the equation of an ellipse

Elliptical trammel

Scotch Yoke Mechanism



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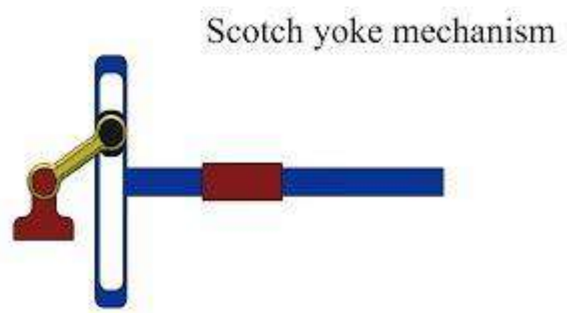

Scotch yoke mechanism animation

- This mechanism is used for converting rotary motion into a reciprocating motion.
- The inversion is obtained by fixing either the link 1 or link 3. link 1 is fixed.
- In this mechanism, when the link 2 (crank) rotates about *B as centre, the link* 4 (frame) reciprocates.
- The fixed link 1 guides the frame.

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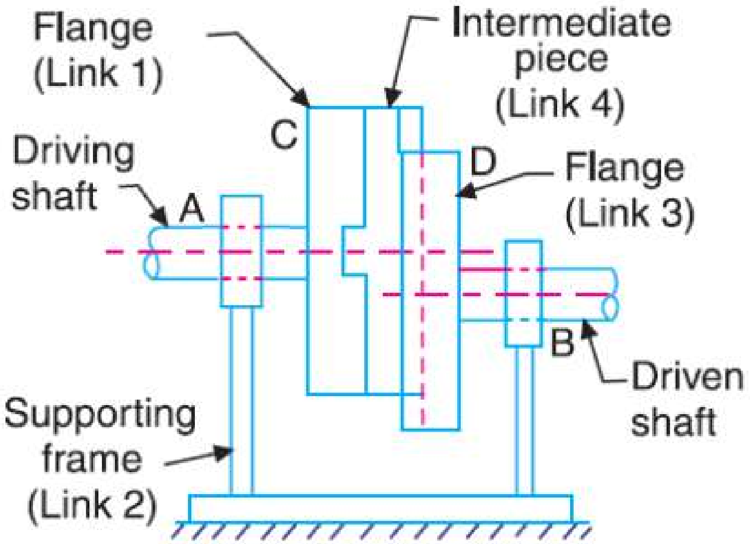
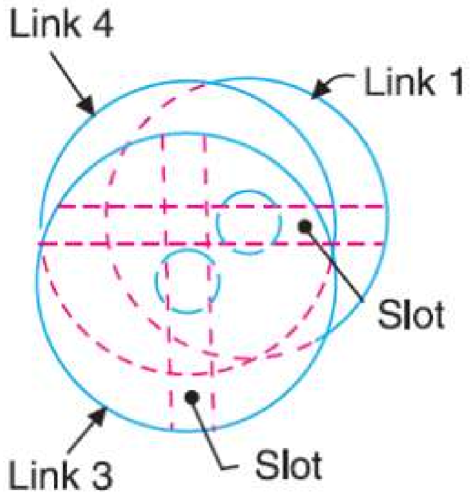
3. Oldham's coupling.

- **An oldham's coupling is** used for connecting two parallel shafts whose axes are at a small distance apart. The shafts are coupled in such a way that if one shaft rotates, the other shaft also rotates at the same speed.
- This inversion is obtained by fixing the link 2, as shown in Fig (a).
- The shafts to be connected have two flanges (link 1 and link 3) rigidly fastened at their ends by forging.

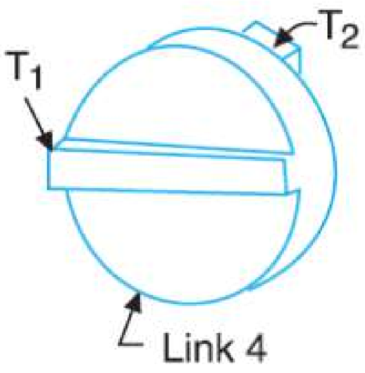
https://youtu.be/0KHbV7dIxgs



(a)

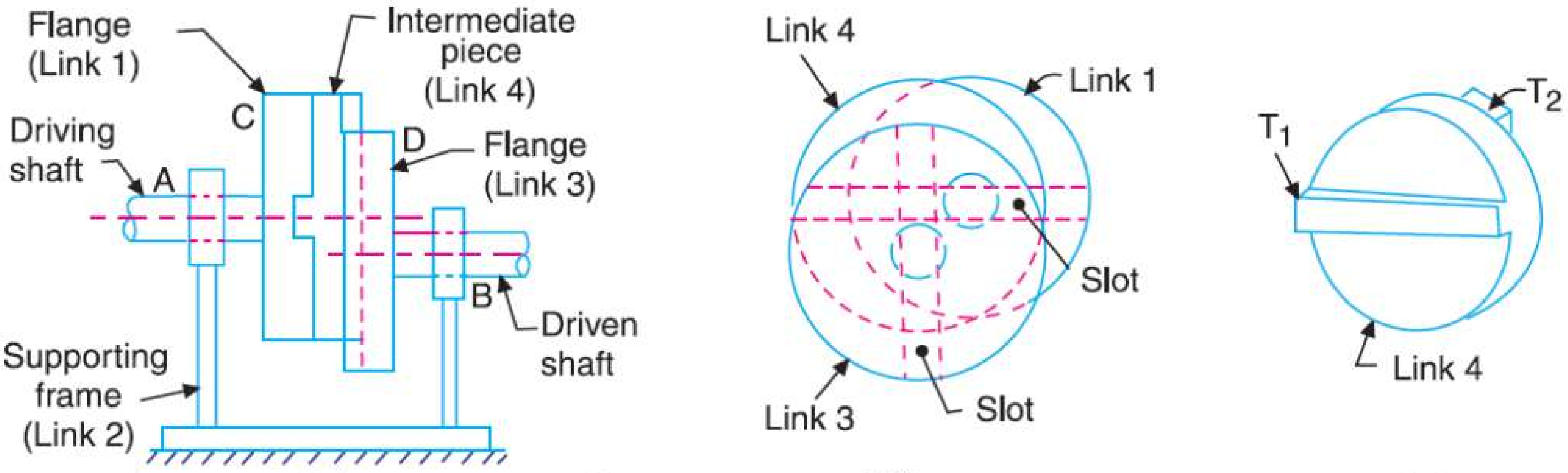


(b)



(c)

- The link 1 and link 3 form turning pairs with link 2.
  - These flanges have diametrical slots cut in their inner faces, as shown in Fig.(b).
  - The intermediate piece (link 4) which is a circular disc, have two tongues (i.e. diametrical projections) T1 and T2 on each face at right angles to each other, as shown in Fig. 5.36 (c).
  - The tongues on the link 4 closely fit into the slots in the two flanges (link 1 and link3). The link 4 can slide or reciprocate in the slots in the flanges.



(a) (b) (c)

- If the distance between the axes of the shafts is constant, the centre of intermediate piece will describe a circle of radius equal to the distance between the axes of the two shafts. Therefore, the maximum sliding speed of each tongue along its slot is equal to the peripheral velocity of the centre of the disc along its circular path

Let ω = Angular velocity of each shaft in rad/s, and

r = Distance between the axes of the shafts in metres

Maximum sliding speed of each tongue (in m/s),

$$v = \omega . r$$

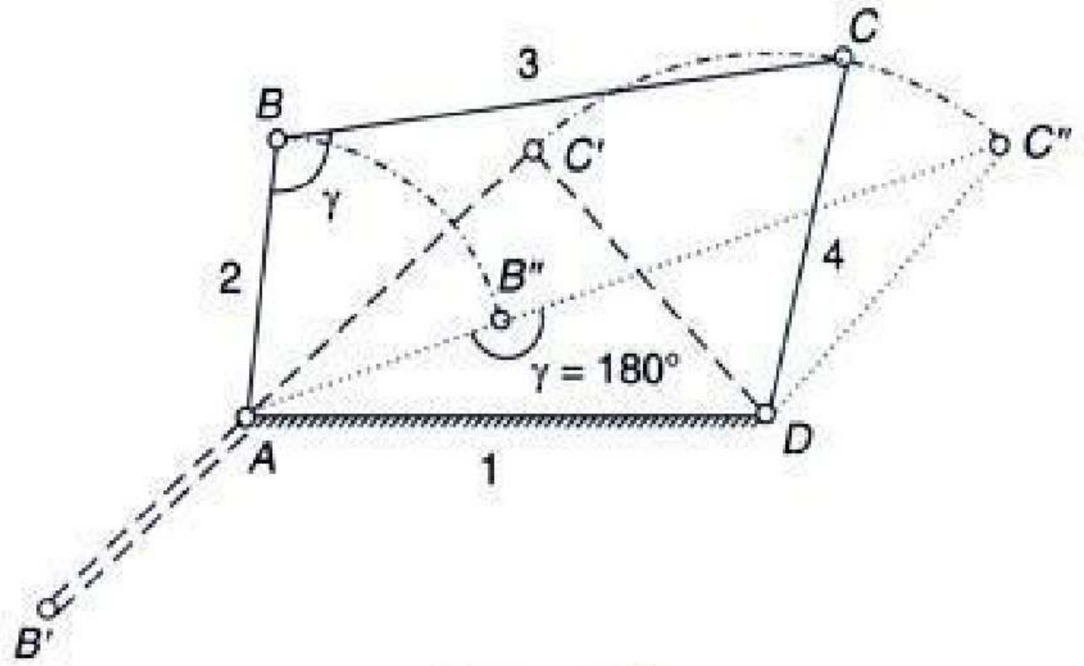
Mechanical Advantage(M A)

The *mechanical advantage* (MA) of a mechanism is the ratio of the output force or torque to the input force or torque at any instant. Thus for the linkage of Fig. 1.42, if friction and inertia forces are ignored and the input torque T_2 is applied to the link 2 to drive the output link 4 with a resisting torque T_4 then

Power input = Power output

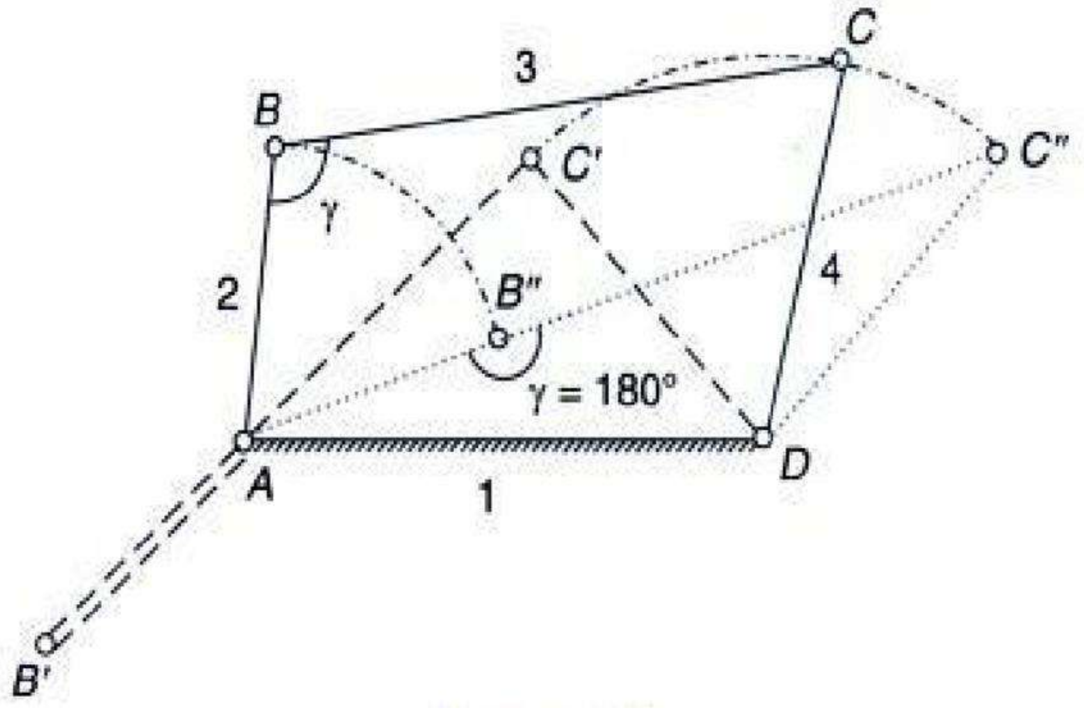
$$T_2\,\omega_2 = T_4\,\omega_4$$

or $$\text{MA} = \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4}$$



Mechanical Advantage(M A) is the reciprocal of velocity ratio

Thus, it is the reciprocal of the velocity ratio. In case of crank-rocker mechanisms, the velocity ω_4 of the output link DC (rocker) becomes zero at the extreme positions ($AB'C''D$ and $AB''C''D$), i.e., when the input link AB is in line with the coupler BC and the angle γ between them is either zero or 180°, it makes the mechanical advantage to be infinite at such positions. Only a small input torque can overcome a large output torque load. The extreme positions of the linkage are known as *toggle* positions.



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ADDITIONAL CONCEPT

↓

Transmission Angle

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Transmission Angle

The angle μ between the output link and the coupler is known as *transmission angle*. In Fig. 1.43, if the link *AB* is the input link, the force applied to the output link *DC* is transmitted through the coupler *BC*. For a particular value of force in the coupler rod, the torque transmitted to the output link (about the point *D*) is maximum when the transmission angle μ is 90°. If links *BC* and *DC* become coincident, the *transmission angle* is zero and the mechanism would lock or jam. If μ deviates significantly from 90°, the torque on the output link decreases. Sometimes, it may not be sufficient to overcome the friction in the system and the mechanism may be locked or jammed. Hence μ is usually kept more than 45°. The best mechanisms, therefore, have a transmission angle that does not deviate much from 90°.

Applying cosine law to triangles *ABD* and *BCD* (Fig. 1.43),

$$a^2 + d^2 - 2ad\cos\theta = k^2 \qquad \text{(i)}$$

and $$b^2 + c^2 - 2bc\cos\mu = k^2 \qquad \text{(ii)}$$

From (i) and (ii),

$$a^2 + d^2 - 2ad\cos\theta = b^2 + c^2 - 2bc\cos\mu$$

or $$a^2 + d^2 - b^2 - c^2 - 2ad\cos\theta + 2bc\cos\mu = 0$$

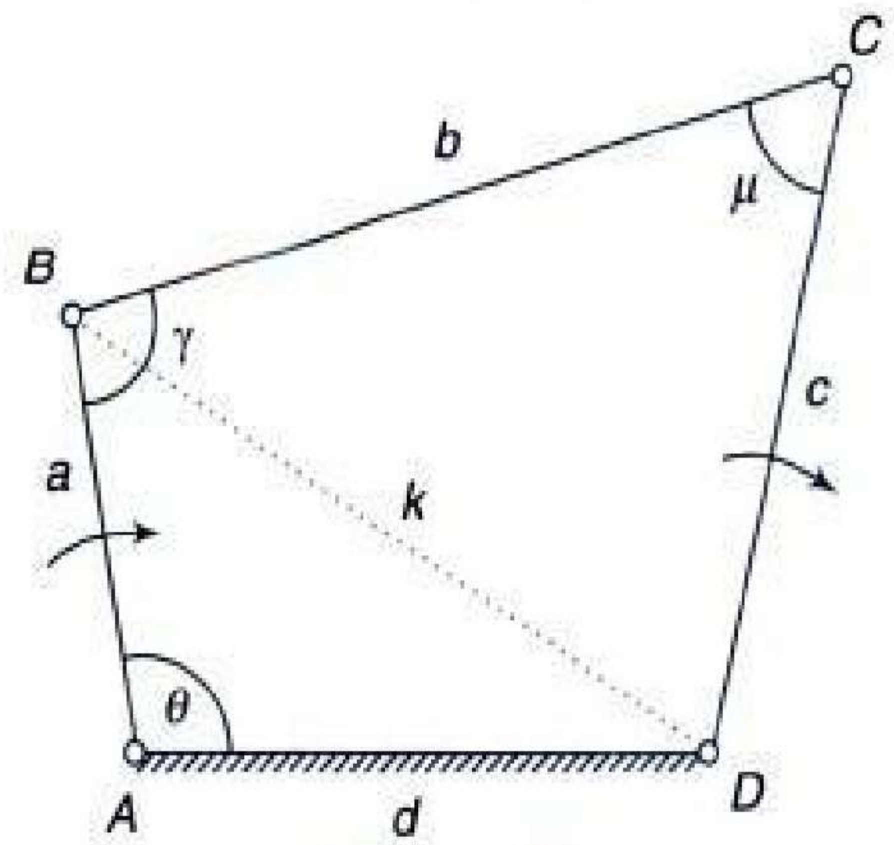


Fig. 1.43

The maximum or minimum values of the transmission angle can be found by putting $d\mu/d\theta$ equal to zero.

Differentiating the above equation with respect to θ,

$$2\,ad\sin\theta - 2bc\sin\mu.\frac{d\mu}{d\theta} = 0$$

or $$\frac{d\mu}{d\theta} = \frac{ad\sin\theta}{bc\sin\mu}$$

Thus, if $d\mu/d\theta$ is to be zero, the term $ad\sin\theta$ has to be zero which means θ is either 0° or 180°. It can be seen that μ is maximum when θ is 180° and minimum when θ is 0°. However, this would be applicable to the mechanisms in which the link a is able to assume these angles, i.e., in double-crank or crank-rocker mechanisms. Figures 1.44(a) and (b) show a crank-rocker mechanism indicating the positions of the maximum and the minimum transmission angles. Figures 1.45(a) and (b) show the maximum and the minimum transmission angles for a double-rocker mechanism.

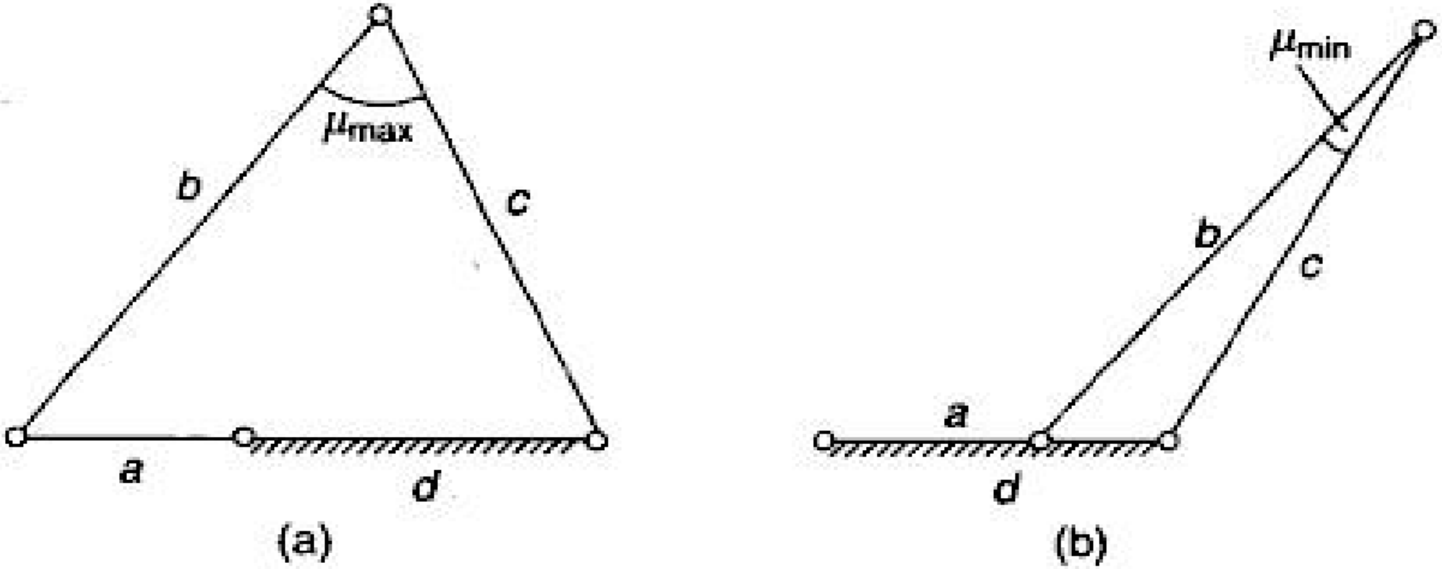


Fig. 1.44

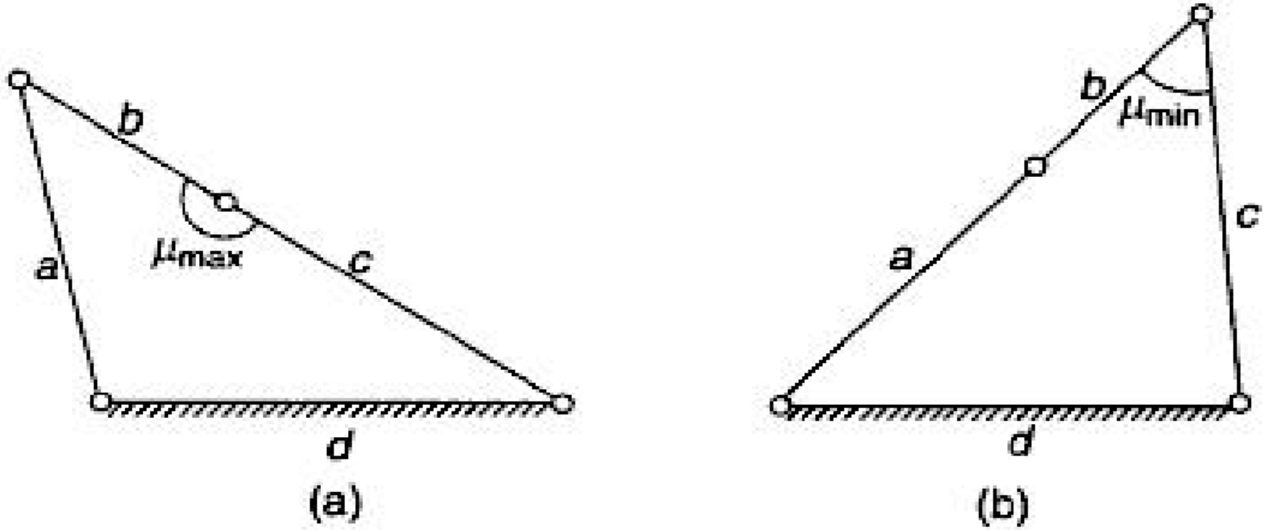


Fig. 1.45

UNIT I BIT BANK

1. In a reciprocating steam engine, which of the following forms a kinematic link ? (c)

(a) cylinder and piston (b) piston rod and connecting rod

(c) crank shaft and flywheel (d) flywheel and engine frame

2. The motion of a piston in the cylinder of a steam engine is an example of (a)

(a) completely constrained motion (b) incompletely constrained motion

(c) successfully constrained motion (d) none of these

3. The motion transmitted between the teeth of gears in mesh is (d)

(a) sliding (b) rolling (c) may be rolling or sliding depending upon the shape of teeth (d) partly sliding and partly rolling

4. The cam and follower without a spring forms a (c)

(a) lower pair (b) higher pair (c) self closed pair (d) force closed pair

5. A ball and a socket joint forms a (d)

(a) turning pair (b) rolling pair (c) sliding pair (d) spherical pair

6.The lead screw of a lathe with nut forms a (c)

(a) sliding pair (b) rolling pair (c) screw pair (d) turning pair

7. When the elements of the pair are kept in contact by the action of external forces, the pair is said to be a (d)

(a) lower pair (b) higher pair (c) self closed pair (d) force closed pair

8. Which of the following is a turning pair ? (b)

(a) Piston and cylinder of a reciprocating steam engine

(b) Shaft with collars at both ends fitted in a circular hole

(c) Lead screw of a lathe with nut (d) Ball and socket joint

9. A combination of kinematic pairs, joined in such a way that the relative motion between the links is completely constrained, is called a (c)

(a) structure (b) mechanism (c) kinematic chain (d) inversion

10. The relation between the number of pairs (p) forming a kinematic chain and the number of links (l) is (c)

(a) $l = 2p - 2$ (b) $l = 2p - 3$ (c) $l = 2p - 4$ (d) $l = 2p - 5$

11. The relation between the number of links (l) and the number of binary joints (j) for a kinematic chain having constrained motion is given by j= (3/2)l -2. If the left hand side of this equation is greater than right hand side, then the chain is (a)

(a) locked chain (b) completely constrained chain

(c) successfully constrained chain (d) incompletely constrained chain

12. In a kinematic chain, a quaternary joint is equivalent to (c)

(a) one binary joint (b) two binary joints (c) three binary joints (d) four binary joints

13. If n links are connected at the same joint, the joint is equivalent to (a)

(a) (n – 1) binary joints (b) (n – 2) binary joints (c) (2n – 1) binary joints (d) none of these

14. In a 4 – bar linkage, if the lengths of shortest, longest and the other two links are denoted by s, l, p and q, then it would result in Grashof's linkage provided that (b)

(a) $l + p < s + q$ (b) $l + s < p + q$ (c) $l + p = s + q$ (d) none of these

15. A kinematic chain is known as a mechanism when (b)

(a) none of the links is fixed (b) one of the links is fixed

(c) two of the links are fixed (d) all of the links are fixed

16. The Grubler's criterion for determining the degrees of freedom (n) of a mechanism having plane motion is (c)

(a) n = (l – 1) – j (b) n = 2 (l – 1) – 2j (c) n = 3 (l – 1) – 2j (d) n = 4 (l – 1) – 3j

where l = Number of links, and j = Number of binary joints.

17. The mechanism forms a structure, when the number of degrees of freedom (n) is equal to (a)

(a) 0 (b) 1 (c) 2 (d) – 1

18. In a four bar chain or quadric cycle chain (a)

(a) each of the four pairs is a turning pair (b) one is a turning pair and three are sliding pairs

(c) three are turning pairs and one is sliding pair (d) each of the four pairs is a sliding pair.

19. Which of the following is an inversion of single slider crank chain ? (d)

(a) Beam engine (b) Watt's indicator mechanism

(c) Elliptical trammels (d) Whitworth quick return motion mechanism

20. Which of the following is an inversion of double slider crank chain ? (c)

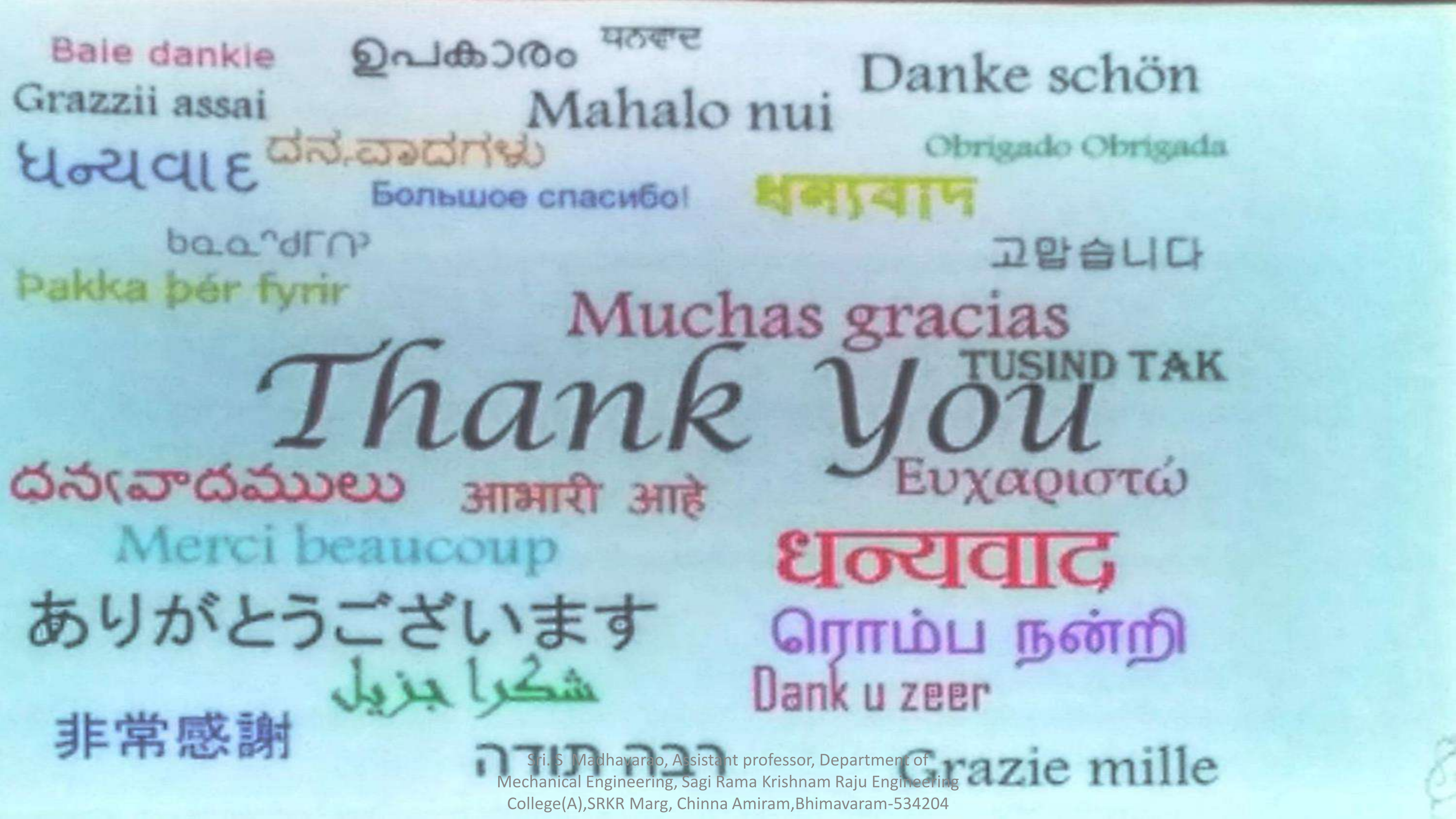
(a) Coupling rod of a locomotive (b) Pendulum pump

(c) Elliptical trammels (d) Oscillating cylinder engine

References:

1. Theory of Machines, Rattan, Tata McGraw-Hill Education, 2009.

2. Theory of Machines, R S Kurmi, Eurasia Publishing House, 2005



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UNIT II
Kinematics

(Velocity Analysis & Acceleration Analysis)

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UNIT II

- **Kinematics:** Velocity and acceleration – Motion of link in machine – Determination of Velocity and acceleration – Graphical method – Application of relative velocity method.
- **Plane motion of body:** Instantaneous center of rotation– Three centers in line theorem –Graphical determination of instantaneous center, determination of angular velocity of links by instantaneous center method.
- **Analysis of Mechanisms:** Analysis of slider crank chain for displacement- velocity

  and acceleration of slider – Acceleration diagram for a given mechanism.

  Coriolis acceleration - determination of Coriolis component of acceleration-

  Kliens construction

Velocity Analysis

Velocity analysis of any mechanism can be carried out by various methods.

1. By graphical method
2. By relative velocity method
3. By instantaneous method

ABSOLUTE AND RELATIVE MOTIONS

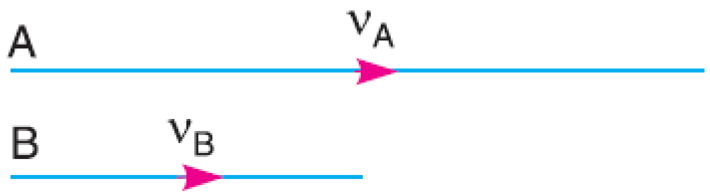
Strictly speaking, all motions are relative since an arbitrary set of axes or planes is required to define a motion. Usually, the earth is taken to be a fixed reference plane and all motions relative to it are termed absolute motions.

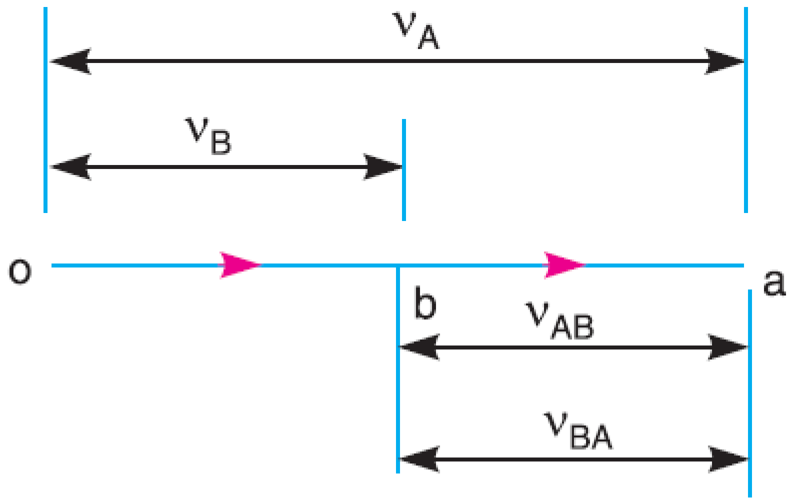
If a train moves in a particular direction, the motion of the train is referred as the absolute motion of the train or motion of the train relative to the earth. Now, suppose a man moves inside the train. Then, the motion of the man will be described in two different ways with different meanings:

1. Motion of the man relative to the train—it is equivalent to the motion of the man assuming the train to be stationary.
2. Motion of the man or absolute motion of the man or motion of the man relative to the earth = motion of man relative to the train + Motion of train relative to the earth.

Relative Velocity of Two Bodies Moving in Straight Lines

(i)





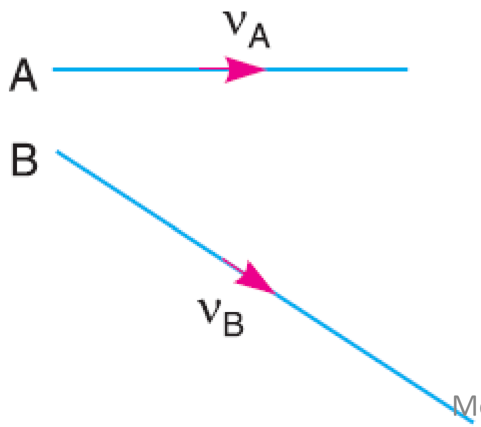
$$v_{AB}\,(\text{Velocity at A w.r.t B}) = \bar{v}_A - \bar{v}_B$$

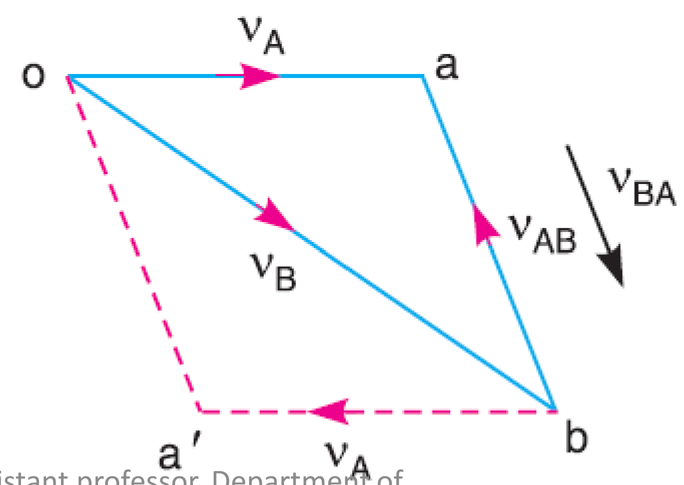
$$\overline{ba} = \overline{oa} - \overline{ob}$$

$$v_{BA}\,(\text{Velocity at B w.r.t A}) = \bar{v}_B - \bar{v}_A$$

$$\overline{ab} = \overline{ob} - \overline{oa}$$

(ii)





$$v_{BA} = \bar{v}_B - \bar{v}_A$$

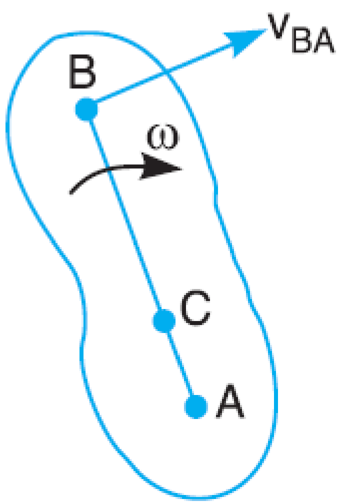
$$\overline{ab} = \overline{ob} - \overline{oa}$$

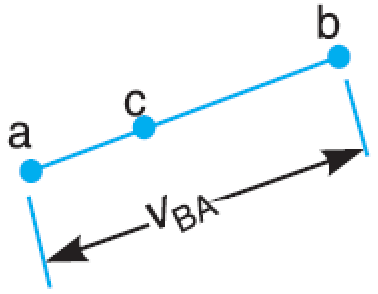
$$v_{AB} = \bar{v}_A - \bar{v}_B$$

$$\overline{ba} = \overline{oa} - \overline{ob}$$

Motion of a Link

velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram



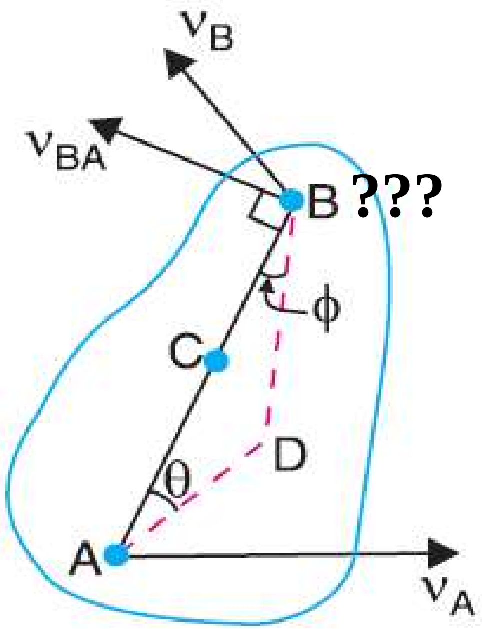


$$v_{BA} = \overline{ab} = \omega . AB$$

$$v_{CA} = \overline{ac} = \omega . AC$$

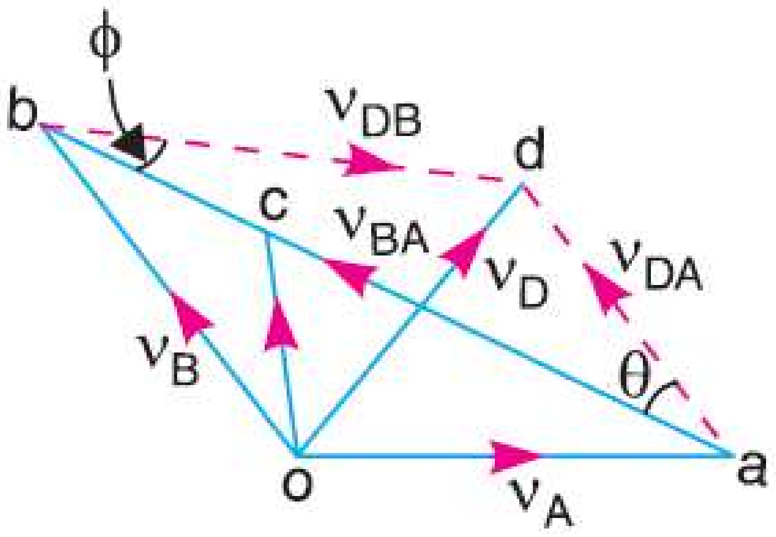
$$\frac{v_{CA}}{v_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{\omega . AC}{\omega . AB} = \frac{AC}{AB}$$

Velocity of a Point on a Link by Relative Velocity Method



Only direction Known

Magnitude And Direction Known



Steps:

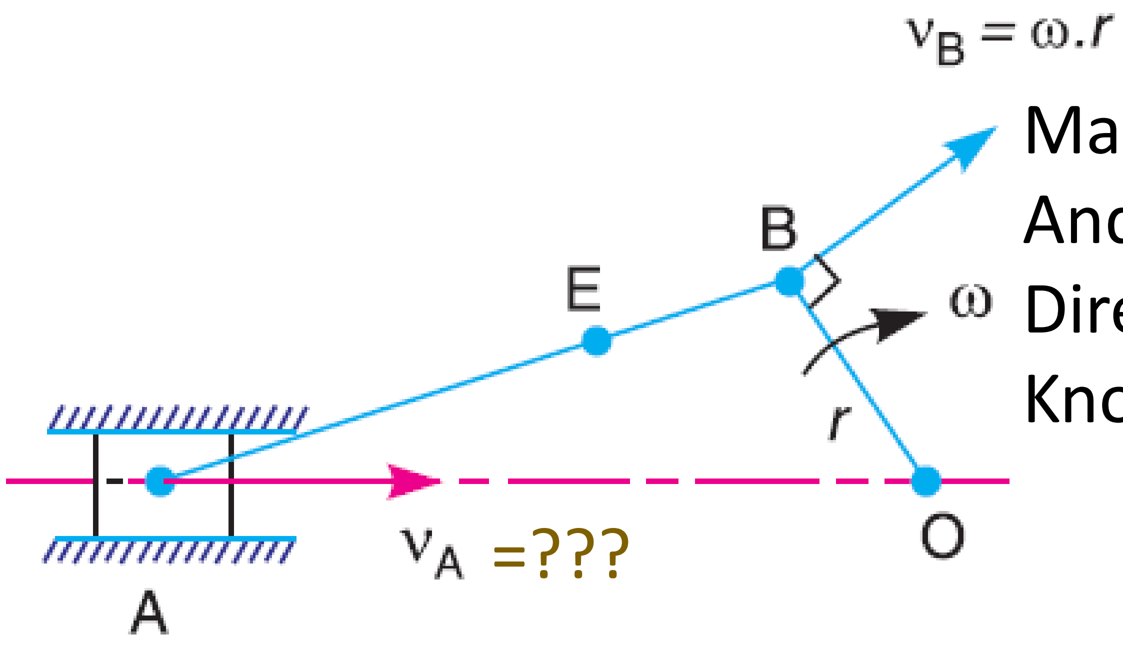
1. Take some convenient point o, known as the pole.
2. Through o, draw oa parallel and equal to V_A, to some suitable scale.
3. Through a, draw a line perpendicular to AB. This line will represent the velocity of B with respect to A, i.e. V_{BA}.
4. Through o, draw a line parallel to V_B intersecting the line of V_{BA} at b.
5. Measure ob, which gives the required velocity of point B (V_B), to the scale.

By Graphical Method

- The following points are to be considered while solving problems by this method.

1. Draw the configuration design to a suitable scale.
2. Locate all fixed point in a mechanism as a common point in velocity diagram.
3. Choose a suitable scale for the vector diagram velocity.
4. The velocity vector of each rotating link is $\perp^r$ to the link.
5. Velocity of each link in mechanism has both magnitude and direction. Start from a point whose magnitude and direction is known.
6. The points of the velocity diagram are indicated by small letters.

Velocities in Slider Crank Mechanism



(a) Slider crank mechanism.

Magnitude
And
Direction
Known

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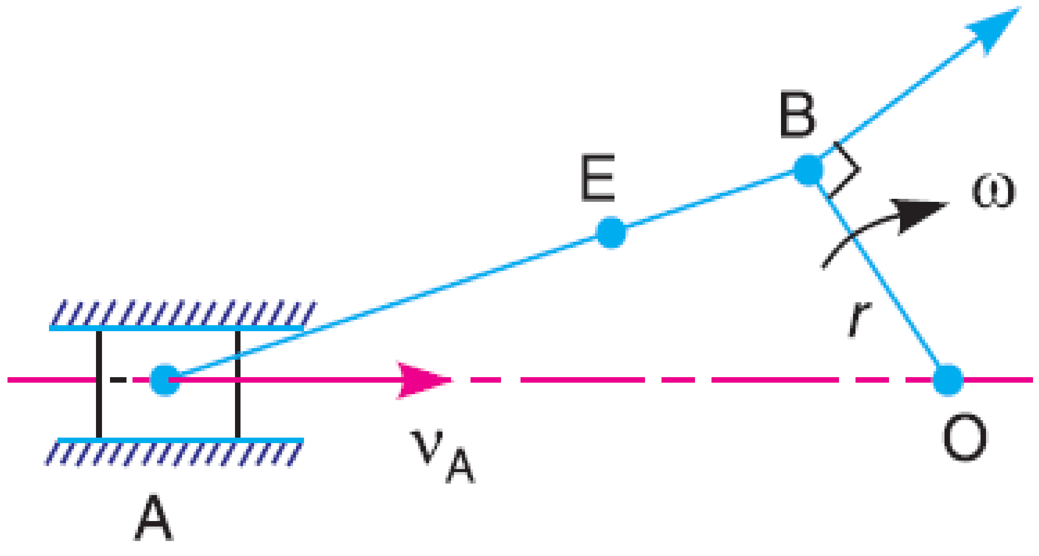
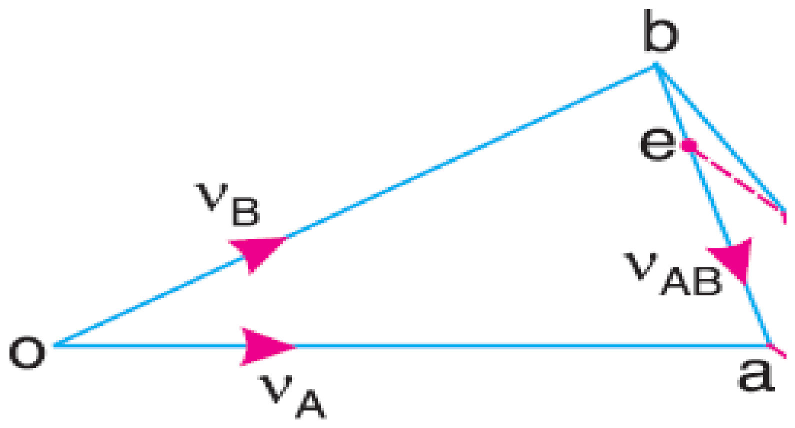
Velocities in Slider Crank Mechanism





Velocity Diagram

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB} \qquad \text{(Anticlockwise about A)}$$

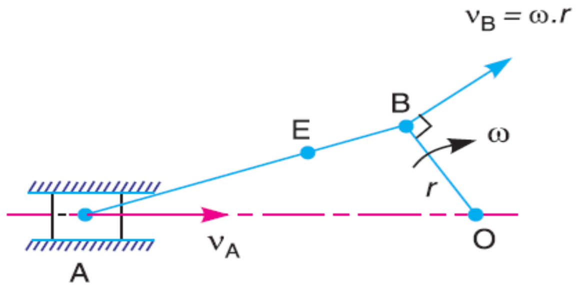
Steps:

1. From any point o, draw vector ob parallel to the direction of V_B (or perpendicular to OB) such that ob = V_B = ωxr, to some suitable scale.
2. Since AB is a rigid link, therefore the velocity of A relative to B is perpendicular to AB. Now draw vector ba perpendicular to AB to represent the velocity of A with respect to B i.e. V_{AB}.
3. From point o, draw vector oa parallel to the path of motion of the slider A (which is along AO only). The vectors ba and oa intersect at a. Now oa represents the velocity of the slider A i.e. V_A, to the scale.

To Determine Velocity of Rubbing

- Two links of a mechanism having turning point will be connected by pins. When the links are motion they rub against pin surface. The velocity of rubbing of pins depends on the angular velocity of links relative to each other as well as direction.
- For example: In a four bar mechanism we have pins at points A, B, C and D

$$\therefore V_{ra} = \omega_{ab} \text{ x ratios of pin A } (r_{pa})$$



(*a*) Slider crank mechanism.

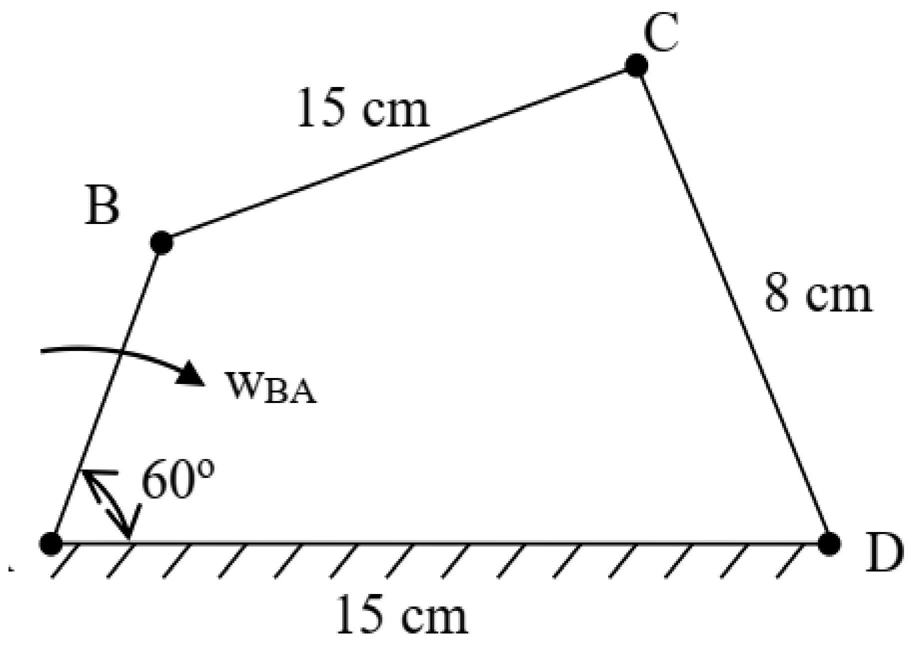
- + sign is used .Therefore ωab is CW and ωbc is CCW i.e. when angular velocities are in opposite directions use + sign when angular velocities are in some directions use - ve sign

$$V_{rC} = (\omega_{bc} + \omega_{cd}) \text{ radius } r$$

$$V_{rD} = \omega_{cd} \; r_{pd}$$

1. Four – Bar Mechanism:

- In a four bar chain ABCD link AD is fixed and in 15 cm long. The crank AB is 4 cm long rotates at 180 rpm (cw) while link CD rotates about D is 8 cm long BC = AD and $\lfloor$ BAD = 60 degrees. Find angular velocity of link CD.



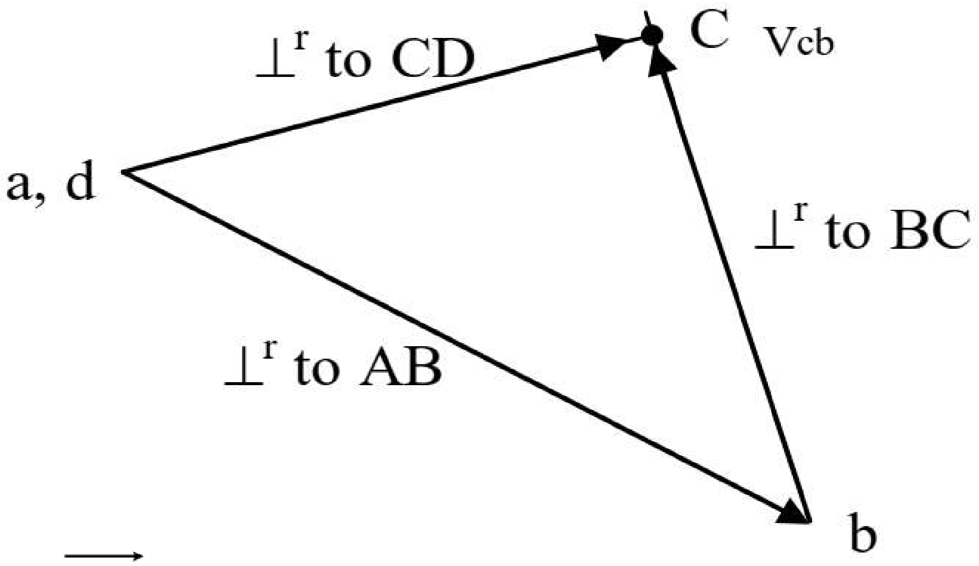
Configuration Diagram

<u>Velocity vector diagram</u>

$$\mathbf{V_b = \omega r = \omega_{ba} \times AB} = \frac{2\pi \times 120}{60} \times 4 \mathbf{= 50.24\ cm/sec}$$

Choose a suitable scale

1 cm = 20 m/s = $\vec{ab}$



$$V_{cb} = \vec{bc}$$

$$V_c = \vec{dc} = 38 \text{ cm/s} = V_{cd}$$

We know that **V = ωR**

$$V_{cd} = \omega_{cD} \times CD$$

$$W_{cD} = \frac{V_{cd}}{CD} = \frac{38}{8} = 4.75 \text{ rad/s (cw)}$$

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Q. The crank and connecting rod of a theoretical steam engine are 0.5 m and 2 m long respectively. The crank makes 180 r.p.m. in the clockwise direction. When it has turned 45° from the inner dead centre position, determine : **1.** velocity of piston, **2.** angular velocity of connecting rod, **3.** velocity of point E on the connecting rod 1.5 m from the gudgeon pin, **4.** velocities of rubbing at the pins of the crank shaft, crank and crosshead when the diameters of their pins are 50 mm, 60 mm and 30 mm respectively, **5.** position and linear velocity of any point G on the connecting rod which has the least velocity relative to crank shaft (Hint: theoretical steam engine means reciprocation motion to rotary motion)

Solution. Given : $N_{BO} = 180$ r.p.m. or $\omega_{BO} = 2\pi \times 180/60 = 18.852$ rad/s

Since the crank length $OB = 0.5$ m, therefore linear velocity of B with respect to O or velocity of B (because O is a fixed point),

$$v_{BO} = v_B = \omega_{BO} \times OB = 18.852 \times 0.5 = 9.426 \text{ m/s}$$

. . . (Perpendicular to BO)

$$\text{vector } ob = v_{BO} = v_B = 9.426 \text{ m/s}$$

$$v_P = \text{vector } op = 8.15 \text{ m/s} \quad \textbf{Ans.}$$

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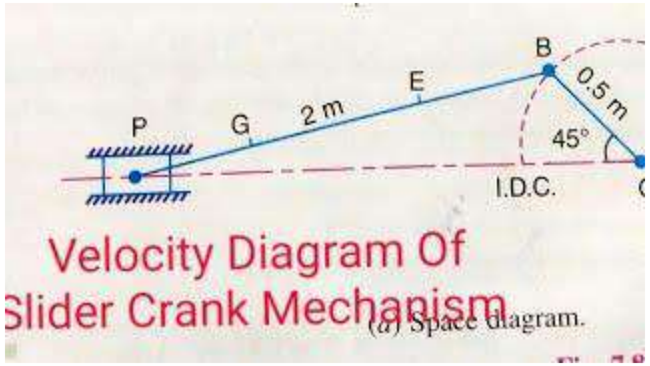
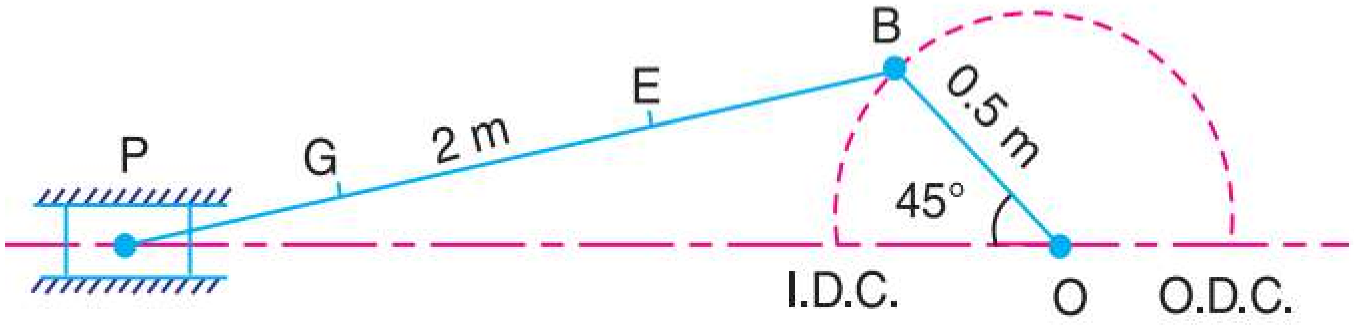
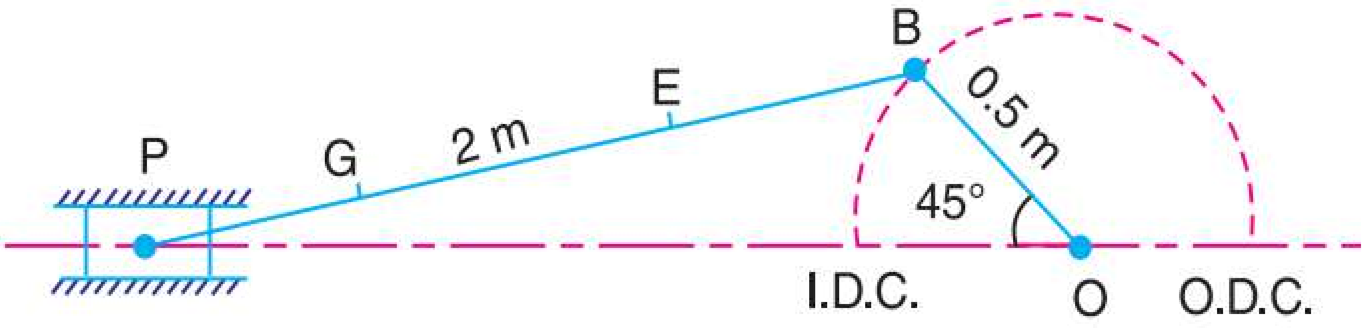


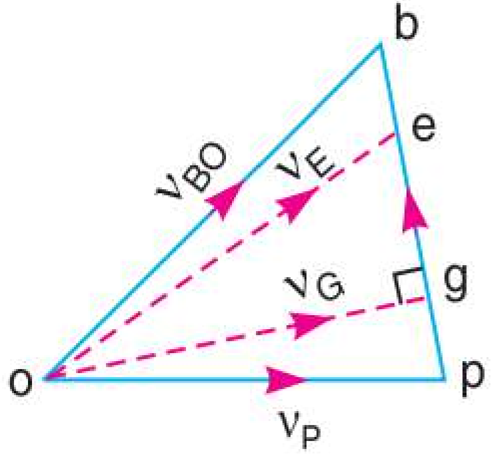


(*a*) Space diagram.

v_P = vector $op = 8.15$ m/s **Ans.**



(*a*) Space diagram.



(*b*) Velocity diagram.

2. *Angular velocity of connecting rod*

From the velocity diagram, we find that the velocity of P with respect to B,

$$v_{PB} = \text{vector } bp = 6.8 \text{ m/s}$$

Since the length of connecting rod PB is 2 m, therefore angular velocity of the connecting rod,

$$\omega_{PB} = \frac{v_{PB}}{PB} = \frac{6.8}{2} = 3.4 \text{ rad/s (Anticlockwise)} \quad \textbf{Ans.}$$

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3. *Velocity of point E on the connecting rod*

The velocity of point E on the connecting rod 1.5 m from the gudgeon pin (*i.e.* PE = 1.5 m) is determined by dividing the vector bp at e in the same ratio as E divides PB in Fig. 7.8 (*a*). This is done in the similar way as discussed in Art 7.6. Join oe. The vector oe represents the velocity of E. By measurement, we find that velocity of point E,

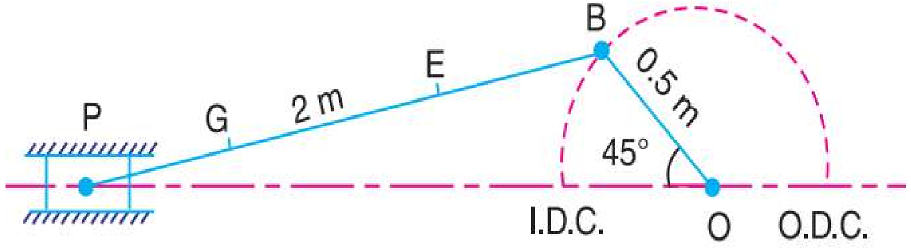
$$v_E = \text{vector } oe = 8.5 \text{ m/s} \quad \textbf{Ans.}$$

Note : The point e on the vector bp may also be obtained as follows :

$$\frac{BE}{BP} = \frac{be}{bp} \quad \text{or} \quad be = \frac{BE \times bp}{BP}$$

v_P = vector op = 8.15 m/s **Ans.**

4. *Velocity of rubbing*



We know that diameter of crank-shaft pin at O,

$$d_O = 50 \text{ mm} = 0.05 \text{ m}$$

Diameter of crank-pin at B,

$$d_B = 60 \text{ mm} = 0.06 \text{ m}$$

and diameter of cross-head pin,

$$d_C = 30 \text{ mm} = 0.03 \text{ m}$$

We know that velocity of rubbing at the pin of crank-shaft

$$= \frac{d_O}{2} \times \omega_{BO} = \frac{0.05}{2} \times 18.85 = 0.47 \text{ m/s} \quad \textbf{Ans.}$$

Velocity of rubbing at the pin of crank

$$= \frac{d_B}{2}(\omega_{BO} + \omega_{PB}) = \frac{0.06}{2}(18.85 + 3.4) = 0.6675 \text{ m/s} \quad \textbf{Ans.}$$

...($\because$ ω_{BO} is clockwise and ω_{PB} is anticlockwise.)

and velocity of rubbing at the pin of cross-head

$$= \frac{d_C}{2} \times \omega_{PB} = \frac{0.03}{2} \times 3.4 = 0.051 \text{ m/s} \quad \textbf{Ans.}$$

...($\because$ At the cross-head, the slider does not rotate and only the connecting rod has angular motion.)

...($\because$ At the cross-head, the slider does not rotate and only the connecting rod has angular motion.)

5. *Position and linear velocity of point G on the connecting rod which has the least velocity relative to crank-shaft*

The position of point *G* on the connecting rod which has the least velocity relative to crank-shaft is determined by drawing perpendicular from *o* to vector *bp*. Since the length of *og* will be the least, therefore the point *g* represents the required position of *G* on the connecting rod.

By measurement, we find that

$$\text{vector } bg = 5 \text{ m/s}$$

The position of point *G* on the connecting rod is obtained as follows:

$$\frac{bg}{bp} = \frac{BG}{BP} \quad \text{or} \quad BG = \frac{bg}{bp} \times BP = \frac{5}{6.8} \times 2 = 1.47 \text{ m} \quad \textbf{Ans.}$$

By measurement, we find that the linear velocity of point *G*,

$$v_G = \text{vector } og = 8 \text{ m/s} \quad \textbf{Ans.}$$

Example 7.3. *In Fig. 7.9, the angular velocity of the crank OA is 600 r.p.m. Determine the linear velocity of the slider D and the angular velocity of the link BD, when the crank is inclined at an angle of 75° to the vertical. The dimensions of various links are : OA = 28 mm ; AB = 44 mm ; BC 49 mm ; and BD = 46 mm. The centre distance between the centres of rotation O and C is 65 mm. The path of travel of the slider is 11 mm below the fixed point C. The slider moves along a horizontal path and OC is vertical.*

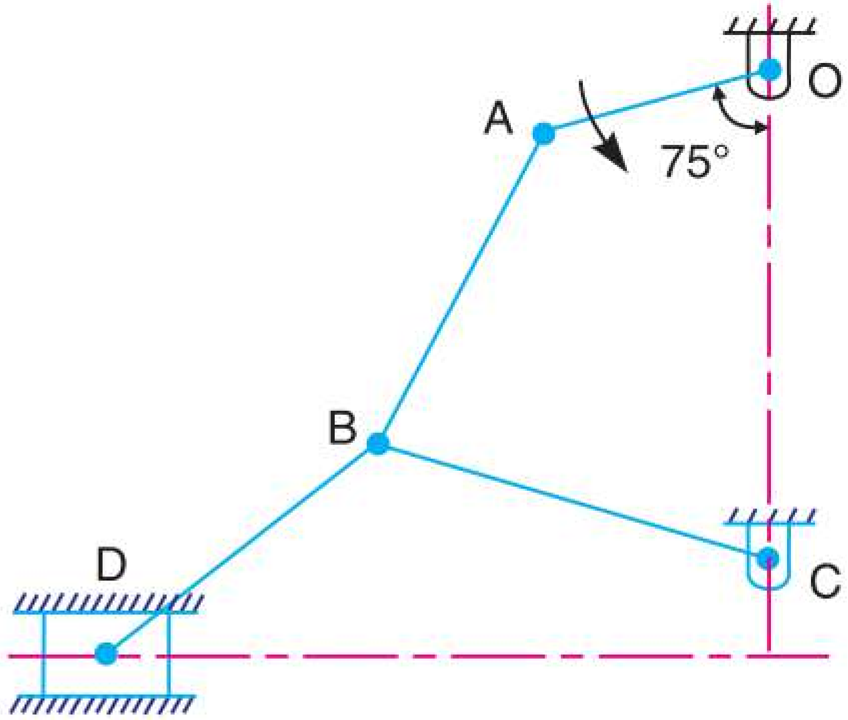


Fig. 7.9

Solution. Given: N_{AO} = 600 r.p.m. or

$\omega_{AO} = 2\pi \times 600/60 = 62.84$ rad/s

Since OA = 28 mm = 0.028 m, therefore velocity of A with respect to O or velocity of A (because O is a fixed point),

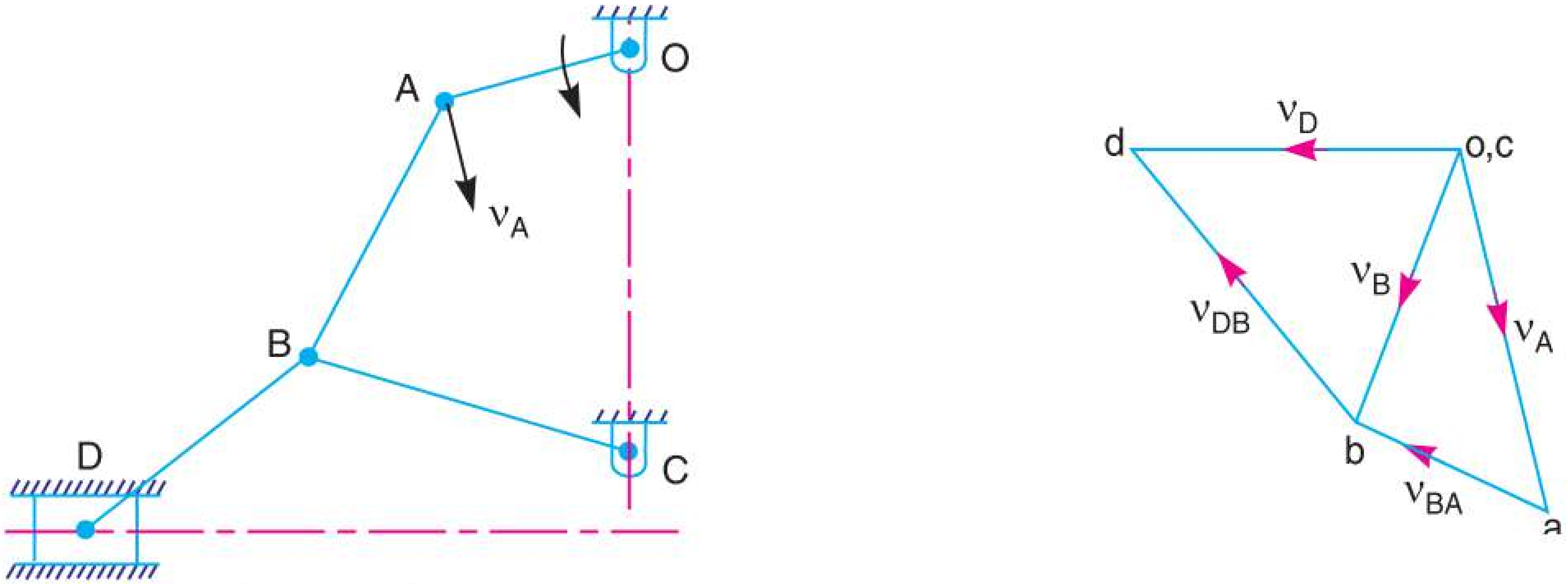
$$v_{AO} = v_A = \omega_{AO} \times OA = 62.84 \times 0.028 = 1.76 \text{ m/s}$$

. . . (Perpendicular to OA)

Linear velocity of the slider D

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.10 (*a*). Now the velocity diagram, as shown in Fig. 7.10 (*b*), is drawn as discussed below :

$$\text{vector } oa = v_{AO} = v_A = 1.76 \text{ m/s}$$



(*a*) Space diagram. (*b*) Velocity diagram.

Fig. 7.10

$$v_D = \text{vector } od = 1.6 \text{ m/s} \quad \textbf{Ans.}$$

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Angular velocity of the link BD

By measurement from velocity diagram, we find that velocity of D with respect to B,

$$v_{DB} = \text{vector } bd = 1.7 \text{ m/s}$$

Since the length of link $BD = 46$ mm $= 0.046$ m, therefore angular velocity of the link BD,

$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{1.7}{0.046} = 36.96 \text{ rad/s (Clockwise about } B\text{)}$$ **Ans.**

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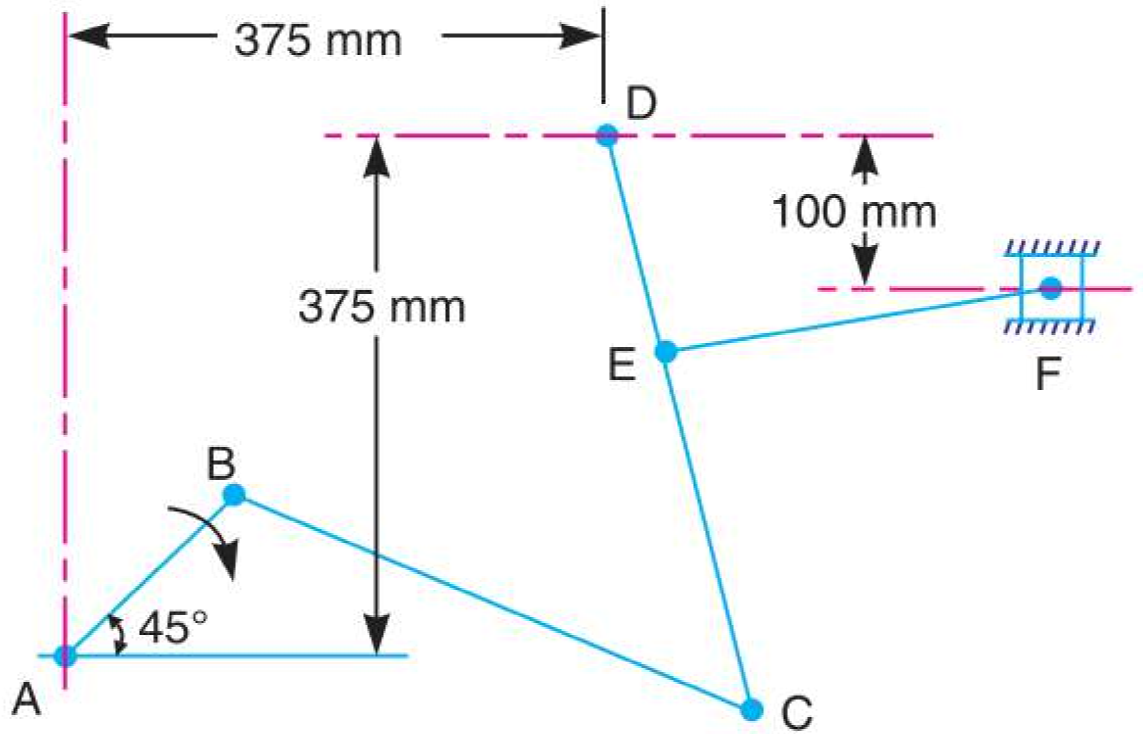
Example 7.4. *The mechanism, as shown in Fig. 7.11, has the dimensions of various links as follows :*

AB = DE = 150 mm ; BC = CD = 450 mm ; EF = 375 mm.



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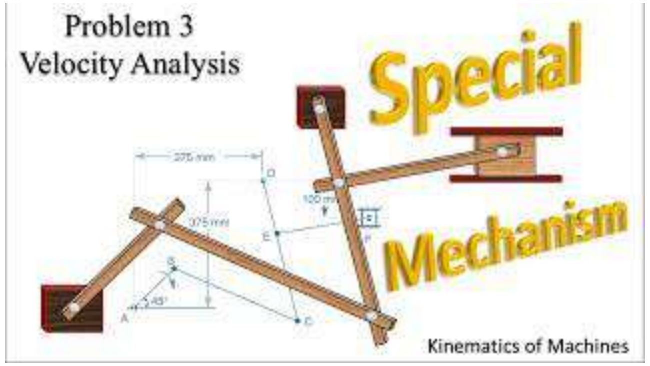


Fig. 7.11

The crank AB makes an angle of 45° with the horizontal and rotates about A in the clockwise direction at a uniform speed of 120 r.p.m. The lever DC oscillates about the fixed point D, which is connected to AB by the coupler BC.

The block F moves in the horizontal guides, being driven by the link EF. Determine: **1.** *velocity of the block F,* **2.** *angular velocity of DC, and* **3.** *rubbing speed at the pin C which is 50 mm in diameter.*

Solution. Given : $N_{BA} = 120$ r.p.m. or $\omega_{BA} = 2\pi \times 120/60 = 4\pi$ rad/s

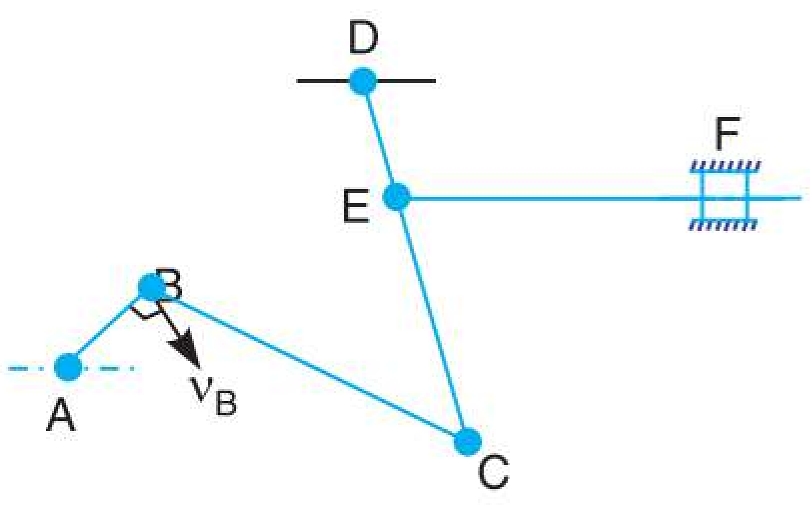
Since the crank length $AB = 150$ mm $= 0.15$ m, therefore velocity of B with respect to A or simply velocity of B (because A is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 4\pi \times 0.15 = 1.885 \text{ m/s}$$

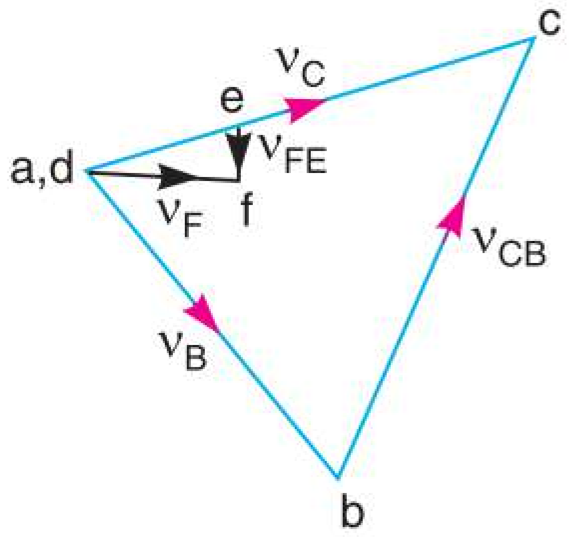
... (Perpendicular to AB)

1. *Velocity of the block F*

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.12 (*a*). Now the velocity diagram, as shown in Fig. 7.12 (*b*), is drawn as discussed below:



(*a*) Space diagram.



(*b*) Velocity diagram.

Fig. 7.12

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$$\text{vector } ab = v_{BA} = v_B = 1.885 \text{ m/s}$$

3. Since the point *E* lies on *DC*, therefore divide vector *dc* in *e* in the same ratio as *E* divides *CD* in Fig. 7.12 (*a*). In other words

$$ce/cd = CE/CD$$

By measurement, we find that velocity of the block *F*,

$$v_F = \text{vector } df = 0.7 \text{ m/s} \quad \textbf{Ans.}$$

2. *Angular velocity of DC*

By measurement from velocity diagram, we find that velocity of *C* with respect to *D*,

$$v_{CD} = \text{vector } dc = 2.25 \text{ m/s}$$

Since the length of link *DC* = 450 mm = 0.45 m, therefore angular velocity of *DC*,

$$\omega_{DC} = \frac{v_{CD}}{DC} = \frac{2.25}{0.45} = 5 \text{ rad/s} \quad \text{. . . (Anticlockwise about } D\text{)}$$

3. *Rubbing speed at the pin C*

We know that diameter of pin at C,

$$d_C = 50 \text{ mm} = 0.05 \text{ m} \quad \text{or} \quad \text{Radius}, r_C = 0.025 \text{ m}$$

From velocity diagram, we find that velocity of C with respect to B,

$$v_{CB} = \text{vector } bc = 2.25 \text{ m/s} \qquad \text{. . . (By measurement)}$$

$$\text{Length } BC = 450 \text{ mm} = 0.45 \text{ m}$$

∴ Angular velocity of BC,

$$\omega_{CB} = \frac{v_{CB}}{BC} = \frac{2.25}{0.45} = 5 \text{ rad/s} \qquad \text{... (Anticlockwise about } B\text{)}$$

We know that rubbing speed at the pin C

$$= (\omega_{CB} - \omega_{CD})\, r_C = (5 - 5)\, 0.025 = 0$$ **Ans.**

Acceleration Analysis

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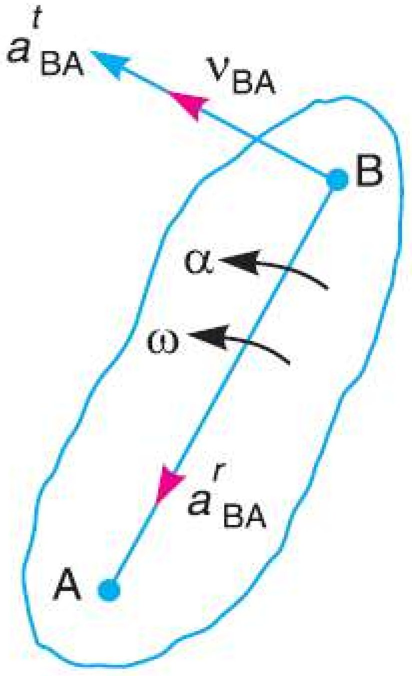
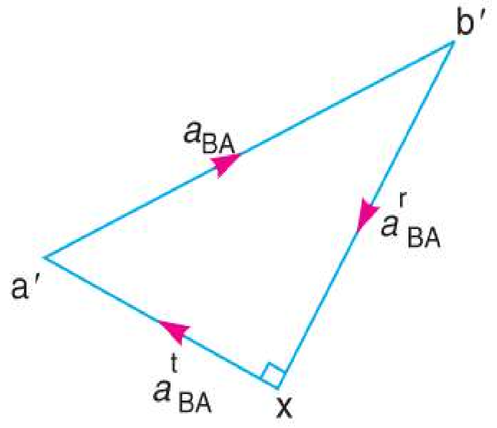
Acceleration Diagram for a Link



(*a*) Link.



(*b*) Acceleration diagram.

Acceleration of a particle whose velocity changes both in magnitude and direction at any instant has the following two components

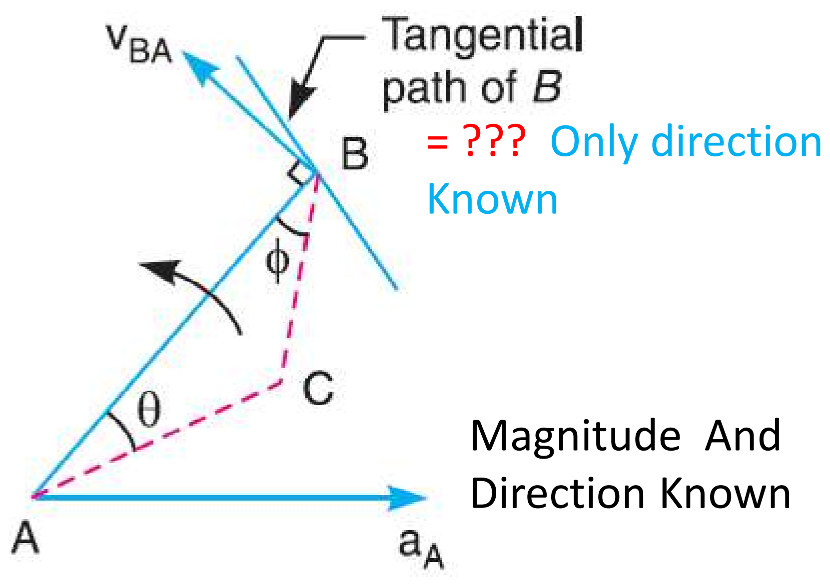
(i) Centripetal or radial component which is perpendicular to the velocity of the particle at the given instant.(i.e parallel to link)

(ii) Tangential component which is parallel to the velocity of the particle at the given instant.(i.e perpendicular to link)

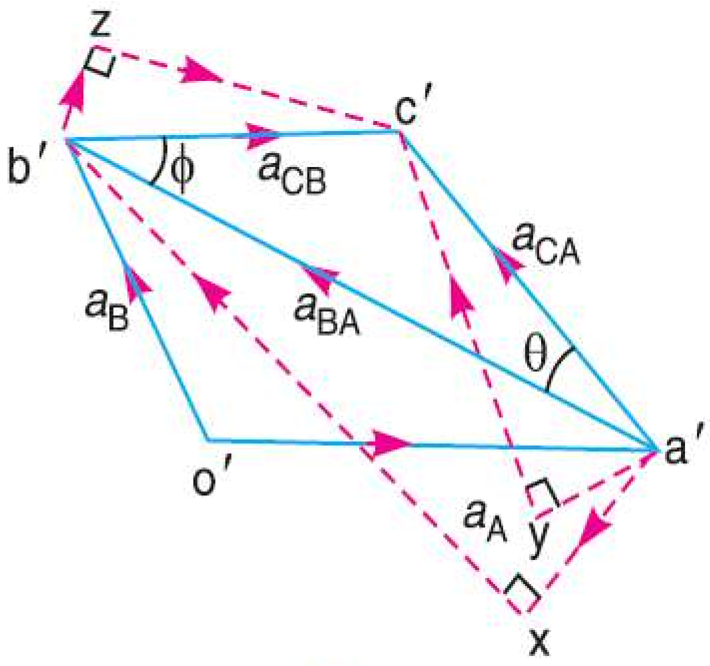
$$a^r_{BA} = \omega^2 \times \text{Length of link } AB = \omega^2 \times AB = v^2_{BA} / AB$$

$$a^t_{BA} = \alpha \times \text{Length of the link } AB = \alpha \times AB$$

Acceleration of a Point on a Link



(a) Points on a Link.



(b) Acceleration diagram.

Steps:

1. From any point o', draw vector o'a' parallel to the direction of absolute acceleration at point A i.e. aA , to some suitable scale, as shown in Fig. (b).

2. We know that the acceleration of B with respect to A i.e. aBA has the following two components:

    1. Radial component of the acceleration of B with respect to A i.e. a^r_{BA} and

    2. Tangential component of the acceleration B with respect to A i.e. a^t_{BA}. These two components are mutually perpendicular.

3. Draw vector a'x parallel to the link AB (because radial component of the acceleration of B withrespect to A will pass through AB), such that vector a’x= a’BA =V^2_{BA} / AB.

Acceleration of a Point on a Link

4. From point x, draw vector xb' perpendicular to AB or vector a'x (because tangential component of B with respect to A i.e. a^r_{BA} , is perpendicular to radial component a^r_{BA}) and through o' draw a line parallel to the path of B to represent the absolute acceleration of B i.e. a_B . The vectors xb' and o' b' intersect at b'. Now the values of a B and a^t_{BA} at may be measured, to the scale.

5. By joining the points a' and b' we may determine the total acceleration of B with respect to A i.e. a_{BA}. The vector a' b' is known as acceleration image of the link AB.

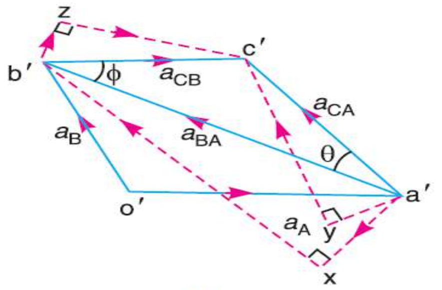
6. For any other point C on the link, draw triangle a' b' c' similar to triangle ABC. Now vector b'c' represents the acceleration of C with respect to B i.e. a_{CB} , and vector a'c' represents the acceleration of C with respect to A i.e. a_{CA}. As discussed above, a_{CB} and a_{CA} will each have two components as follows

1. a_{CB} has two components; a^r_{CB} and a^t_{BA} as shown by triangle b'zc' in Fig. (b), in which b'zis parallel to BC and zc' is perpendicular to b'z or BC.

2. aCA has two components ; a^r_{CA} and a^t_{CA} as shown by triangle a'yc' in Fig. (b), in which a'y is parallel to AC and yc' is perpendicular to a'y or AC.

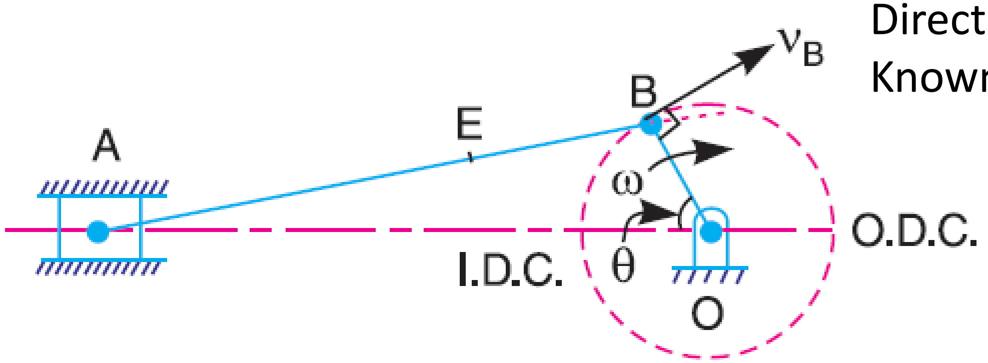
7. The angular acceleration of the link AB is obtained by dividing the tangential compc the acceleration of B with respect to A (a^t_{BA}) to the length of the link. Mathematically, acceleration of the link AB

$$\alpha_{AB} = a^t_{BA} / AB$$



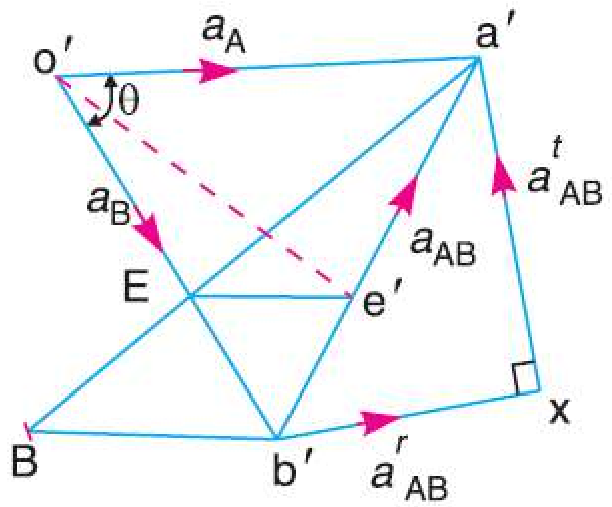
(b) Acceleration diagram.

Acceleration In Slider Crank Mechanisms



Magnitude
And
Direction
Known

(*a*) Slider crank mechanism.



Steps:

1. Draw vector $o'\,b'$ parallel to BO and set off equal in magnitude of $a^r_{BO} = a_B$, to some suitable scale.

2. From point b', draw vector $b'x$ parallel to BA. The vector $b'x$ represents the radial component of the acceleration of A with respect to B whose magnitude is given by :

$$a^r_{AB} = v^2_{AB} / BA$$

Since the point B moves with constant angular velocity, therefore there will be ***no tangential*** component of the acceleration.

Acceleration in Slider Crank Mechanisms

Steps: **3.** From point x, draw vector xa' perpendicular to $b'x$ (or AB). The vector xa' represents the tangential component of the acceleration of A with respect to B *i.e.* a^t_{AB}.

Note: When a point moves along a straight line, it has **no centripetal or radial** component of the acceleration.

4. Since the point A reciprocates along AO, therefore the acceleration must be parallel to velocity. Therefore from o', draw $o'a'$ parallel to AO, intersecting the vector xa' at a'.

Now the acceleration of the piston or the slider A (a_A) and a^t_{AB} may be measured to the scale.

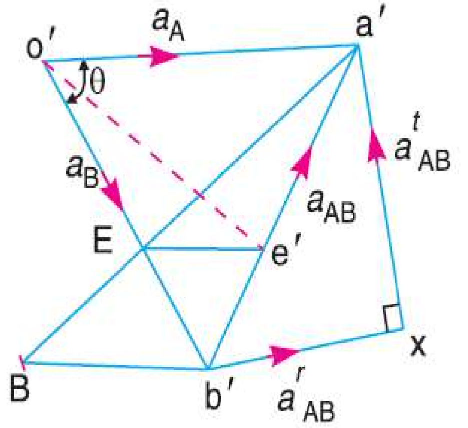
5. The vector $b'a'$, which is the sum of the vectors $b'x$ and xa', represents the total acceleration of A with respect to B *i.e.* a_{AB}. The vector $b'a'$ represents the acceleration of the connecting rod AB.

6. The acceleration of any other point on AB such as E may be obtained by dividing the vector $b'a'$ at e' in the same ratio as E divides AB in Fig. 8.3 (a). In other words

$$a'e' / a'b' = AE / AB$$

7. The angular acceleration of the connecting rod AB may be obtaine[d by dividing the] tangential component of the acceleration of A with respect to B (a^t_{AB}) to the leng[th ...] words, angular acceleration of AB,

$$\alpha_{AB} = a^t_{AB} / AB \text{ (Clockwise about } B\text{)}$$



Example 8.4. *PQRS is a four bar chain with link PS fixed. The lengths of the links are PQ = 62.5 mm ; QR = 175 mm ; RS = 112.5 mm ; and PS = 200 mm. The crank PQ rotates at 10 rad/s clockwise. Draw the velocity and acceleration diagram when angle QPS = 60° and Q and R lie on the same side of PS. Find the angular velocity and angular acceleration of links QR and RS.*

Solution. Given : $\omega_{QP} = 10$ rad/s; $PQ = 62.5$ mm $= 0.0625$ m ; $QR = 175$ mm $= 0.175$ m ; $RS = 112.5$ mm $= 0.1125$ m ; $PS = 200$ mm $= 0.2$ m

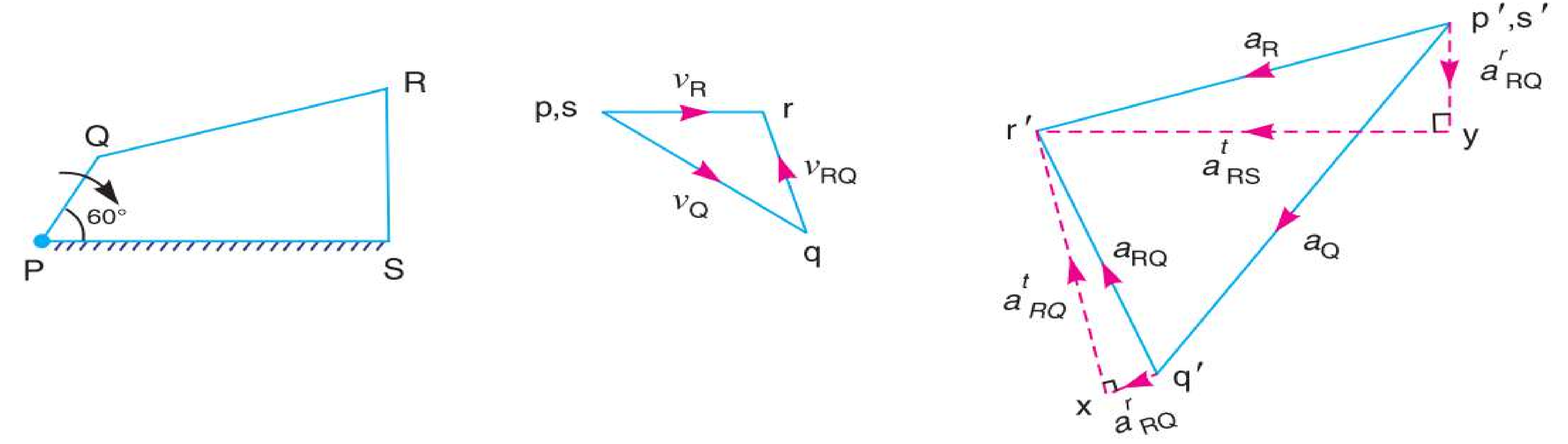
We know that velocity of Q with respect to P or velocity of Q,

$$v_{QP} = v_Q = \omega_{QP} \times PQ = 10 \times 0.0625 = 0.625 \text{ m/s}$$

...(Perpendicular to PQ)

Angular velocity of links QR and RS

First of all, draw the space diagram of a four bar chain, to some suitable scale, as shown in Fig. 8.9 (a). Now the velocity diagram as shown in Fig. 8.9 (b), is drawn as discussed below:



(a) Space diagram. (b) Velocity diagram. (c) Acceleration diagram.

vector $pq = v_{QP} = v_Q = 0.625$ m/s

v_{RQ} = vector $qr = 0.333$ m/s, and $v_{RS} = v_R$ = vector $sr = 0.426$ m/s

We know that angular velocity of link QR,

https://youtu.be/te6IhV7KfZc

$$\omega_{QR} = \frac{v_{RQ}}{RQ} = \frac{0.333}{0.175} = 1.9 \text{ rad/s (Anticlockwise)} \quad \textbf{Ans.}$$

and angular velocity of link RS,

$$\omega_{RS} = \frac{v_{RS}}{SR} = \frac{0.426}{0.1125} = 3.78 \text{ rad/s (Clockwise)} \quad \textbf{Ans.}$$

Angular acceleration of links QR and RS

Since the angular acceleration of the crank PQ is not given, therefore there will be no tangential component of the acceleration of Q with respect to P.

We know that radial component of the acceleration of Q with respect to P (or the acceleration of Q),

$$a^r_{QP} = a_{QP} = a_Q = \frac{v_{QP}^2}{PQ} = \frac{(0.625)^2}{0.0625} = 6.25 \text{ m/s}^2$$

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Radial component of the acceleration of R with respect to Q,

$$a_{RQ}^r = \frac{v_{RQ}^2}{QR} = \frac{(0.333)^2}{0.175} = 0.634 \text{ m/s}^2$$

and radial component of the acceleration of R with respect to S (or the acceleration of R),

$$a_{RS}^r = a_{RS} = a_R = \frac{v_{RS}^2}{SR} = \frac{(0.426)^2}{0.1125} = 1.613 \text{ m/s}^2$$

The acceleration diagram, as shown in Fig. 8.9 (c) is drawn as follows :

1. Since P and S are fixed points, therefore these points lie at one place in the acceleration diagram. Draw vector $p'q'$ parallel to PQ, to some suitable scale, to represent the radial component of acceleration of Q with respect to P or acceleration of Q *i.e* a_{QP}^r or a_Q such that

$$\text{vector } p'q' = a_{QP}^r = a_Q = 6.25 \text{ m/s}^2$$

2. From point q', draw vector $q'x$ parallel to QR to represent the radial component of acceleration of R with respect to Q *i.e.* a_{RQ}^r such that

$$\text{vector } q'x = a_{RQ}^r = 0.634 \text{ m/s}^2$$

3. From point x, draw vector xr' perpendicular to QR to represent the tangential component of acceleration of R with respect to Q *i.e* a^t_{RQ} whose magnitude is not yet known.

4. Now from point s', draw vector $s'y$ parallel to SR to represent the radial component of the acceleration of R with respect to S *i.e.* a^r_{RS} such that

$$\text{vector } s'y = a^r_{RS} = 1.613 \text{ m/s}^2$$

5. From point y, draw vector yr' perpendicular to SR to represent the tangential component of acceleration of R with respect to S *i.e.* a^t_{RS}.

6. The vectors xr' and yr' intersect at r'. Join $p'r$ and $q'r'$. By measurement, we find that

$$a^t_{RQ} = \text{vector } xr' = 4.1 \text{ m/s}^2 \text{ and } a^t_{RS} = \text{vector } yr' = 5.3 \text{ m/s}^2$$

We know that angular acceleration of link QR,

$$\alpha_{QR} = \frac{a^t_{RQ}}{QR} = \frac{4.1}{0.175} = 23.43 \text{ rad/s}^2 \text{ (Anticlockwise)}$$ **Ans.**

and angular acceleration of link RS,

$$\alpha_{RS} = \frac{a^t_{RS}}{SR} = \frac{5.3}{0.1125} = 47.1 \text{ rad/s}^2 \text{ (Anticlockwise)}$$ **Ans.**

Example 8.1. *The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine : **1.** linear velocity and acceleration of the midpoint of the connecting rod, and **2.** angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.*

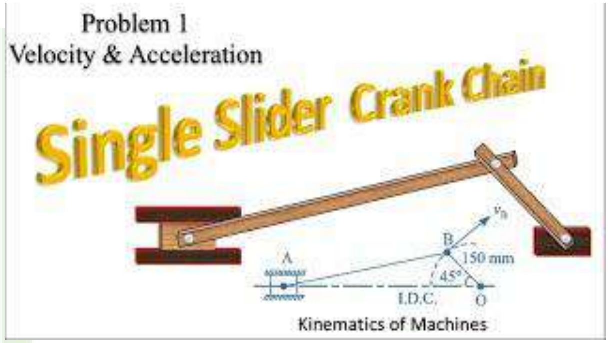
Solution. Given : N_{BO} = 300 r.p.m. or $\omega_{BO} = 2\,\pi \times 300/60 = 31.42$ rad/s; OB = 150 mm = 0.15 m ; BA = 600 mm = 0.6 m

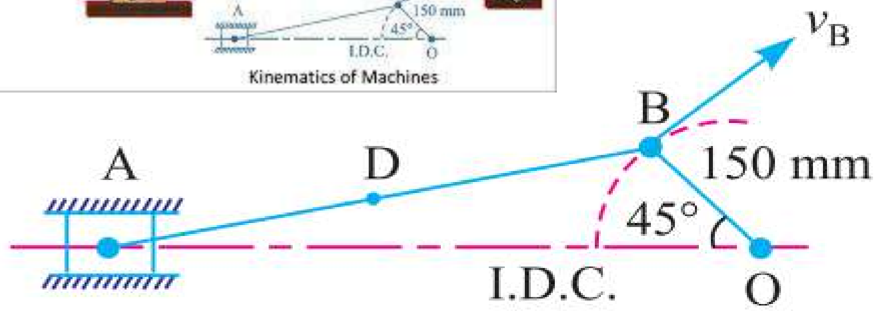
We know that linear velocity of B with respect to O or velocity of B,

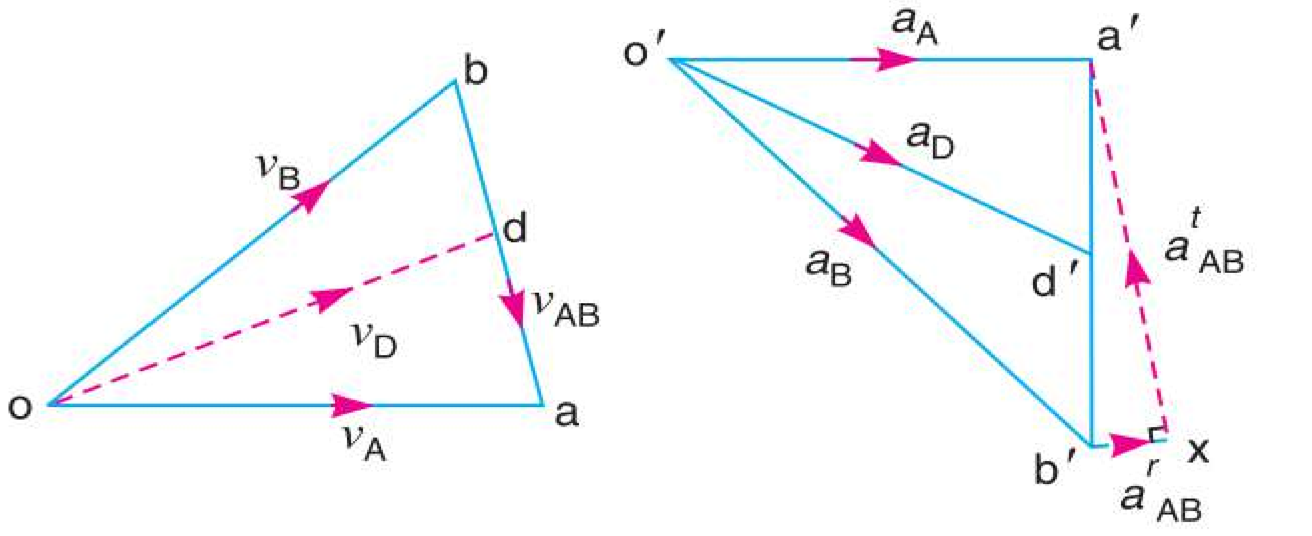
$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713 \text{ m/s}$$

https://youtu.be/D5jUncwm1gY

...(Perpendicular to BO)







(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

1. Linear velocity of the midpoint of the connecting rod

1. Draw vector *ob* perpendicular to *BO*, to some suitable scale, to represent the velocity of *B* with respect to *O* or simply velocity of *B* *i.e.* v_{BO} or v_B, such that

$$\text{vector } ob = v_{BO} = v_B = 4.713 \text{ m/s}$$

By measurement, we find that velocity of *A* with respect to *B*,

$$v_{AB} = \text{vector } ba = 3.4 \text{ m/s}$$

$$\text{Velocity of } A,\ v_A = \text{vector } oa = 4 \text{ m/s}$$

3. In order to find the velocity of the midpoint *D* of the connecting rod *AB*, divide the vector *ba* at *d* in the same ratio as *D* divides *AB*, in the space diagram. In other words,

$$bd / ba = BD/BA$$

Note: Since *D* is the midpoint of *AB*, therefore *d* is also midpoint of vector *ba*.

4. Join *od*. Now the vector *od* represents the velocity of the midpoint *D* of the connecting rod *i.e.* v_D.

By measurement, we find that

$$v_D = \text{vector } od = 4.1 \text{ m/s} \quad \textbf{Ans.}$$

Acceleration of the midpoint of the connecting rod

We know that the radial component of the acceleration of B with respect to O or the acceleration of B,

$$a^r_{BO} = a_B = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

and the radial component of the acceleraiton of A with respect to B,

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.4 (*c*) is drawn as discussed below:

1. Draw vector $o'b'$ parallel to BO, to some suitable scale, to represent the radial component of the acceleration of B with respect to O or simply acceleration of B *i.e.* a^r_{BO} or a_B, such that

$$\text{vector } o'b' = a^r_{BO} = a_B = 148.1 \text{ m/s}^2$$

2. The acceleration of A with respect to B has the following two components:

(*a*) The radial component of the acceleration of A with respect to B *i.e.* a^r_{AB}, and

(*b*) The tangential component of the acceleration of A with respect to B *i.e.* a^t_{AB}. These two components are mutually perpendicular.

Therefore from point b', draw vector $b'x$ parallel to AB to represent $a^r_{AB} = 19.3 \text{ m/s}^2$ and from point x draw vector xa' perpendicular to vector $b'x$ whose magnitude is yet unknown.

4. In order to find the acceleration of the midpoint *D* of the connecting rod *A B*, divide the vector *a′ b′* at *d′* in the same ratio as *D* divides *A B*. In other words

$$b'd' / b'a' = BD / BA$$

Note: Since *D* is the midpoint of *A B*, therefore *d′* is also midpoint of vector *b′ a′*.

5. Join *o′ d′*. The vector *o′ d′* represents the acceleration of midpoint *D* of the connecting rod *i.e.* a_D.

By measurement, we find that

$$a_D = \text{vector } o'd' = 117 \text{ m/s}^2$$ **Ans.**

2. *Angular velocity of the connecting rod*

We know that angular velocity of the connecting rod *A B*,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \text{ rad/s}^2 \text{ (Anticlockwise about } B\text{)}$$ **Ans.**

Angular acceleration of the connecting rod

From the acceleration diagram, we find that

$$a^t_{AB} = 103 \text{ m/s}^2 \qquad \text{...(By measurement)}$$

We know that angular acceleration of the connecting rod *A B*,

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67 \text{ rad/s}^2 \text{ (Clockwise about } B\text{)}$$ **Ans.**

Example 8.2. *An engine mechanism is shown in Fig. 8.5. The crank CB = 100 mm and the connecting rod BA = 300 mm with centre of gravity G, 100 mm from B. In the position shown, the crankshaft has a speed of 75 rad/s and an angular acceleration of 1200 rad/s². Find:* **1.** *velocity of G and angular velocity of AB, and* **2.** *acceleration of G and angular acceleration of AB.*

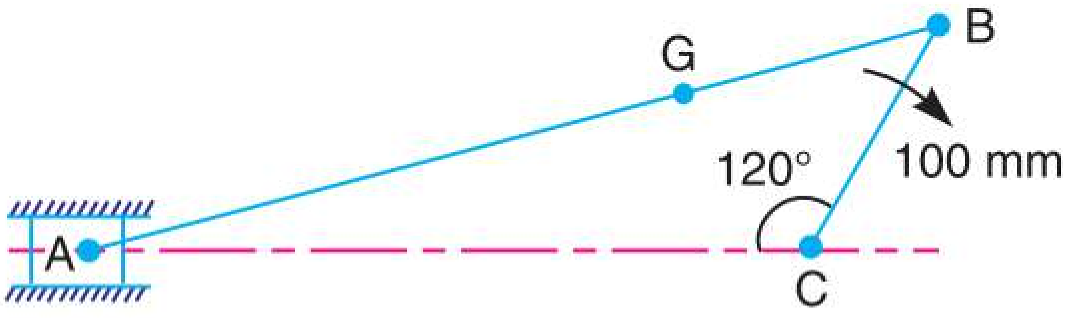


Fig. 8.5

Solution. Given : $\omega_{BC} = 75$ rad/s ; $\alpha_{BC} = 1200$ rad/s², $CB = 100$ mm $= 0.1$ m; $BA = 300$ mm $= 0.3$ m

We know that velocity of B with respect to C or velocity of B,

$$v_{BC} = v_B = \omega_{BC} \times CB = 75 \times 0.1 = 7.5 \text{ m/s} \qquad \text{...(Perpendicular to } BC)$$

Since the angular acceleration of the crankshaft, $\alpha_{BC} = 1200$ rad/s², therefore tangential component of the acceleration of B with respect to C,

$$a^t_{BC} = \alpha_{BC} \times CB = 1200 \times 0.1 = 120 \text{ m/s}^2$$

1. *Velocity of G and angular velocity of AB*

First of all, draw the space diagram, to some suitable scale, as shown in Fig. 8.6 (*a*). Now the velocity diagram, as shown in Fig. 8.6 (*b*), is drawn as discussed below:

1. Draw vector *cb* perpendicular to *CB*, to some suitable scale, to represent the velocity of *B* with respect to *C* or velocity of *B* (*i.e.* v_{BC} *or* v_B), such that

$$\text{vector } cb = v_{BC} = v_B = 7.5 \text{ m/s}$$

2. From point *b*, draw vector *ba* perpendicular to *BA* to represent the velocity of *A* with respect to *B* *i.e.* v_{AB}, and from point *c*, draw vector *ca* parallel to the path of motion of *A* (which is along *AC*) to represent the velocity of *A* *i.e.* v_A. The vectors *ba* and *ca* intersect at *a*.

3. Since the point *G* lies on *AB*, therefore divide vector *ab* at *g* in the same ratio as *G* divides *AB* in the space diagram. In other words,

$$ag / ab = AG / AB$$

The vector *cg* represents the velocity of *G*.

By measurement, we find that velocity of *G*,

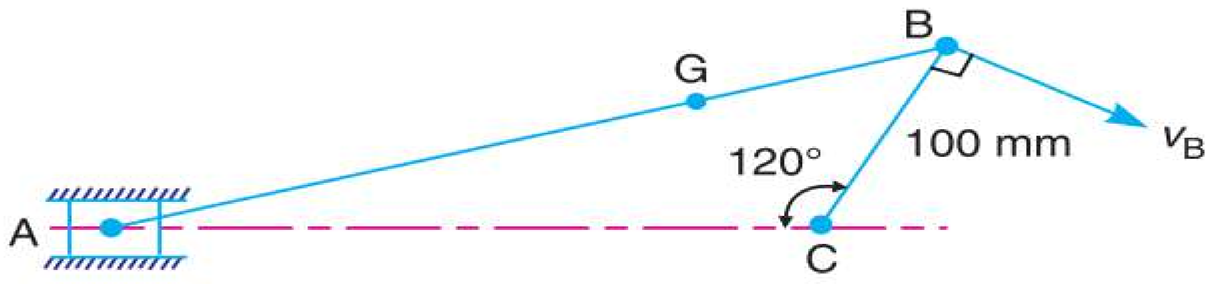
$$v_G = \text{vector } cg = 6.8 \text{ m/s} \quad \textbf{Ans.}$$

From velocity diagram, we find that velocity of A with respect to B,

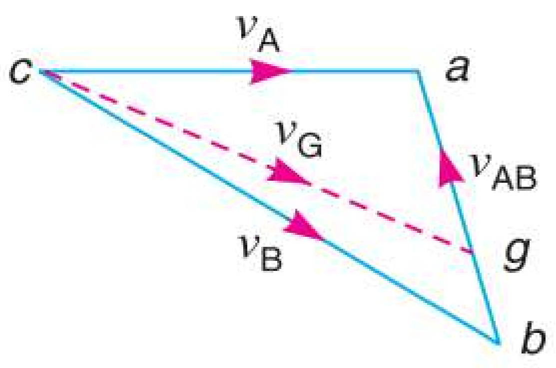
$$v_{AB} = \text{vector } ba = 4 \text{ m/s}$$

We know that angular velocity of AB,

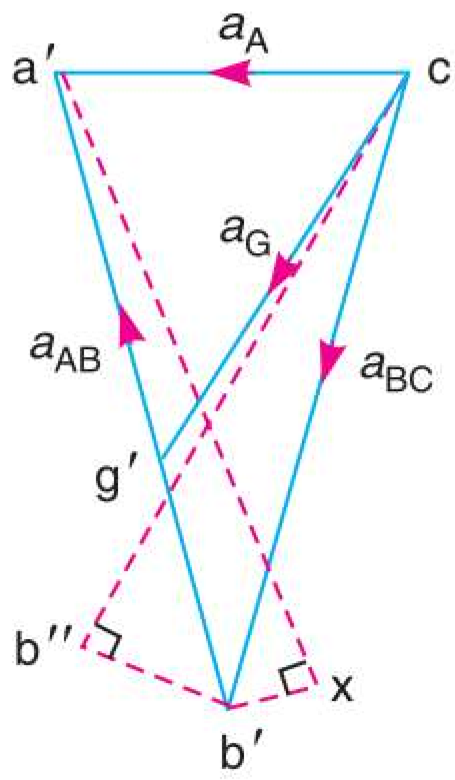
$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{4}{0.3} = 13.3 \text{ rad/s (Clockwise)}$$ **Ans.**



(*a*) Space diagram.



(*b*) Velocity diagram.



(*c*) Acceleration diagram.

2. *Acceleration of G and angular acceleration of AB*

We know that radial component of the acceleration of B with respect to C,

$$* \quad a^r_{BC} = \frac{v^2_{BC}}{CB} = \frac{(7.5)^2}{0.1} = 562.5 \text{ m/s}^2$$

and radial component of the acceleration of A with respect to B,

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{4^2}{0.3} = 53.3 \text{ m/s}^2$$

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1. Draw vector $c'\,b''$ parallel to CB, to some suitable scale, to represent the radial component of the acceleration of B with respect to C, *i.e.* a^r_{BC}, such that

$$\text{vector } c'b'' = a^r_{BC} = 562.5 \text{ m/s}^2$$

2. From point b'', draw vector $b''\,b'$ perpendicular to vector $c'\,b''$ or CB to represent the tangential component of the acceleration of B with respect to C *i.e.* a^t_{BC}, such that

$$\text{vector } b''b' = a^t_{BC} = 120 \text{ m/s}^2 \qquad \text{... (Given)}$$

3. Join $c'b'$. The vector $c'b'$ represents the total acceleration of B with respect to C *i.e.* a_{BC}.

4. From point b', draw vector $b'\,x$ parallel to BA to represent radial component of the acceleration of A with respect to B *i.e.* a^r_{AB} such that

$$\text{vector } b'x = a^r_{AB} = 53.3 \text{ m/s}^2$$

By measurement, we find that acceleration of G,

$$a_G = \text{vector } c'g' = 414 \text{ m/s}^2 \textbf{ Ans.}$$

From acceleration diagram, we find that tangential component of the acceleration of A with respect to B,

$$a^t_{AB} = \text{vector } xa' = 546 \text{ m/s}^2 \quad \text{...(By measurement)}$$

$\therefore$ Angular acceleration of AB,

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{546}{0.3} = 1820 \text{ rad/s}^2 \text{ (Clockwise) } \textbf{Ans.}$$

Q4.Find out the acceleration of the slider D and the angular acceleration of link CD for the engine mechanism shown fig. The crank OA rotates uniformly at 180 r.p.m. in clockwise direction. The various lengths are: OA = 150mm ; AB = 450 mm; PB = 240 mm ; BC = 210 mm ; CD = 660 mm.

Solution. Given: $N_{AO} = 180$ r.p.m., or $\omega_{AO} = 2\pi \times 180/60 = 18.85$ rad/s ; $OA = 150$ mm = 0.15 m ; $AB = 450$ mm = 0.45 m ; $PB = 240$ mm = 0.24 m ; $CD = 660$ mm = 0.66 m

We know that velocity of A with respect to O or velocity of A,

$$v_{AO} = v_A = \omega_{AO} \times OA$$
$$= 18.85 \times 0.15 = 2.83 \text{ m/s}$$

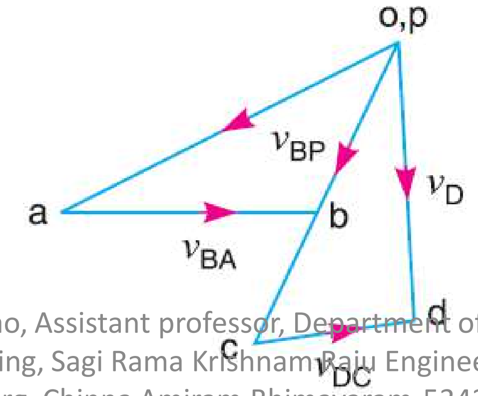
vector $oa = v_{AO} = v_A = 2.83$ m/s ...(Perpendicular to OA)

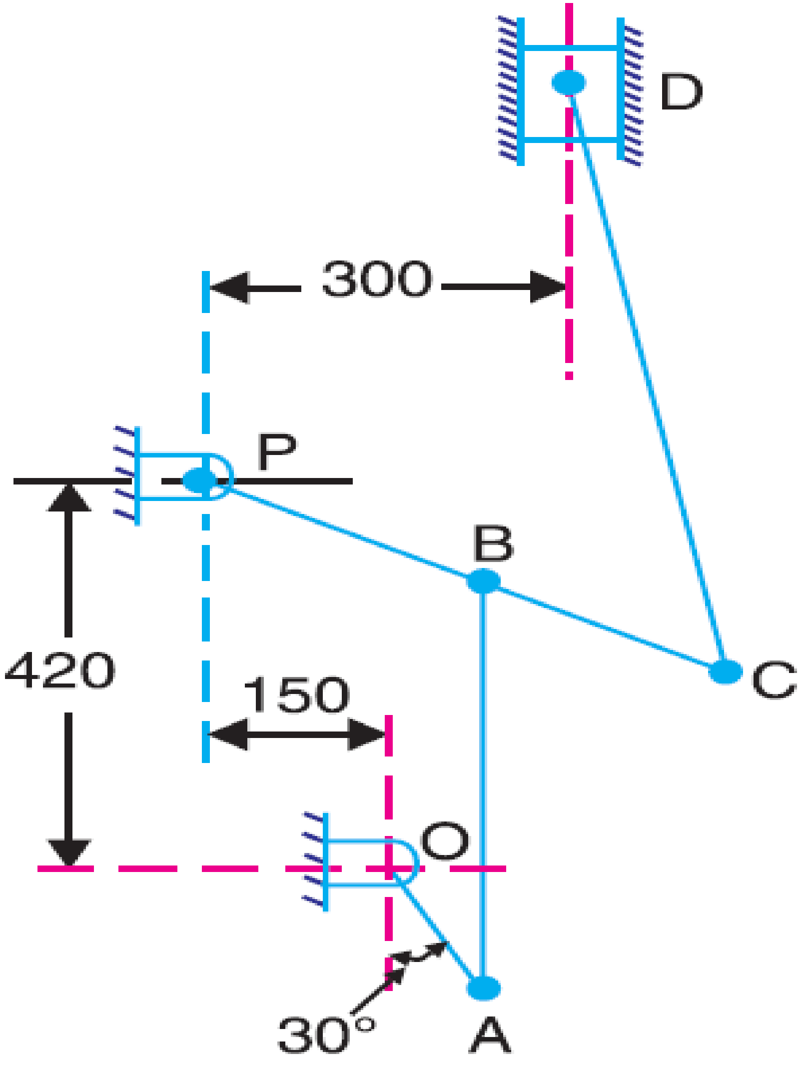
v_D = vector od = 2.36 m/s

v_{DC} = vector cd = 1.2 m/s

v_{BA} = vector ab = 1.8 m/s

v_{BP} = vector pb = 1.5 m/s





All dimensions in mm.

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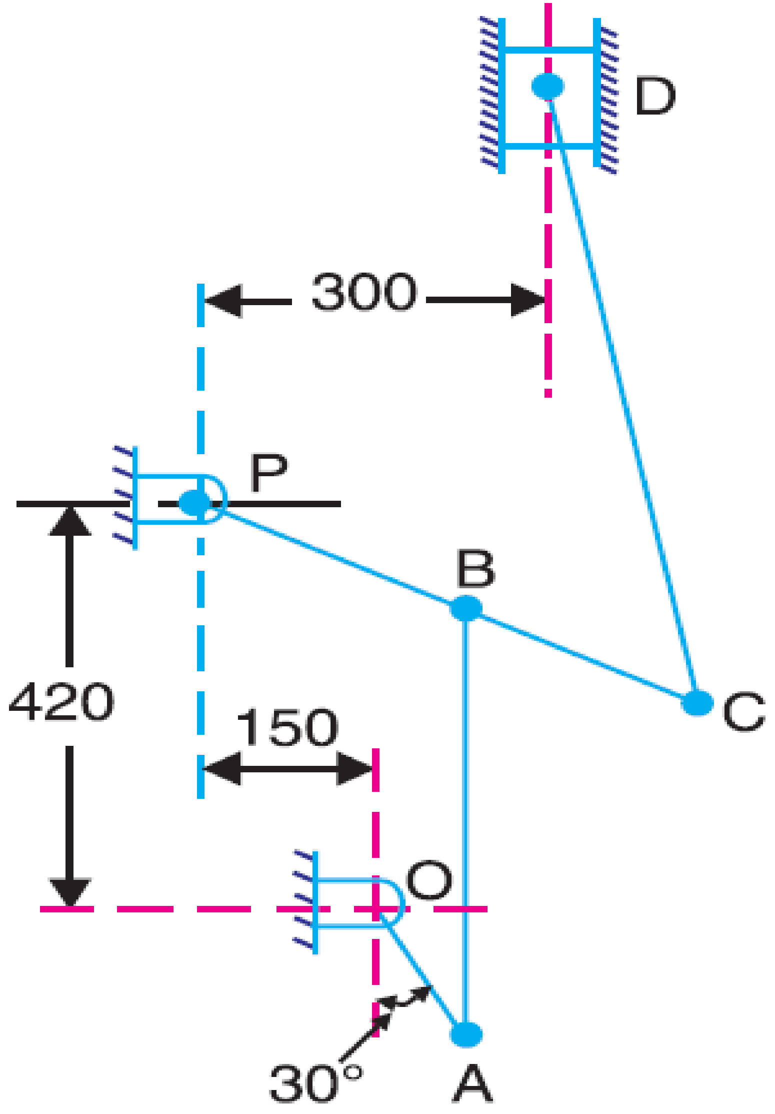
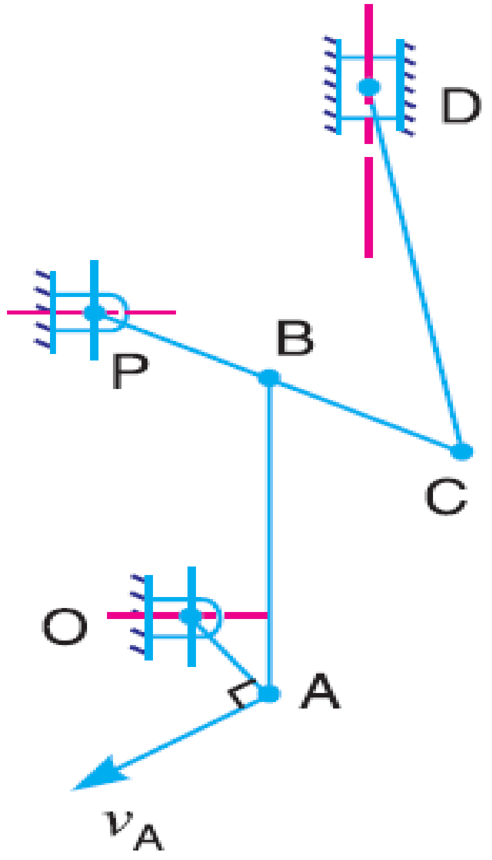
Scale 100 mm : 1 cm

Scale 2.83 m/s : 2.83 cm



All dimensions in mm.



$$v_{AO} = v_A = \omega_{AO} \times OA$$

$$= 18.85 \times 0.15 = 2.83 \text{ m/s}$$

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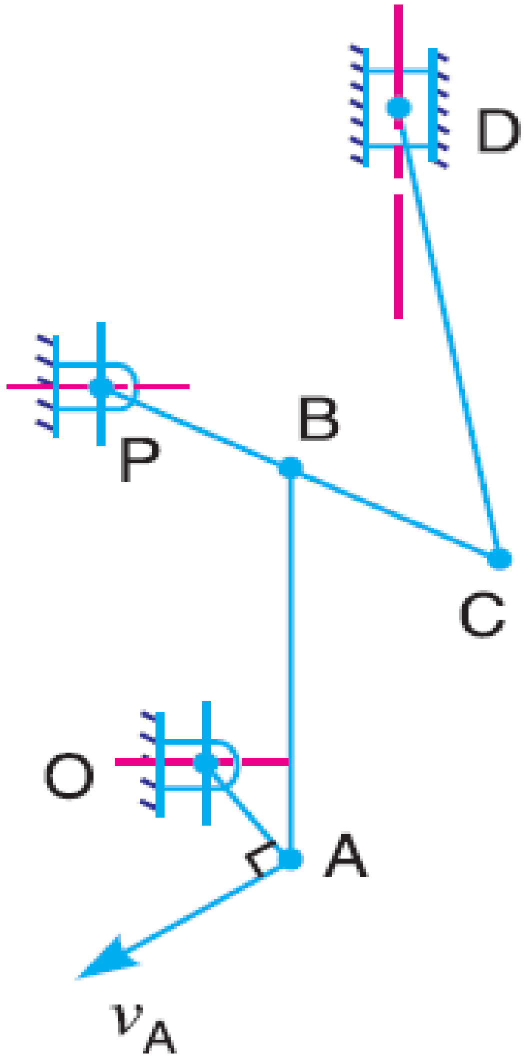
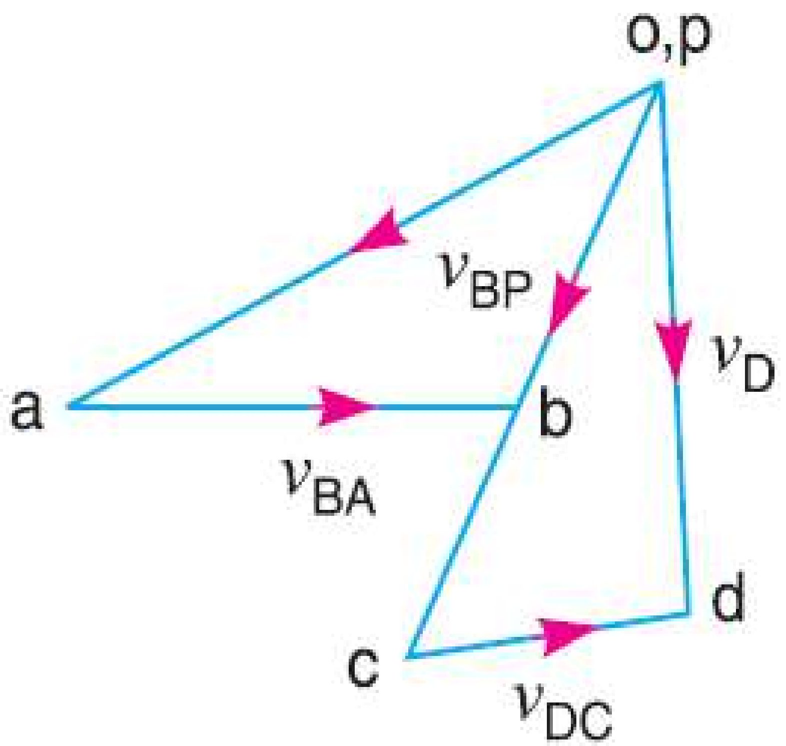
Velocity diagram





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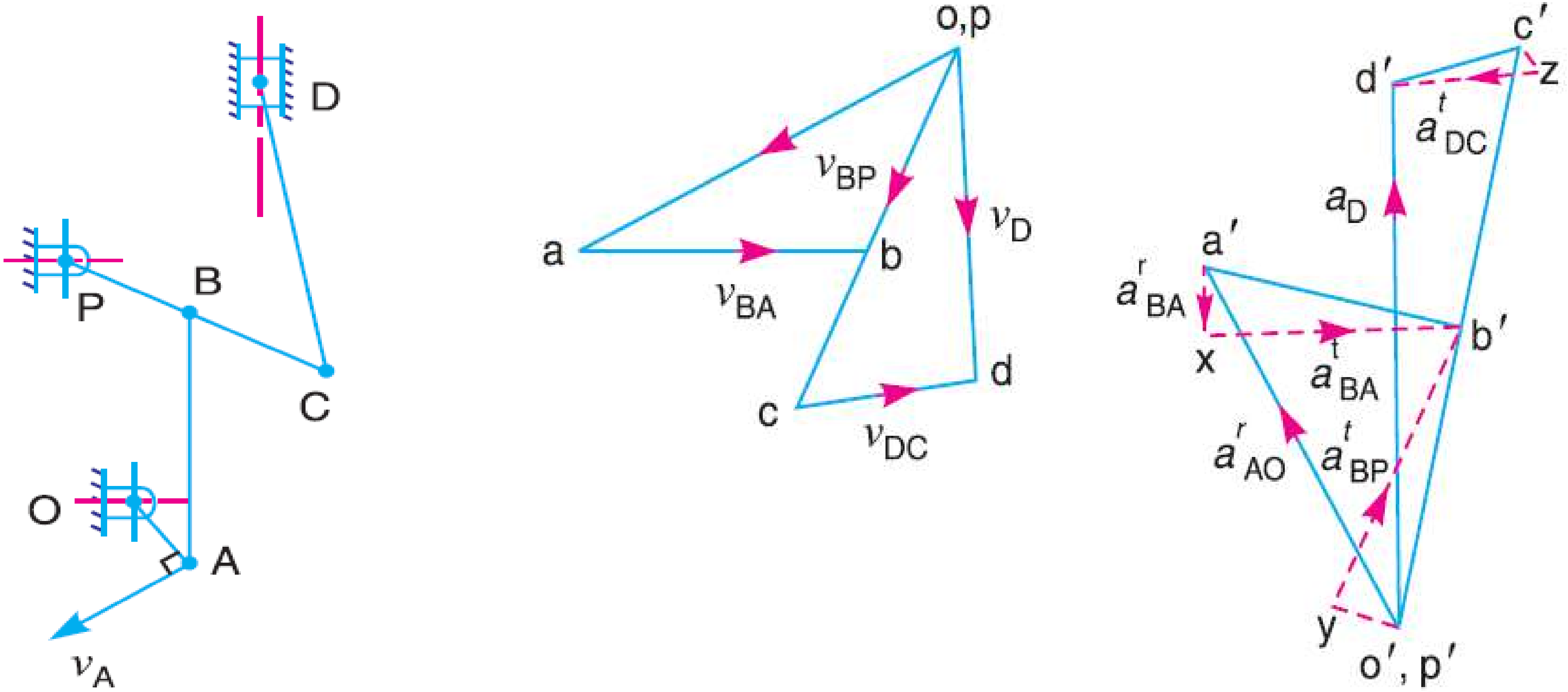
Acceleration diagram



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Acceleration of the slider D

We know that radial component of the acceleration of A with respect to O or acceleration of A,

$$a^r_{AO} = a_A = \omega^2_{AO} \times AO = (18.85)^2 \times 0.15 = 53.3 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to A,

$$a^r_{BA} = \frac{v^2_{BA}}{AB} = \frac{(1.8)^2}{0.45} = 7.2 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to P,

$$a^r_{BP} = \frac{v^2_{BP}}{PB} = \frac{(1.5)^2}{0.24} = 9.4 \text{ m/s}^2$$

Radial component of the acceleration of D with respect to C,

$$a^r_{DC} = \frac{v^2_{DC}}{CD} = \frac{(1.2)^2}{0.66} = 2.2 \text{ m/s}^2$$

$$\text{vector } o'a' = a^r_{AO} = a_A = 53.3 \text{ m/s}^2$$

$$\text{vector } a'x = a^r_{BA} = 7.2 \text{ m/s}^2$$

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$$\text{vector } p'y = a^r_{BP} = 9.4 \text{ m/s}^2$$

$$\text{vector } c'z = a^r_{DC} = 2.2 \text{ m/s}^2$$

$$a_D = \text{vector } o'd' = 69.6 \text{ m/s}^2$$ **Ans.**

Angular acceleration of CD

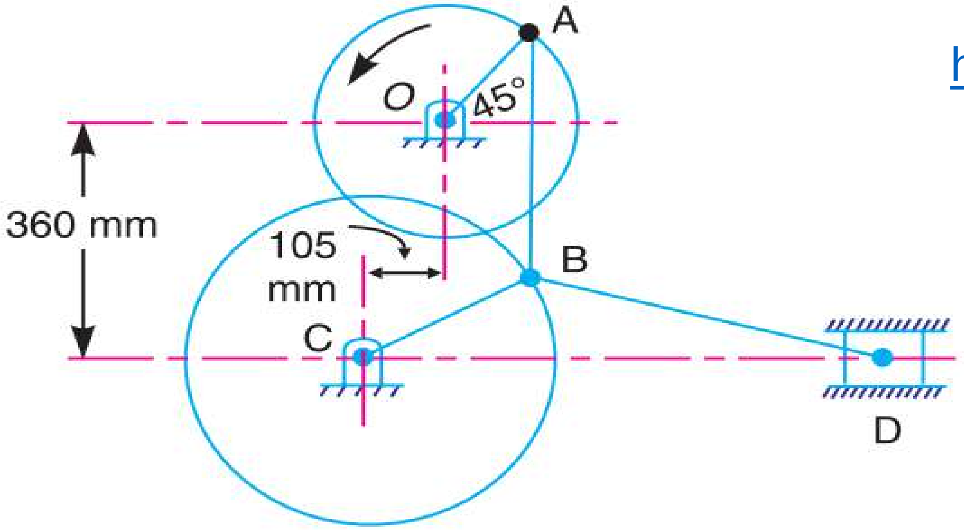
From acceleration diagram, we find that tangential component of the acceleration of *D* with respect to *C*,

$$a^t_{DC} = \text{vector } zd' = 17.4 \text{ m/s}^2 \qquad \text{...(By measurement)}$$

We know that angular acceleration of *CD*,

$$\alpha_{CD} = \frac{a^t_{DC}}{CD} = \frac{17.4}{0.66} = 26.3 \text{ rad/s}^2 \text{ (Anticlockwise)}$$ **Ans.**

Example 8.8. *In the toggle mechanism shown in Fig. 8.16, the slider D is constrained to move on a horizontal path. The crank OA is rotating in the counter-clockwise direction at a speed*



https://youtu.be/VIHXs61WzQQ

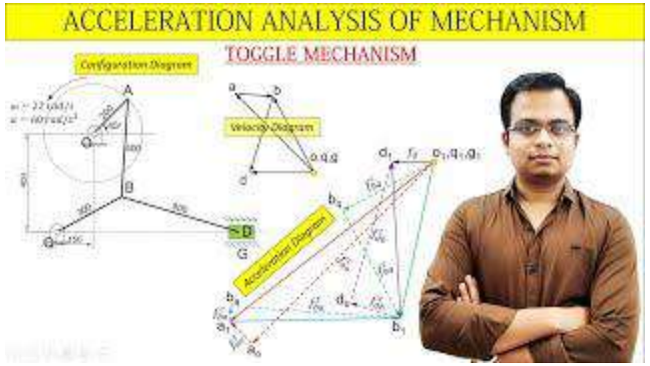


Fig. 8.16

of 180 r.p.m. increasing at the rate of 50 rad/s². The dimensions of the various links are as follows:

OA = 180 mm ; CB = 240 mm ; AB = 360 mm ; and BD = 540 mm.

For the given configuration, find **1.** *Velocity of slider D and angular velocity of BD, and* **2.** *Acceleration of slider D and angular acceleration of BD.*

Solution. Given : $N_{AO} = 180$ r.p.m. or $\omega_{AO} = 2\pi \times 180/60 = 18.85$ rad/s ; $OA = 180$ mm $= 0.18$ m ; $CB = 240$ mm $= 0.24$ m ; $AB = 360$ mm $= 0.36$ m ; $BD = 540$ mm $= 0.54$ m

We know that velocity of A with respect to O or velocity of A,

$$v_{AO} = v_A = \omega_{AO} \times OA = 18.85 \times 0.18 = 3.4 \text{ m/s}$$

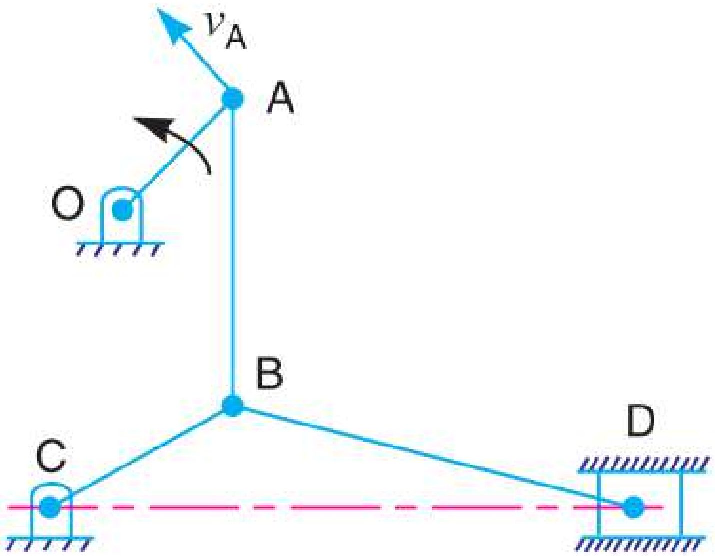
...(Perpendicular to OA)

1. ***Velocity of slider D and angular velocity of BD***

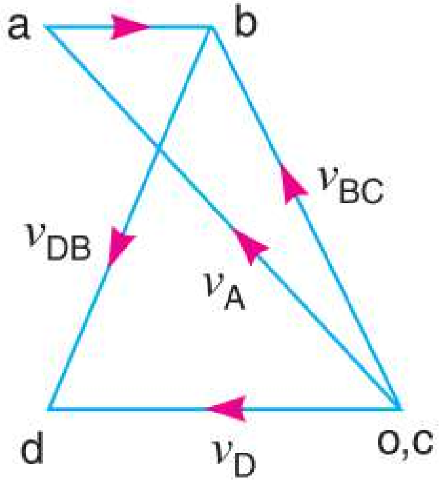
First of all, draw the space diagram to some suitable scale, as shown in Fig. 8.17 (*a*). Now the velocity diagram, as shown in Fig. 8.17 (*b*), is drawn as discussed below:

1. Since *O* and *C* are fixed points, therefore these points lie at one place in the velocity diagram. Draw vector *oa* perpendicular to *OA*, to some suitable scale, to represent the velocity of *A* with respect to *O* or velocity of *A* *i.e.* v_{AO} or v_A, such that

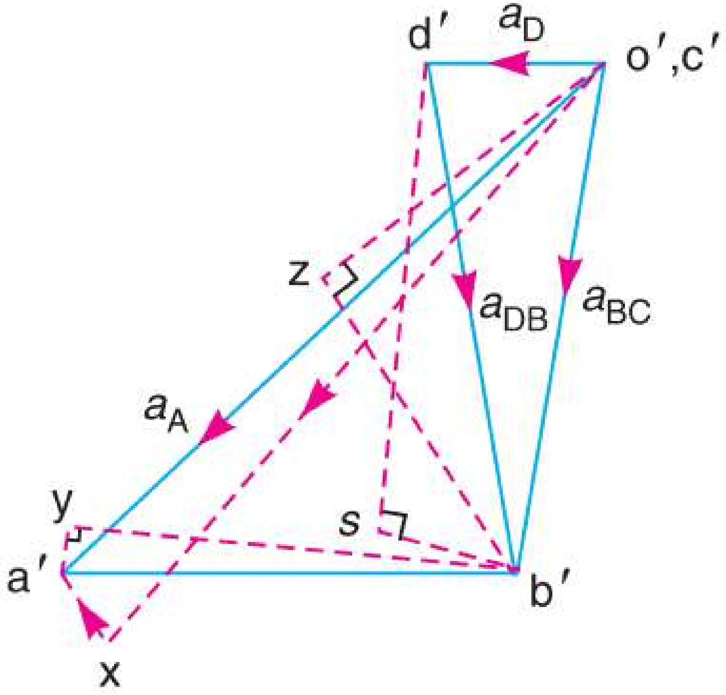
$$\text{vector } oa = v_{AO} = v_A = 3.4 \text{ m/s}$$



(*a*) Space diagram.



(*b*) Velocity diagram.



(*c*) Acceleration diagram.

Fig. 8.17

2. Since B moves with respect to A and also with respect to C, therefore draw vector ab perpendicular to AB to represent the velocity of B with respect to A *i.e.* v_{BA}, and draw vector cb perpendicular to CB to represent the velocity of B with respect to C *ie.* v_{BC}. The vectors ab and cb intersect at b.

3. From point b, draw vector bd perpendicular to BD to represent the velocity of D with respect to B *i.e.* v_{DB}, and from point c draw vector cd parallel to CD (*i.e.*, in the direction of motion of the slider D) to represent the velocity of D *i.e.* v_D.

By measurement, we find that velocity of B with respect to A,

$$v_{BA} = \text{vector } ab = 0.9 \text{ m/s}$$

Velocity of B with respect to C,

$$v_{BC} = \text{vector } cb = 2.8 \text{ m/s}$$

Velocity of D with respect to B,

$$v_{DB} = \text{vector } bd = 2.4 \text{ m/s}$$

and velocity of slider D, $v_D = \text{vector } cd = 2.05$ m/s **Ans.**

Angular velocity of BD

We know that the angular velocity of BD,

$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{2.4}{0.54} = 4.5 \text{ rad/s}$$ **Ans.**

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2. Acceleration of slider D and angular acceleration of BD

Since the angular acceleration of OA increases at the rate of 50 rad/s^2, *i.e.* $\alpha_{AO} = 50$ rad/s^2, therefore

Tangential component of the acceleration of A with respect to O,

$$a^t_{AO} = \alpha_{AO} \times OA = 50 \times 0.18 = 9 \text{ m/s}^2$$

Radial component of the acceleration of A with respect to O,

$$a^r_{AO} = \frac{v^2_{AO}}{OA} = \frac{(3.4)^2}{0.18} = 63.9 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to A,

$$a^r_{BA} = \frac{v^2_{BA}}{AB} = \frac{(0.9)^2}{0.36} = 2.25 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to C,

$$a^r_{BC} = \frac{v^2_{BC}}{CB} = \frac{(2.8)^2}{0.24} = 32.5 \text{ m/s}^2$$

and radial component of the acceleration of D with respect to B,

$$a^r_{DB} = \frac{v^2_{DB}}{BD} = \frac{(2.4)^2}{0.54} = 10.8 \text{ m/s}^2$$

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$$\text{vector } o'x = a^r_{AO} = 63.9 \text{ m/s}^2$$

$$\text{vector } xa' = a^t_{AO} = 9 \text{ m/s}^2$$

$$\text{vector } a'y = a^r_{BA} = 2.25 \text{ m/s}^2$$

$$\text{vector } c'z = a^r_{BC} = 32.5 \text{ m/s}^2$$

$$\text{vector } b's = a^r_{DB} = 10.8 \text{ m/s}^2$$

a_D = vector $c'd'$ = 13.3 m/s^2 **Ans.**

8.5. Coriolis Component of Acceleration

When a point on one link is sliding along another rotating link, such as in quick return motion mechanism, then the coriolis component of the acceleration must be calculated.

Consider a link *OA* and a slider *B* as shown in Fig. 8.26 (*a*). The slider *B* moves along the link *OA*. The point *C* is the coincident point on the link *OA*.

Let ω = Angular velocity of the link *OA* at time *t* seconds.

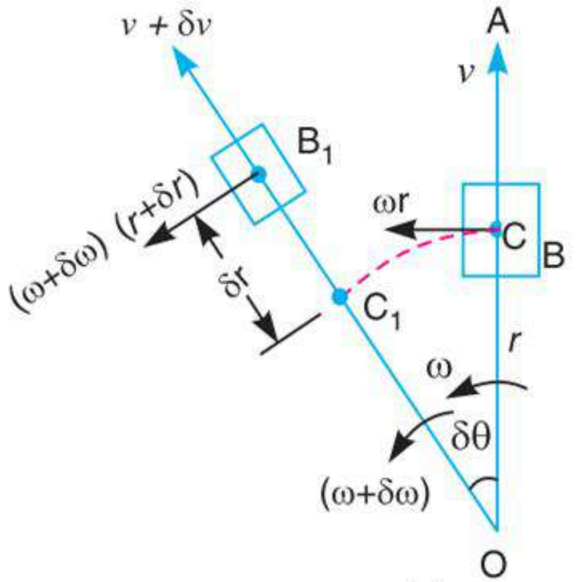
v = Velocity of the slider *B* along the link *OA* at time *t* seconds.

$\omega.r$ = Velocity of the slider *B* with respect to *O* (perpendicular to the link *OA*) at time *t* seconds, and

$(\omega + \delta\omega)$, $(v + \delta v)$ and $(\omega + \delta\omega)\,(r + \delta r)$

= Corresponding values at time $(t + \delta t)$ seconds.

Let us now find out the acceleration of the slider *B* with respect to *O* and with respect to its coincident point *C* lying on the link *OA*.



(*a*)

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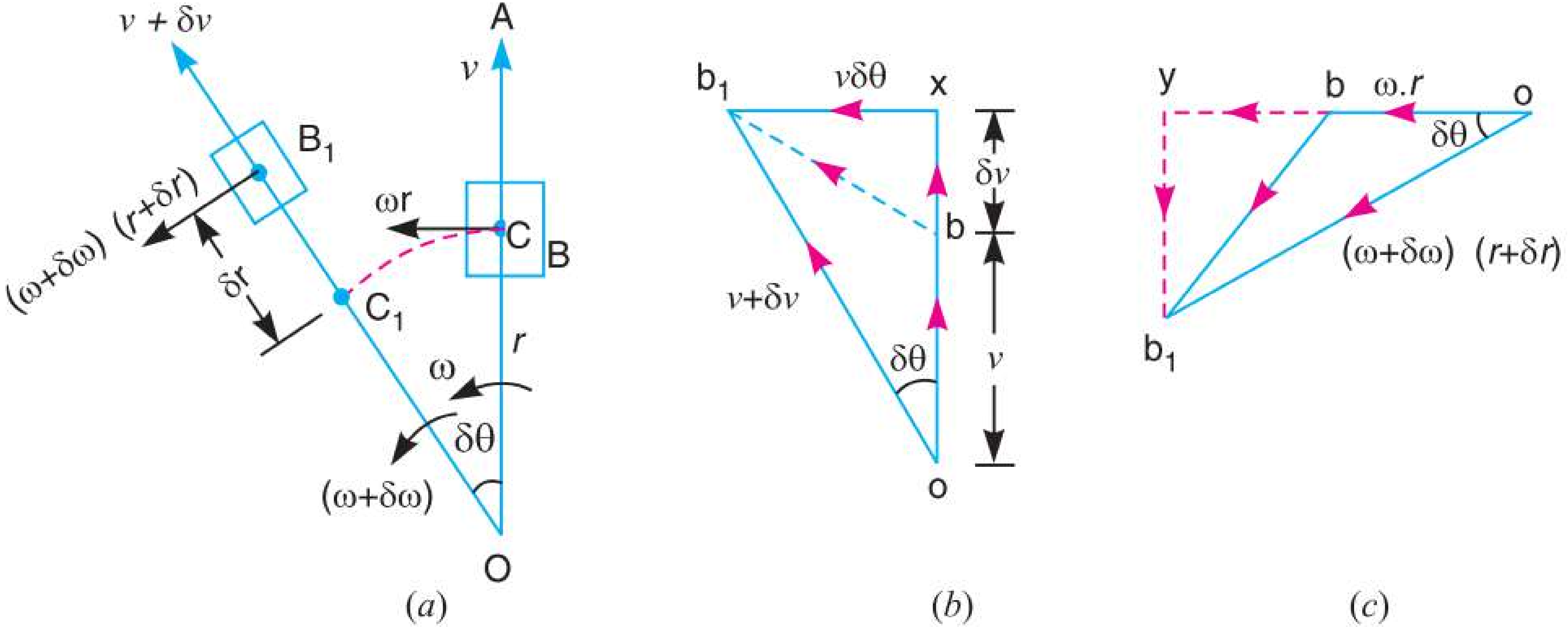
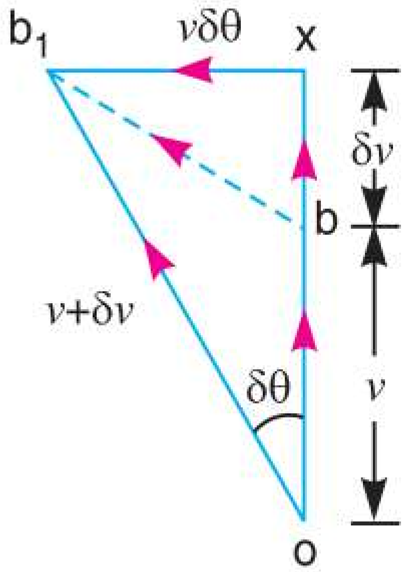


Fig. 8.26 (b) shows the velocity diagram when their velocities v and $(v + \delta v)$ are considered. In this diagram, the vector bb_1 represents the change in velocity in time δt sec ; the vector bx represents the component of change of velocity bb_1 along OA (*i.e.* along radial direction) and vector xb_1 represents the component of change of velocity bb_1 in a direction perpendicular to OA (*i.e.* in tangential direction). Therefore

$$bx = ox - ob = (v + \delta v) \cos \delta\theta - v \uparrow$$



Since δθ is very small, therefore substituting $\cos \delta\theta = 1$, we have

$$bx = (v + \delta v - v) \uparrow = \delta v \uparrow$$

...(Acting radially outwards)

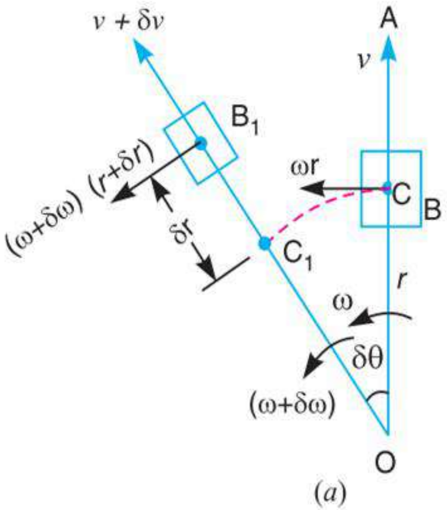
and $$xb_1 = (v + \delta v) \sin \delta\theta$$

Since δθ is very small, therefore substituting $\sin \delta\theta = \delta\theta$, we have

$$xb_1 = (v + \delta v)\, \delta\theta = v.\delta\theta + \delta v.\delta\theta$$

Neglecting $\delta v.\delta\theta$ being very small, therefore

$$xb_1 = v.\overleftarrow{\delta\theta}$$



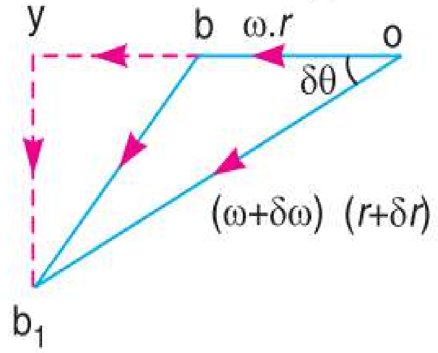


Fig. 8.26 (*c*) shows the velocity diagram when the velocities $\omega.r$ and $(\omega + \delta\omega)(r + \delta r)$ are considered. In this diagram, vector bb_1 represents the change in velocity ; vector yb_1 represents the component of change of velocity bb_1 along OA (*i.e.* along radial direction) and vector by represents the component of change of velocity bb_1 in a direction perpendicular to OA (*i.e.* in a tangential direction). Therefore

$$yb_1 = (\omega + \delta\omega)(r + \delta r) \sin \delta\theta \downarrow$$
$$= (\omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r) \sin \delta\theta$$

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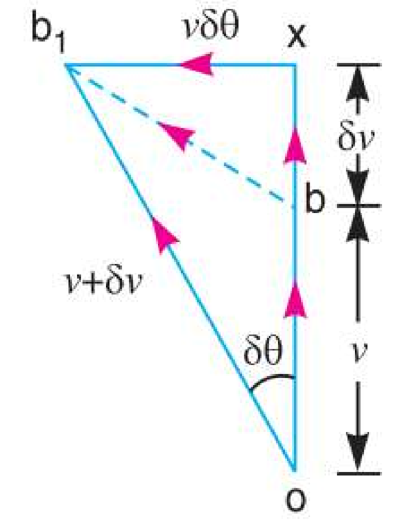
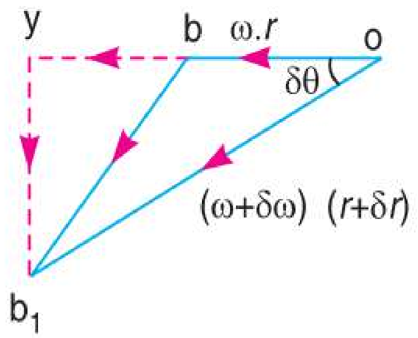




Since δθ is very small, therefore substituting sin δθ = δθ in the above expression, we have

$$yb_1 = \omega.r.\delta\theta + \omega.\delta r.\delta\theta + \delta\omega.r.\delta\theta + \delta\omega.\delta r.\delta\theta$$

$$= \omega.r.\delta\theta \downarrow \text{, acting radially inwards}$$ ...(Neglecting all other quantities)

$$by = oy - ob = (\omega + \delta\omega)(r + \delta r)\cos\delta\theta - \omega.r$$

$$= (\omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r)\cos\delta\theta - \omega.r$$

Since δθ is small, therefore substituting cos δθ = 1, we have

$$by = \omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r - \omega.r = \omega.\delta r + r.\delta\omega$$ ...(Neglecting δω.δr)

...(Perpendicular to *OA* and towards left)

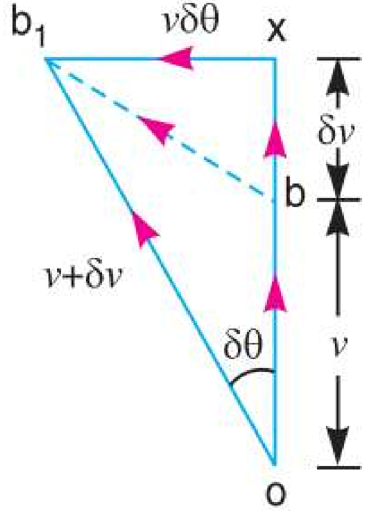
Therefore, total component of change of velocity along radial direction

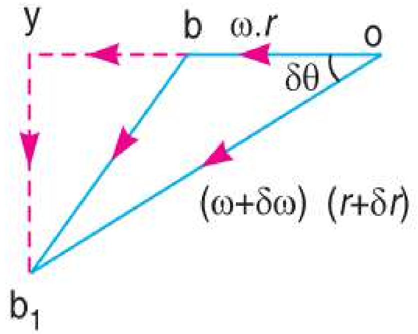
$$= bx - yb_1 = (\delta v - \omega.r.\delta\theta) \uparrow$$ ...(Acting radially outwards from *O* to *A*)

∴ Radial component of the acceleration of the slider *B* with respect to *O* on the link *OA*, acting radially outwards from *O* to *A*,

$$a^r_{BO} = \text{Lt}\,\frac{\delta v - \omega.r.\delta\theta}{\delta t} = \frac{dv}{dt} - \omega.r \times \frac{d\theta}{dt} = \frac{dv}{dt} - \omega^2.r \uparrow$$ ...(*i*)

...(∵ $d\theta/dt = \omega$)





Also, the total component of change of velocity along tangential direction,

$$= xb_1 + by = v.\overleftarrow{\delta}\,\theta + (\omega.\delta r \overleftarrow{+} r.\delta\omega)$$

...(Perpendicular to OA and towards left)

$\therefore$ Tangential component of acceleration of the slider B with respect to O on the link OA, acting perpendicular to OA and towards left,

$$a_{BO}^{t} = \text{Lt}\ \frac{v.\delta\theta + (\omega.\delta r + r.\delta\omega)}{\delta t} = v\ \frac{d\theta}{dt} + \omega\ \frac{dr}{dt} + r\ \frac{d\omega}{dt}$$

$$= v.\omega + \omega.v + r.\alpha = (2\,v.\omega \overleftarrow{+} r.\alpha) \qquad \text{...(ii)}$$

...($\because$ $dr/dt = v$, and $d\omega/dt = \alpha$)

Now radial component of acceleration of the coincident point C with respect to O, acting in a direction from C to O,

$$a_{CO}^{r} = \omega^2.r \uparrow \qquad \text{...(iii)}$$

and tangential component of acceleraiton of the coincident point C with respect to O, acting in a direction perpendicular to CO and towards left,

$$a_{CO}^{t} = \overleftarrow{\alpha}.r \uparrow \qquad \text{...(iv)}$$

Radial component of the slider B with respect to the coincident point C on the link OA, acting radially outwards,

$$a_{BC}^{r} = a_{BO}^{r} - a_{CO}^{r} = \left(\frac{dv}{dt} - \omega^2.r\right) - \left(-\ \omega^2.r\right) = \frac{dv}{dt} \uparrow$$

and tangential component of the slider B with respect to the coincident point C on the link OA acting in a direction perpendicular to OA and towards left,

$$a_{BC}^{t} = a_{BO}^{t} - a_{CO}^{t} = (2\,\omega.v + \alpha.r) - \alpha.r = 2\,\overleftarrow{\omega}.v$$

This tangential component of acceleration of the slider B with respect to the coincident point C on the link is known as ***coriolis component of acceleration*** and is always perpendicualr to the link.

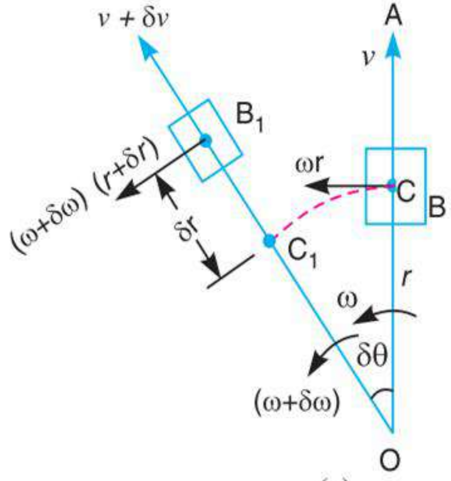
∴ Coriolis component of the acceleration of B with respect of C,

$$a^c_{BC} = a^t_{BC} = 2\,\omega.v$$

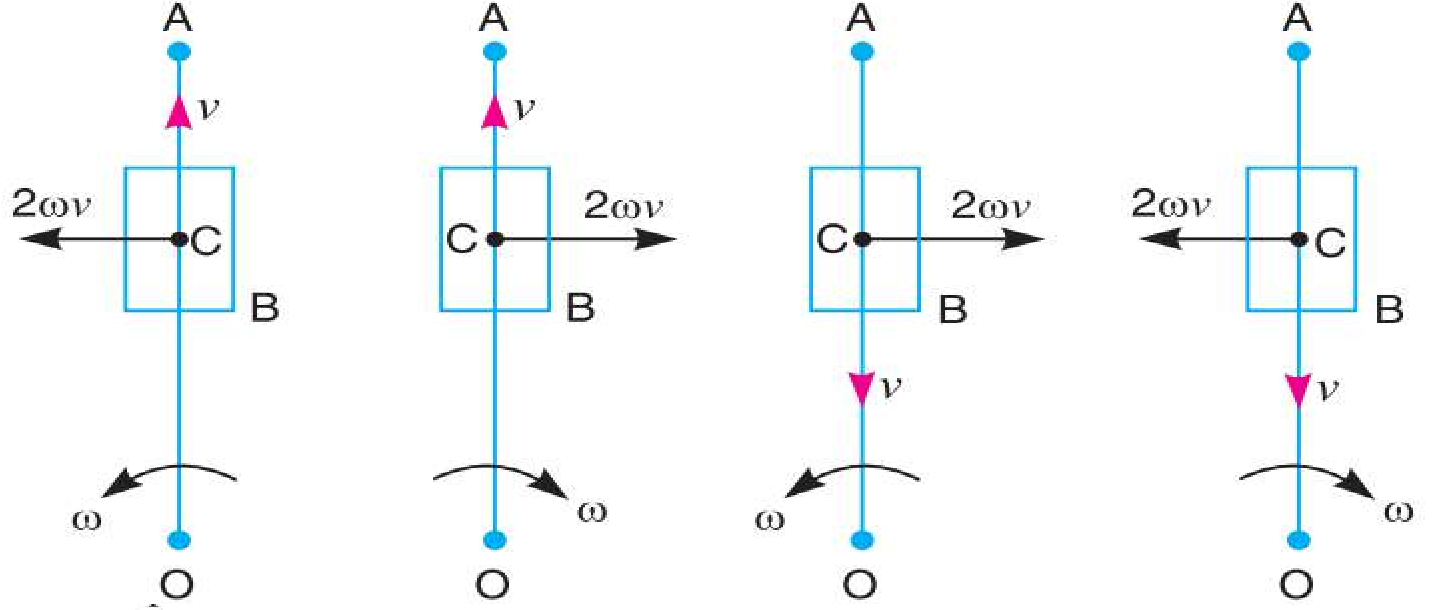
where ω = Angular velocity of the link OA, and

v = Velocity of slider B with respect to coincident point C.

In the above discussion, the anticlockwise direction for ω and the radially outward direction for v are taken as ***positive.*** It may be noted that the direction of coriolis component of acceleration changes sign, if either ω or v is reversed in direction. But the direction of coriolis component of acceleration will not be changed in sign if both ω and v are reversed in direction. It is concluded that the direction of coriolis component of acceleration is obtained by rotating v, at 90°, about its origin in the same direction as that of ω.



(a)



The direction of coriolis component of acceleration ($2\,\omega.v$) for all four possible cases, is shown in Fig. 8.27. The directions of ω and v are given.

Example 8.13. *A mechanism of a crank and slotted lever quick return motion is shown in Fig. 8.28. If the crank rotates counter clockwise at 120 r.p.m., determine for the configuration shown, the velocity and acceleration of the ram D. Also determine the angular acceleration of the slotted lever.*

Crank, AB = 150 mm ; Slotted arm, OC = 700 mm and link CD = 200 mm.

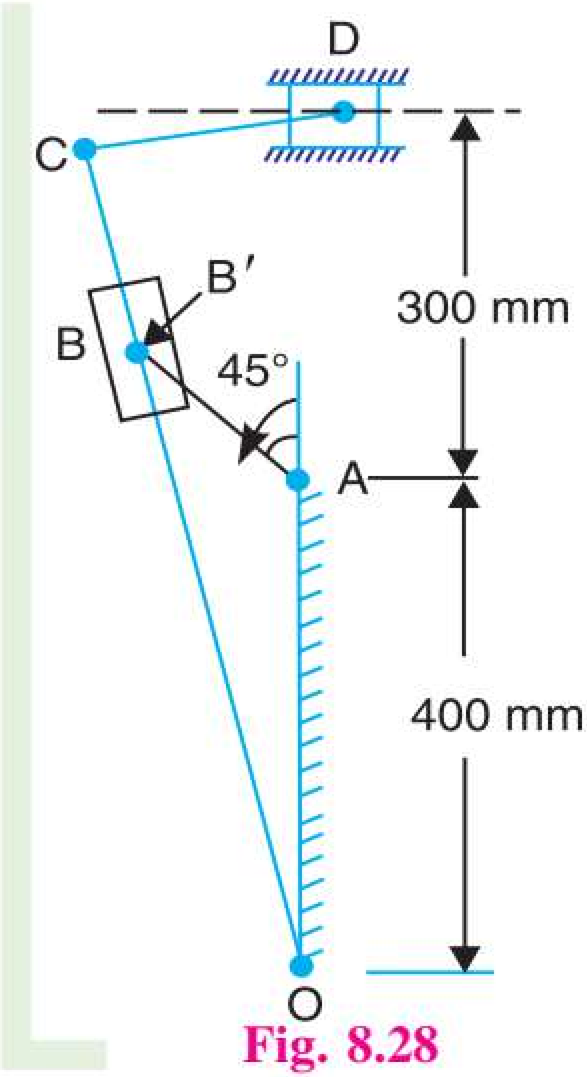


Fig. 8.28

Solution. Given : N_{BA} = 120 r.p.m or $\omega_{BA} = 2\pi \times 120/60$ = 12.57 rad/s ; AB = 150 mm = 0.15 m; OC = 700 mm = 0.7 m; CD = 200 mm = 0.2 m

We know that velocity of B with respect to A,

$$v_{BA} = \omega_{BA} \times AB$$

$$= 12.57 \times 0.15 = 1.9 \text{ m/s}$$

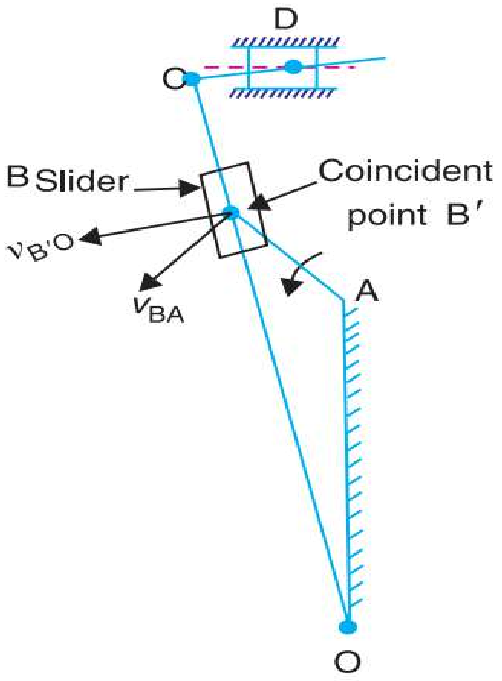
...(Perpendicular to AB)

Velocity of the ram D

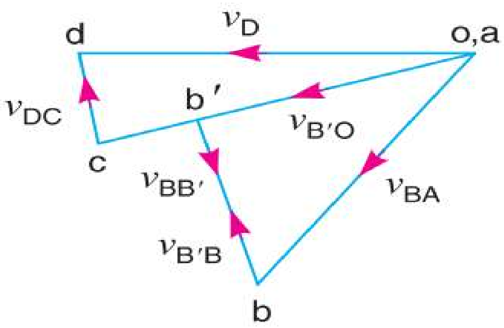
First of all draw the space diagram, to some suitable scale, as shown in Fig. 8.29 (*a*). Now the velocity diagram, as shown in Fig. 8.29

vector $ab = v_{BA} = 1.9$ m/s

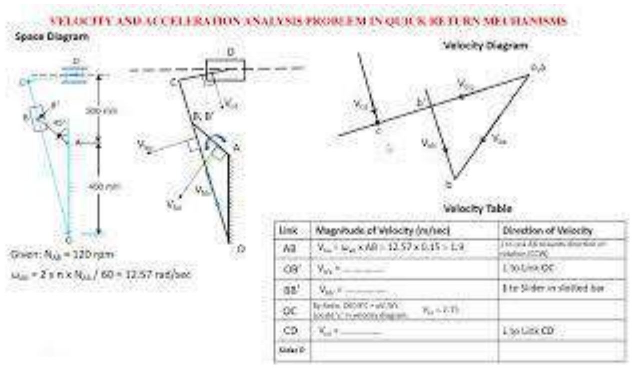
https://youtu.be/9rzaj69hP3M

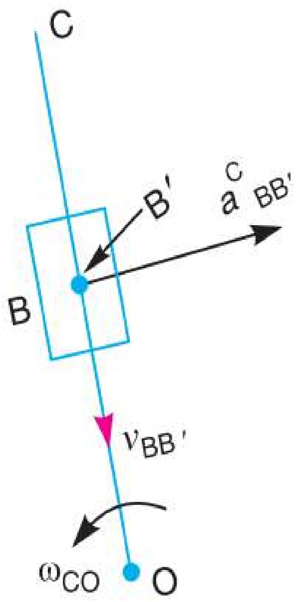


(*a*) Space diagram.

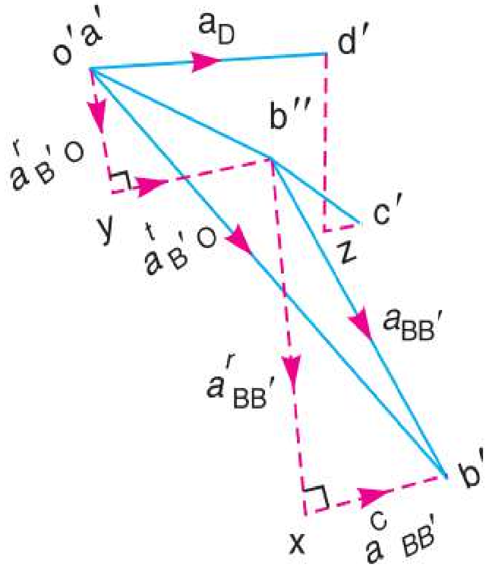


(*b*) Velocity diagram.





(*c*) Direction of coriolis component.



(*d*) Acceleration diagram.

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$$ob' / oc = OB' / OC$$

By measurement, we find that velocity of the ram D,

$$v_D = \text{vector } od = 2.15 \text{ m/s} \quad \textbf{Ans.}$$

From velocity diagram, we also find that

Velocity of B with respect to B',

$$v_{BB'} = \text{vector } b'b = 1.05 \text{ m/s}$$

Velocity of D with respect to C,

$$v_{DC} = \text{vector } cd = 0.45 \text{ m/s}$$

Velocity of B' with respect to O

$$v_{B'O} = \text{vector } ob' = 1.55 \text{ m/s}$$

Velocity of C with respect to O,

$$v_{CO} = \text{vector } oc = 2.15 \text{ m/s}$$

$\therefore$ Angular velocity of the link OC or OB',

$$\omega_{CO} = \omega_{B'O} = \frac{v_{CO}}{OC} = \frac{2.15}{0.7} = 3.07 \text{ rad/s (Anticlockwise)}$$

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Acceleration of the ram D

We know that radial component of the acceleration of B with respect to A,

$$a_{BA}^r = \omega_{BA}^2 \times AB = (12.57)^2 \times 0.15 = 23.7 \text{ m/s}^2$$

Coriolis component of the acceleration of slider B with respect to the coincident point B',

$$a_{BB'}^c = 2\omega.v = 2\omega_{CO}.v_{BB'} = 2 \times 3.07 \times 1.05 = 6.45 \text{ m/s}^2$$

...($\because$ $\omega = \omega_{CO}$ and $v = v_{BB'}$)

Radial component of the acceleration of D with respect to C,

$$a_{DC}^r = \frac{v_{DC}^2}{CD} = \frac{(0.45)^2}{0.2} = 1.01 \text{ m/s}^2$$

Radial component of the acceleration of the coincident point B' with respect to O,

$$a_{B'O}^r = \frac{v_{B'O}^2}{B'O} = \frac{(1.55)^2}{0.52} = 4.62 \text{ m/s}^2$$ ...(By measurement $B'O = 0.52$ m)

$$\text{vector } a'b' = a_{BA}^r = a_B = 23.7 \text{ m/s}^2$$

(*i*) Coriolis component of the acceleration of B with respect to B' *i.e.* $a^c_{BB'}$, and

(*ii*) Radial component of the acceleration of B with respect to B' *i.e.* $a^r_{BB'}$.

These two components are mutually perpendicular. Therefore from point b' draw vector $b'x$ perpendicular to $B'O$ *i.e.* in a direction as shown in Fig. 8.29 (*c*) to represent $a^c_{BB'} = 6.45 \text{ m/s}^2$. The

By measurement, we find that acceleration of the ram D,

$$a_D = \text{vector } o'd' = 8.4 \text{ m/s}^2 \textbf{ Ans.}$$

Angular acceleration of the slotted lever

By measurement from acceleration diagram, we find that tangential component of the coincident point B' with respect to O,

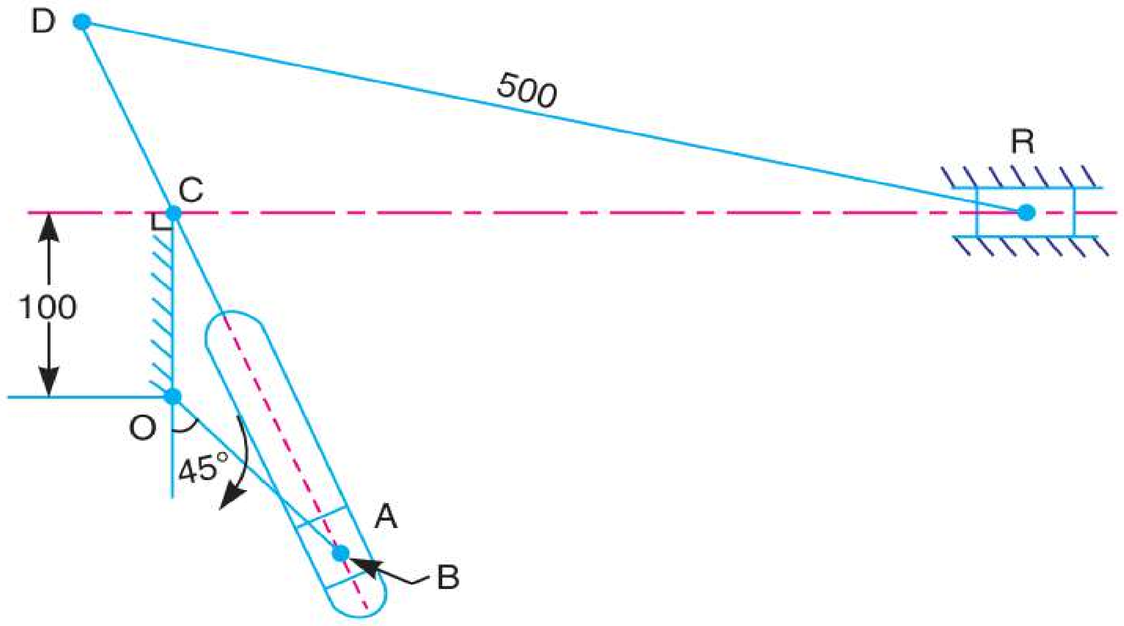
$$a^t_{B'O} = \text{vector } yb'' = 6.4 \text{ m/s}^2$$

We know that angular acceleration of the slotted lever,

$$= \frac{a^t_{B'O}}{OB'} = \frac{6.4}{0.52} = 12.3 \text{ rad/s}^2 \text{ (Anticlockwise) } \textbf{Ans.}$$

Example 8.15. *In a Whitworth quick return motion, as shown in Fig. 8.32. OA is a crank rotating at 30 r.p.m. in a clockwise direction. The dimensions of various links are : OA = 150 mm; OC = 100 mm; CD = 125 mm; and DR = 500 mm.*

Determine the acceleraion of the sliding block R and the angular acceleration of the slotted lever CA.



All dimensions in mm.

Fig. 8.32

Solution. Given : N_{AO} = 30 r.p.m. or $\omega_{AO} = 2\pi \times 30/60 = 3.142$ rad/s ; OA = 150 mm = 0.15 m; OC = 100 mm = 0.1 m ; CD = 125 mm = 0.125 m ; DR = 500 mm = 0.5 m

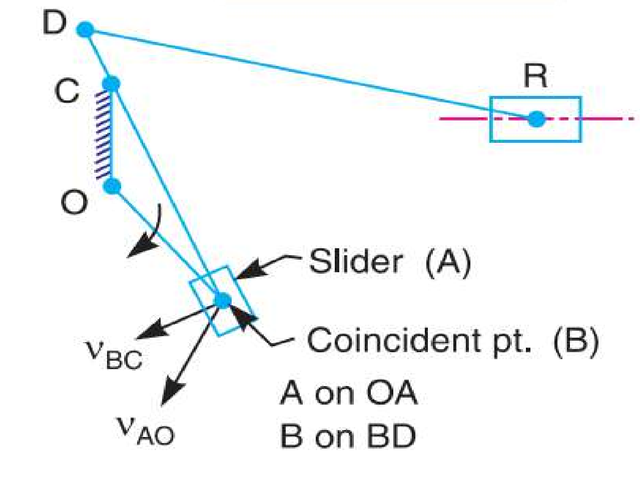
We know that velocity of A with respect to O or velocity of A,

$$v_{AO} = v_A = \omega_{AO} \times OA = 3.142 \times 0.15 = 0.47 \text{ m/s}$$

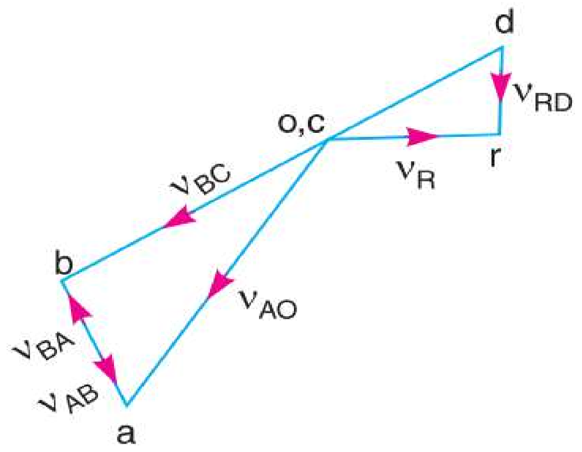
...(Perpendicular to OA)

$$\text{vector } oa = v_{AO} = v_A = 0.47 \text{ m/s}$$

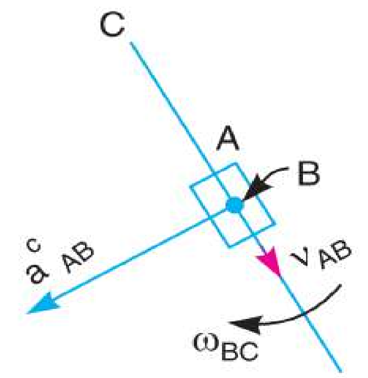
$$bd/bc = BD/BC$$



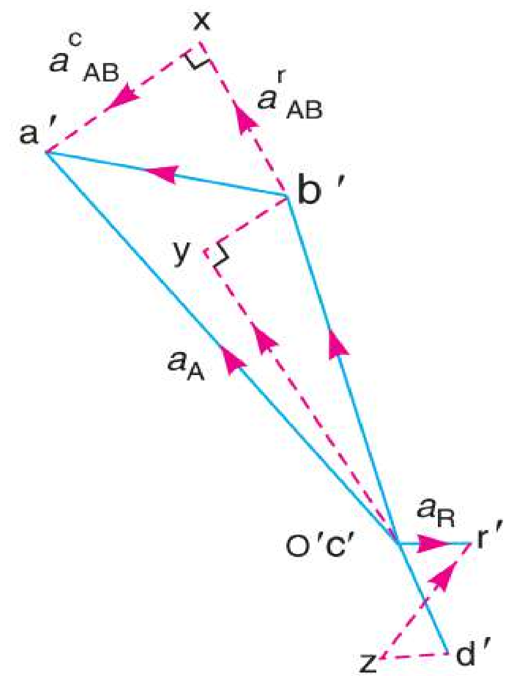
(*a*) Space diagram.



(*b*) Velocity diagram.



(*c*) Direction of coriolis component.



(*d*) Acceleration diagram.

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By measurement, we find that velocity of B with respect to C,

$$v_{BC} = \text{vector } cb = 0.46 \text{ m/s}$$

Velocity of A with respect to B,

$$v_{AB} = \text{vector } ba = 0.15 \text{ m/s}$$

and velocity of R with respect to D,

$$v_{RD} = \text{vector } dr = 0.12 \text{ m/s}$$

We know that angular velocity of the link BC,

$$\omega_{BC} = \frac{v_{BC}}{CB} = \frac{0.46}{0.24} = 1.92 \text{ rad/s} \quad \text{(Clockwise)}$$

...(By measurement, $CB = 0.24$ m)

Acceleration of the sliding block R

We know that the radial component of the acceleration of A with respect to O,

$$a^r_{AO} = \frac{v^2_{AO}}{OA} = \frac{(0.47)^2}{0.15} = 1.47 \text{ m/s}^2$$

Coriolis component of the acceleration of slider A with respect to coincident point B,

$$a^c_{AB} = 2\,\omega_{BC} \times v_{AB} = 2 \times 1.92 \times 0.15 = 0.576 \text{ m/s}^2$$

Radial component of the acceleration of *B* with respect to *C*,

$$a_{BC}^{r} = \frac{v_{BC}^2}{CB} = \frac{(0.46)^2}{0.24} = 0.88 \text{ m/s}^2$$

Radial component of the acceleration of *R* with respect to *D*,

$$a_{RD}^{r} = \frac{v_{RD}^2}{DR} = \frac{(0.12)^2}{0.5} = 0.029 \text{ m/s}^2$$

$$\text{vector } o'a' = a_{AO}^{r} = a_A = 1.47 \text{ m/s}^2$$

$$b'd'/b'c' = BD/BC. \quad a_{RD}^{r} = 0.029 \text{ m/s}^2$$

$$a_R = \text{vector } c'r' = 0.18 \text{ m/s}^2$$ **Ans.**

Angular acceleration of the slotted lever CA

By measurement from acceleration diagram, we find that tangential component of *B* with respect to *C*,

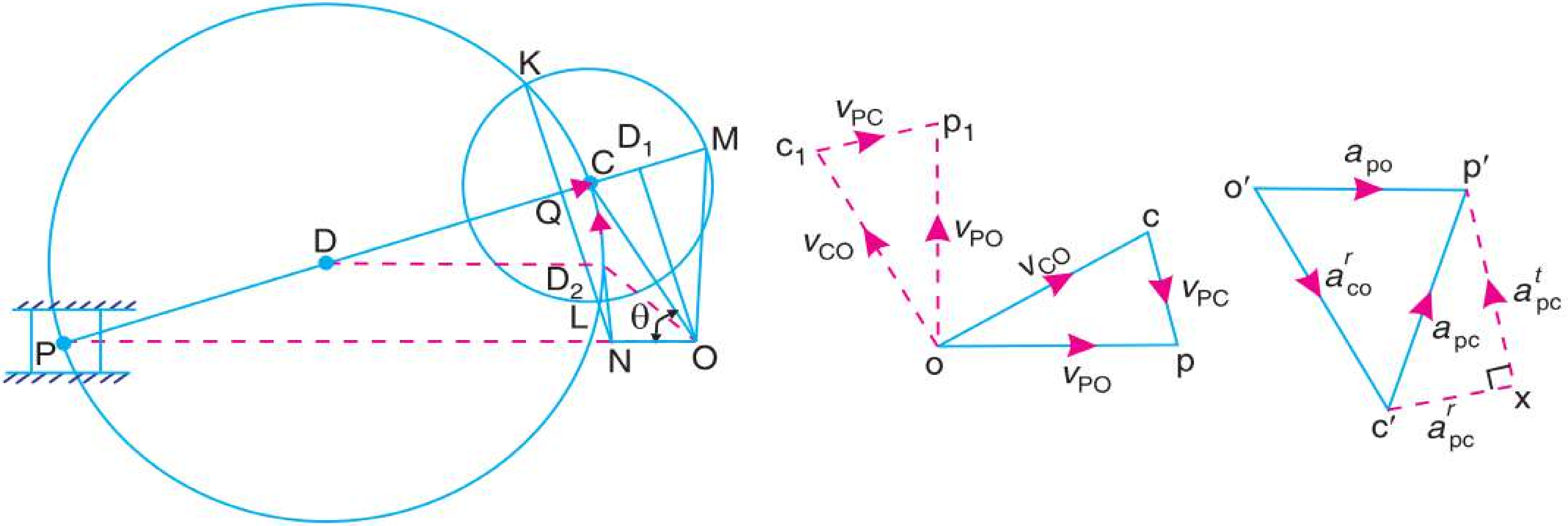
$$a_{BC}^{t} = \text{vector } yb' = 0.14 \text{ m/s}^2$$

We know that angular acceleration of the slotted lever *CA*,

$$\alpha_{CA} = \alpha_{BC} = \frac{a_{CB}^{t}}{BC} = \frac{0.14}{0.24} = 0.583 \text{ rad/s}^2 \text{ (Anticlockwise)}$$ **Ans.**

Klien's Construction

Let OC be the crank and PC the connecting rod of a reciprocating steam engine, as shown in Fig. 15.2 (*a*). Let the crank makes an angle θ with the line of stroke PO and rotates with uniform angular velocity ω rad/s in a clockwise direction. The Klien's velocity and acceleration diagrams are drawn as discussed below:

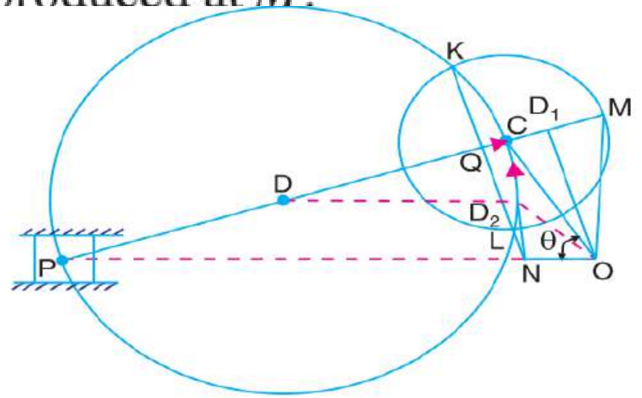


(*a*) Klien's acceleration diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig. 15.2. Klien's construction.

Klien's velocity diagram

First of all, draw OM perpendicular to OP; such that it intersects the line PC produced at M. The triangle OCM is known as ***Klien's velocity diagram***. In this triangle OCM,



OM may be regarded as a line perpendicular to PO,

CM may be regarded as a line parallel to PC, and ...(∵ It i

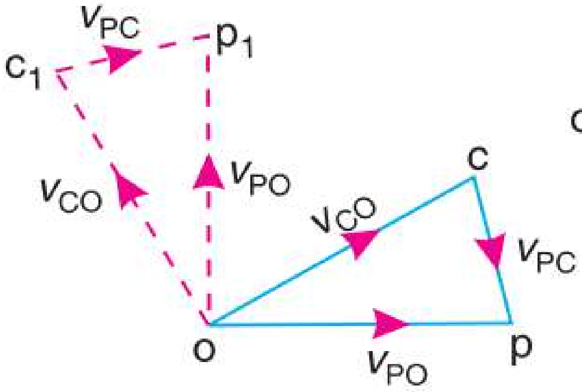
CO may be regarded as a line parallel to CO.

We have already discussed that the velocity diagram for given configuration is a triangle *ocp* as shown in Fig. 15.2 (*b*). If this triangle is revolved through 90°, it will be a triangle $oc_1\,p_1$, in which

oc_1 represents v_{CO} (*i.e.* velocity of C with respect to O or velocity of crank pin C) and is paralel to OC,

op_1 represents v_{PO} (*i.e.* velocity of P with respect to O or velocity of cross-head or piston P) and is perpendicular to OP, and

c_1p_1 represents v_{PC} (*i.e.* velocity of P with respect to C) and is parallel to CP.



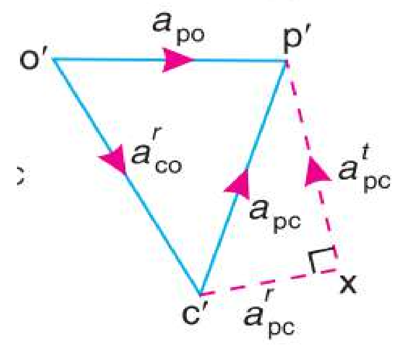
A little consideration will show, that the triangles oc_1p_1 and OCM are similar. Therefore,

$$\frac{oc_1}{OC} = \frac{op_1}{OM} = \frac{c_1p_1}{CM} = \omega \text{ (a constant)}$$

or $$\frac{v_{CO}}{OC} = \frac{v_{PO}}{OM} = \frac{v_{PC}}{CM} = \omega$$

$\therefore$ $$v_{CO} = \omega \times OC \, ; \, v_{PO} = \omega \times OM, \text{ and } v_{PC} = \omega \times CM$$

Thus, we see that by drawing the Klien's velocity diagram, the velocities of various points may be obtained without drawing a separate velocity diagram.



Klien's acceleration diagram

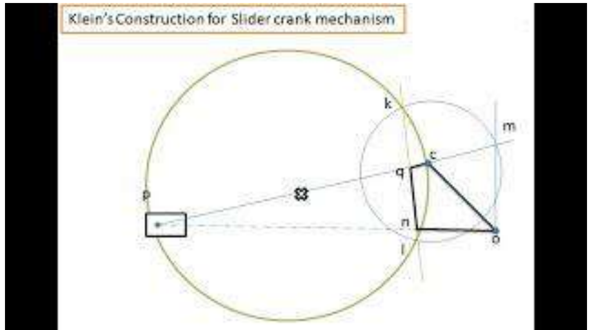
The Klien's acceleration diagram is drawn as discussed below:

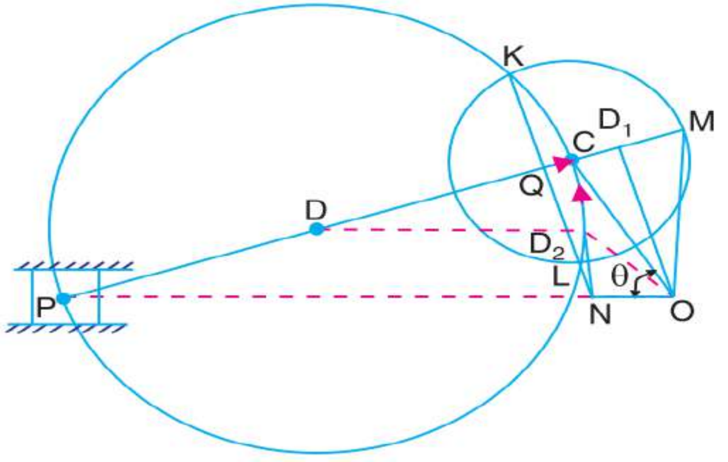
1. First of all, draw a circle with C as centre and CM as radius.

2. Draw another circle with PC as diameter. Let this circle intersect the previous circle at K and L.

3. Join KL and produce it to intersect PO at N. Let KL intersect PC at Q. This forms the quadrilateral CQNO, which is known as **Klien's acceleration diagram.** https://youtu.be/GpxZ1SdutvE

We have already discussed that the acceleration diagram for the given configuration is as shown in Fig. 15. 2 (c). We know that





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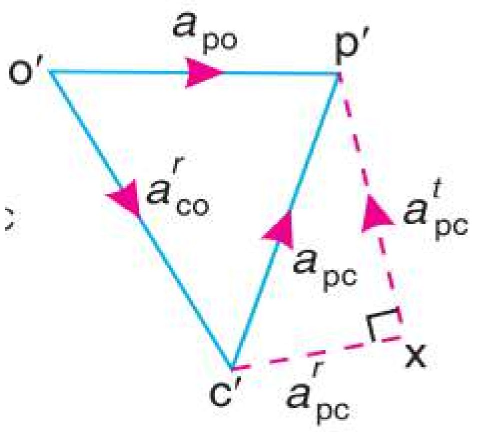
(*i*) $o'c'$ represents a^r_{CO} (*i.e.* radial component of the acceleration of crank pin *C* with respect to *O*) and is parallel to *CO*;

(*ii*) $c'x$ represents a^r_{PC} (*i.e.* radial component of the acceleration of crosshead or piston *P* with respect to crank pin *C*) and is parallel to *CP* or *CQ*;

(*iii*) xp' represents a^t_{PC} (*i.e.* tangential component of the acceleration of *P* with respect to *C*) and is parallel to *QN* (because *QN* is perpendicular to *CQ*); and

(*iv*) $o'p'$ represents a_{PO} (*i.e.* acceleration of *P* with respect to *O* or the acceleration of piston *P*) and is parallel to *PO* or *NO*.

A little consideration will show that the quadrilateral $o'c'x\,p'$ [Fig. 15.2 (*c*)] is similar to quadrilateral *CQNO* [Fig. 15.2 (*a*)]. Therefore,



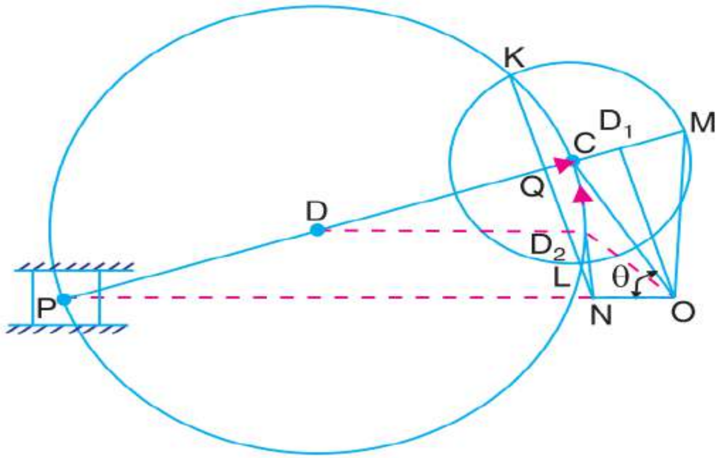
$$\frac{o'c'}{OC} = \frac{c'x}{CQ} = \frac{xp'}{QN} = \frac{o'p'}{NO} = \omega^2 \text{ (a constant)}$$

or

$$\frac{a^r_{CO}}{OC} = \frac{a^r_{PC}}{CQ} = \frac{a^t_{PC}}{QN} = \frac{a_{PO}}{NO} = \omega^2$$

$$a^r_{CO} = \omega^2 \times OC;\ a^r_{PC} = \omega^2 \times CQ$$

$$a^t_{PC} = \omega^2 \times QN;\ \text{and } a_{PO} = \omega^2 \times NO$$



Thus we see that by drawing the Klien's acceleration diagram, the acceleration of various points may be obtained without drawing the separate acceleration diagram.

1. The direction of linear velocity of any point on a link with respect to another point on the same link is

(*a*) parallel to the link joining the points
(*b*) perpendicular to the link joining the points
(*c*) at 45° to the link joining the points
(*d*) none of these

2. The magnitude of linear velocity of a point B on a link AB relative to point A is

(*a*) ωAB
(*b*) $\omega (AB)^2$
(*c*) $\omega^2 . AB$
(*d*) $(\omega . AB)^2$

where ω = Angular velocity of the link AB.

3. The two links OA and OB are connected by a pin joint at O. If the link OA turns with angular velocity ω_1 rad/s in the clockwise direction and the link OB turns with angular velocity ω_2 rad/s in the anti-clockwise direction, then the rubbing velocity at the pin joint O is

(*a*) $\omega_1 . \omega_2 . r$
(*b*) $(\omega_1 - \omega_2)\, r$
(*c*) $(\omega_1 + \omega_2)\, r$
(*d*) $(\omega_1 - \omega_2)\, 2\, r$

where r = Radius of the pin at O.

4. In the above question, if both the links OA and OB turn in clockwise direction, then the rubbing velocity at the pin joint O is

(*a*) $\omega_1 . \omega_2 . r$
(*b*) $(\omega_1 - \omega_2)\, r$
(*c*) $(\omega_1 + \omega_2)\, r$
(*d*) $(\omega_1 - \omega_2)\, 2\, r$

1. (*b*)	**2.** (*a*)	**3.** (*c*)	**4.** (*b*)

5. In a four bar mechanism, as shown in Fig. 7.43, if a force F_A is acting at point A in the direction of its velocity v_A and a force F_B is transmitted to the joint B in the direction of its velocity v_B, then the ideal mechanical advantage is equal to

(*a*) $F_B . v_A$

(*b*) $F_A . v_B$

(*c*) $\frac{F_B}{v_B}$

(*d*) $\frac{F_B}{F_A}$

5. (*d*)

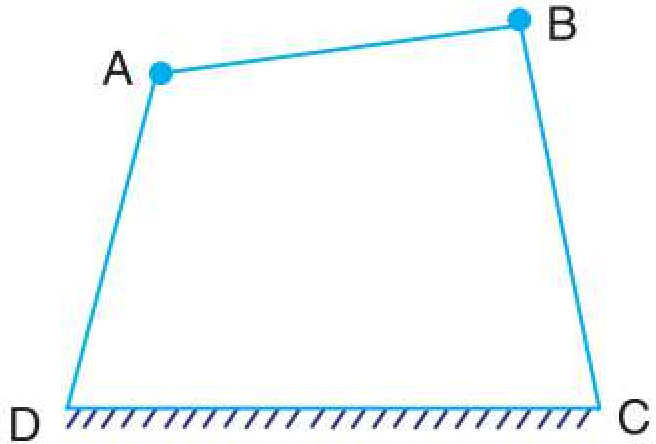


Fig. 7.43

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1. The component of the acceleration, parallel to the velocity of the particle, at the given instant is called

(*a*) radial component (*b*) tangential component

(*c*) coriolis component (*d*) none of these

2. A point B on a rigid link AB moves with respect to A with angular velocity ω rad/s. The radial component of the acceleration of B with respect to A,

(*a*) $v_{BA} \times AB$ (*b*) $v^2_{BA} \times AB$ (*c*) $\frac{v_{BA}}{AB}$ (*d*) $\frac{v^2_{BA}}{AB}$

where v_{BA} = Linear velocity of B with respect to $A = \omega \times AB$

3. A point B on a rigid link AB moves with respect to A with angular velocity ω rad/s. The angular acceleration of the link AB is

(*a*) $\frac{a^r_{BA}}{AB}$ (*b*) $\frac{a^t_{BA}}{AB}$ (*c*) $v_{BA} \times AB$ (*d*) $\frac{v^2_{BA}}{AB}$

4. A point B on a rigid link AB moves with respect to A with angular velocity ω rad/s. The total acceleration of B with respect to A will be equal to

(*a*) vector sum of radial component and coriolis component

(*b*) vector sum of tangential component and coriolis component

(*c*) vector sum of radial component and tangential component

(*d*) vector difference of radial component and tangential component

5. The coriolis component of acceleration is taken into account for

(*a*) slider crank mechanism (*b*) four bar chain mechanism

(*c*) quick return motion mechanism (*d*) none of these

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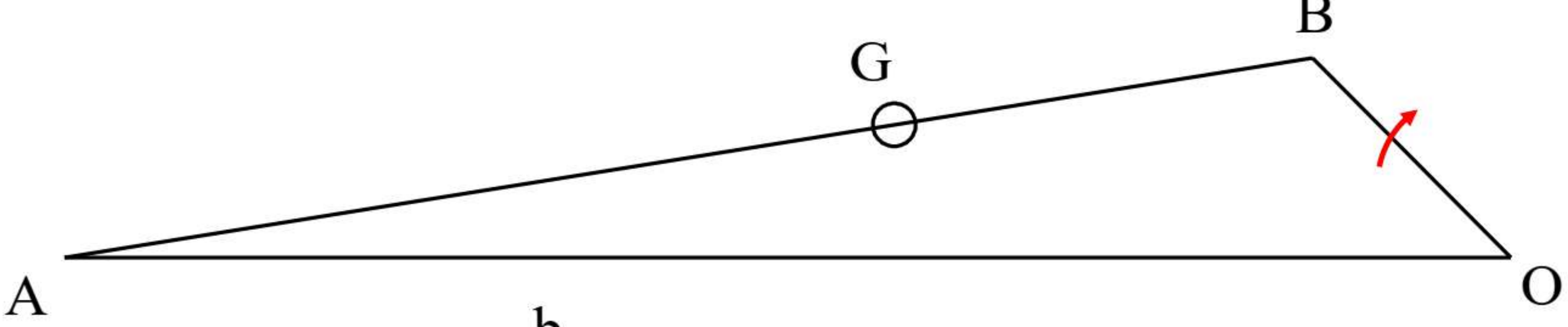
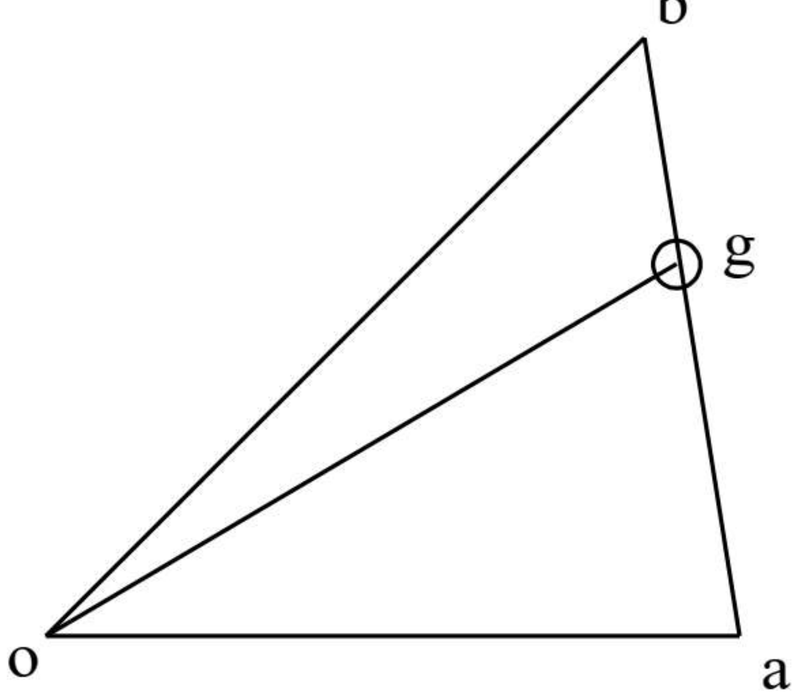
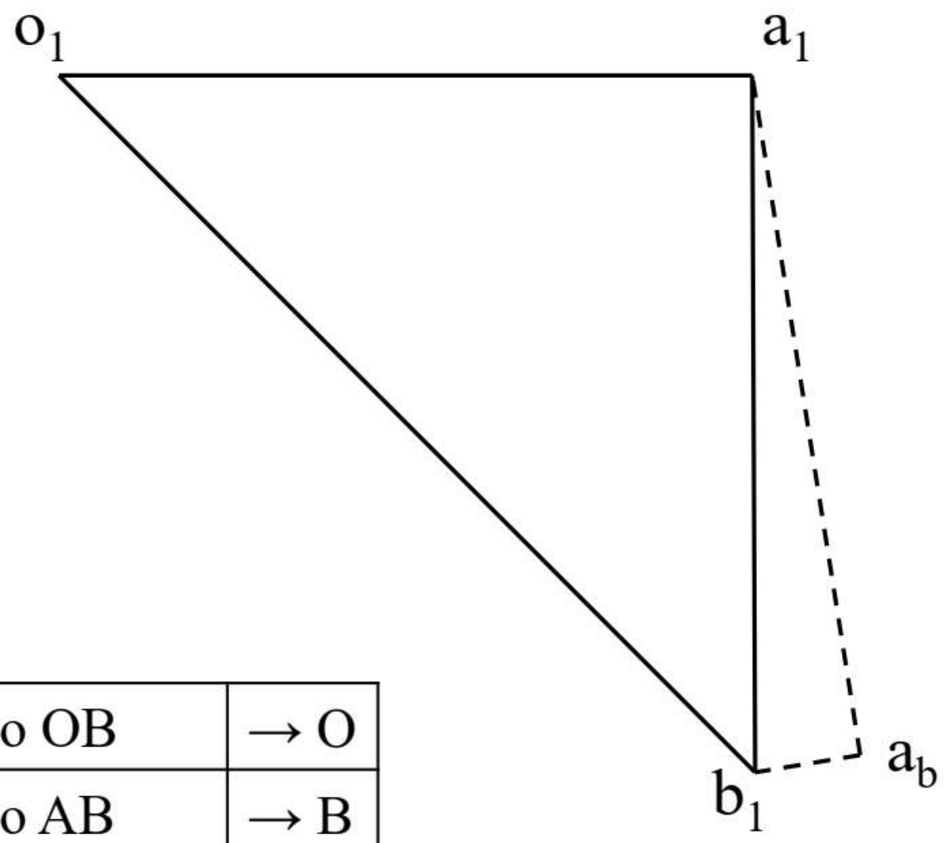
ANSWERS

1. (*b*) 2. (*d*) 3. (*b*) 4. (*c*) 5. (*c*)







f^c_{OB}	V_{ob}^2 / OB	21.93 m/s^2	II to OB	→ O
f^c_{AB}	V_{ab}^2 / AB	2.39 m/s^2	II to AB	→ B
f^t_{AB}			L to AB	
f^t_{A}			II to AO	

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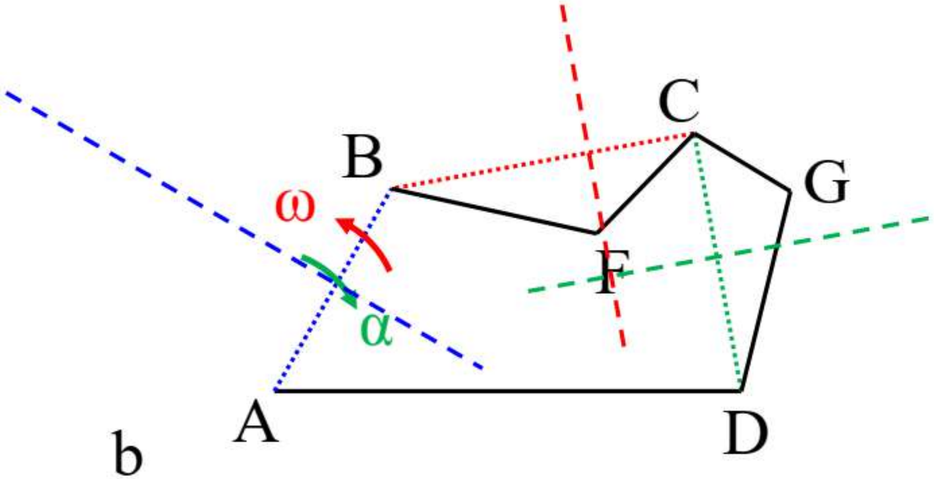
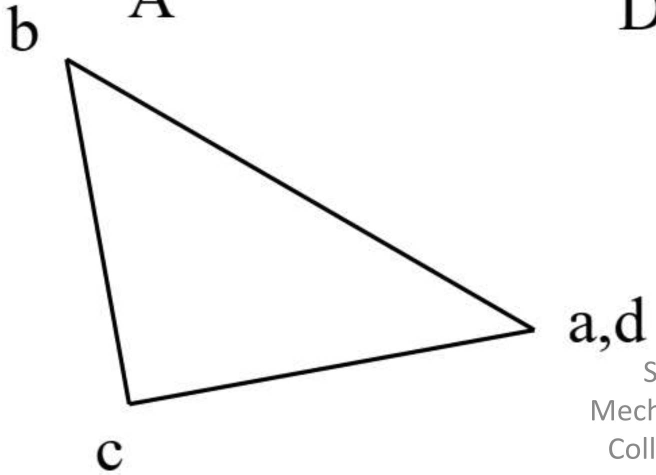
f^c_{AB}	V_{ab}^2 / AB	5.51 m/s^2	II to AB	→ A
f^t_{AB}	r α	1.3 m/s^2	L to AB	
f^c_{BC}	V_{bc}^2 / BC	1.86 m/s^2	II to BC	→ B
f^t_{BC}	------	-----	L to BC	
f^c_{CD}	V_{cd}^2 / CD	2.7 m/s^2	II to CD	→ D
f^t_{CD}	------	-----	L to CD	





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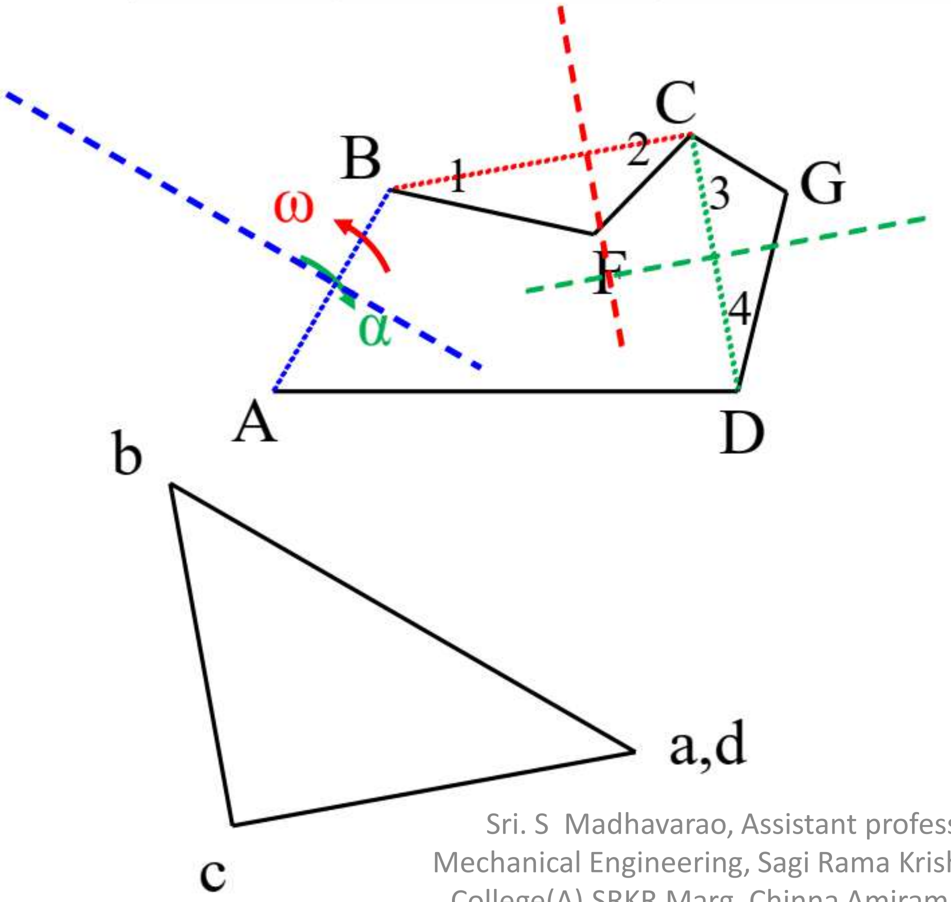
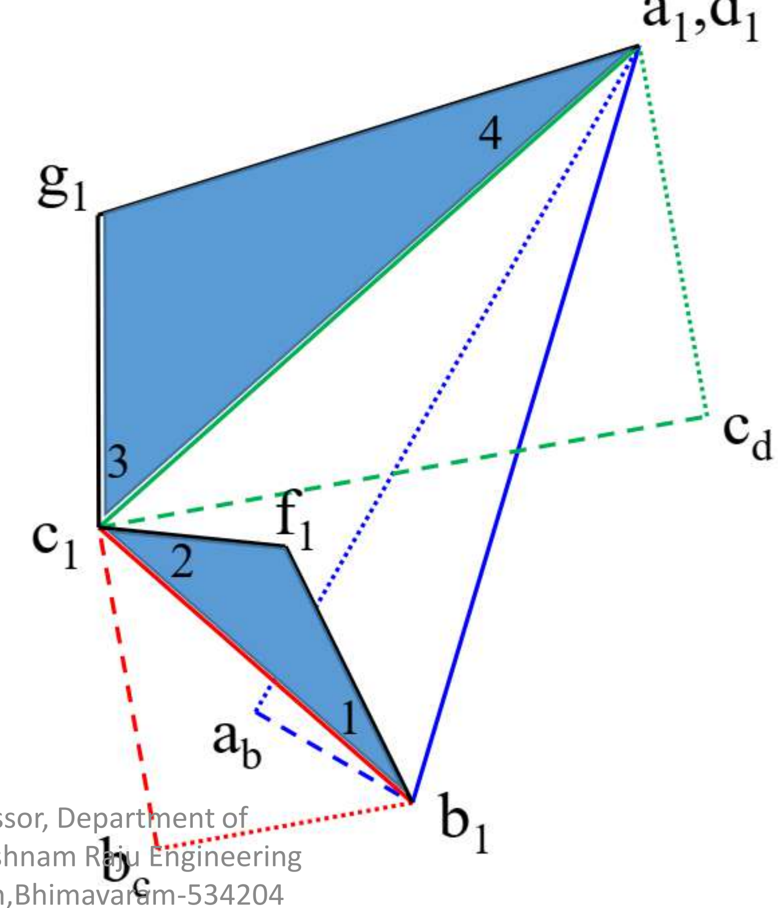
f^c_{AB}	V_{ab}^2 / AB	5.51 m/s²	II to AB	→ A
f^t_{AB}	r α	1.3 m/s²	L to AB	
f^c_{BC}	V_{bc}^2 / BC	1.86 m/s²	II to BC	→ B
f^t_{BC}	------	-----	L to BC	
f^c_{CD}	V_{cd}^2 / CD	2.7 m/s²	II to CD	→ D
f^t_{CD}	------	-----	L to CD	





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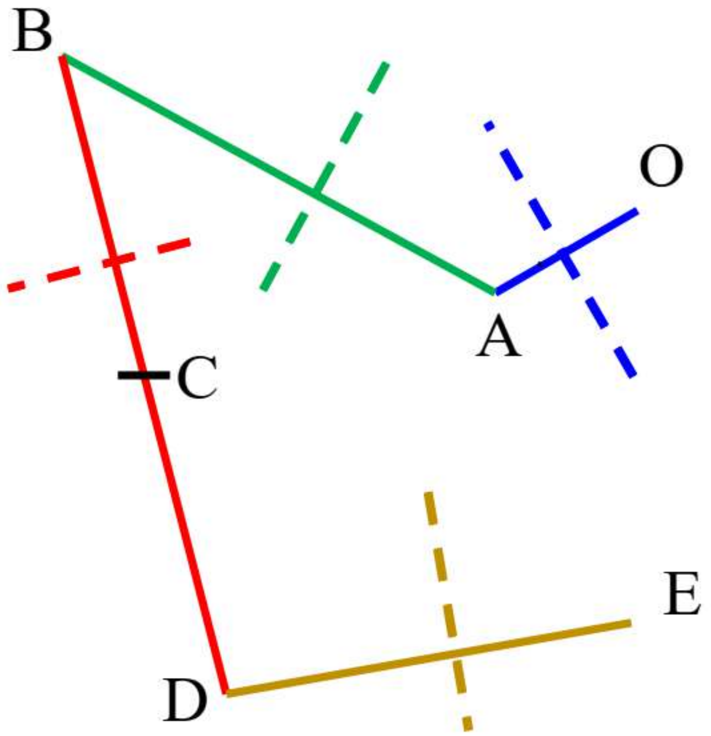
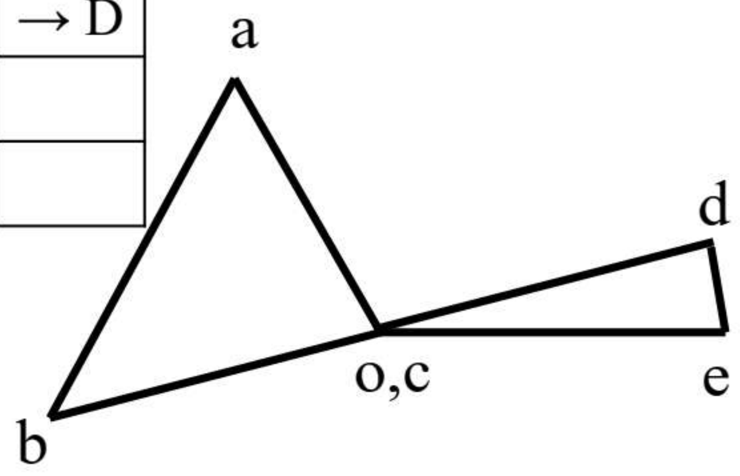
f^c_{OA}	$V_{oa}{}^2$ / OA	10.975 m/s^2	II to OA	→ O
f^c_{AB}	$V_{ab}{}^2$ / AB	6.365 m/s^2	II to AB	→ A
f^t_{AB}	-----	-----	L to AB	
f^c_{BC}	$V_{bc}{}^2$ / BC	7.495 m/s^2	II to BC	→ C
f^t_{BC}	-----	-----	L to BC	
f^c_{DE}	$V_{de}{}^2$ / DE	0.386 m/s^2	II to DE	→ D
f^t_{DE}	-----	-----	L to DE	
f^{total}_E	-----	-----	Horizontal	





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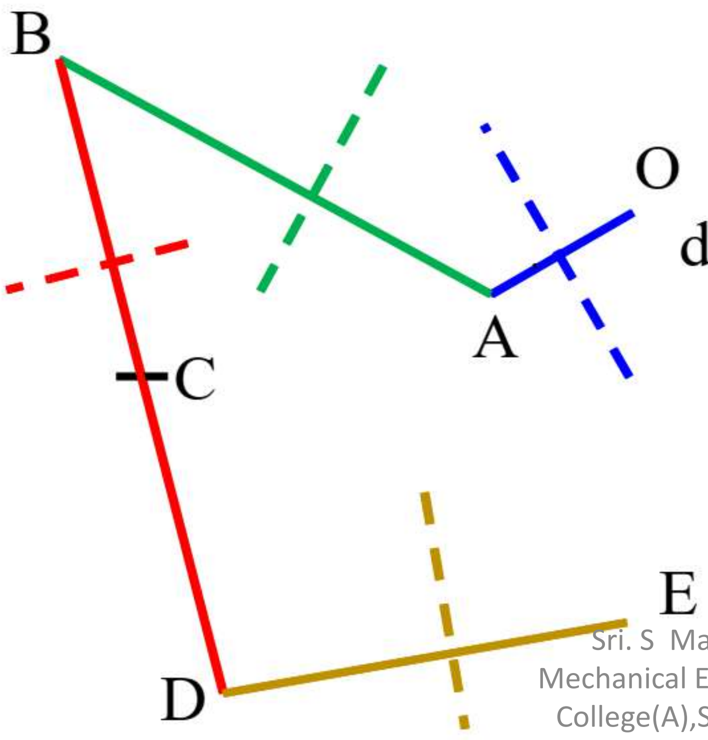
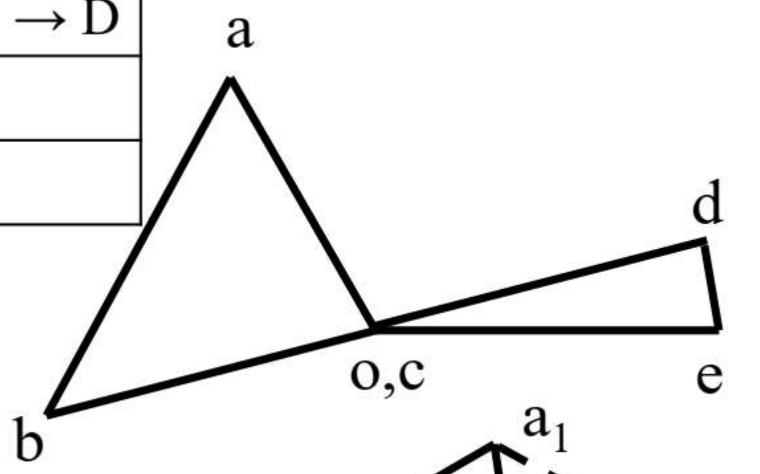
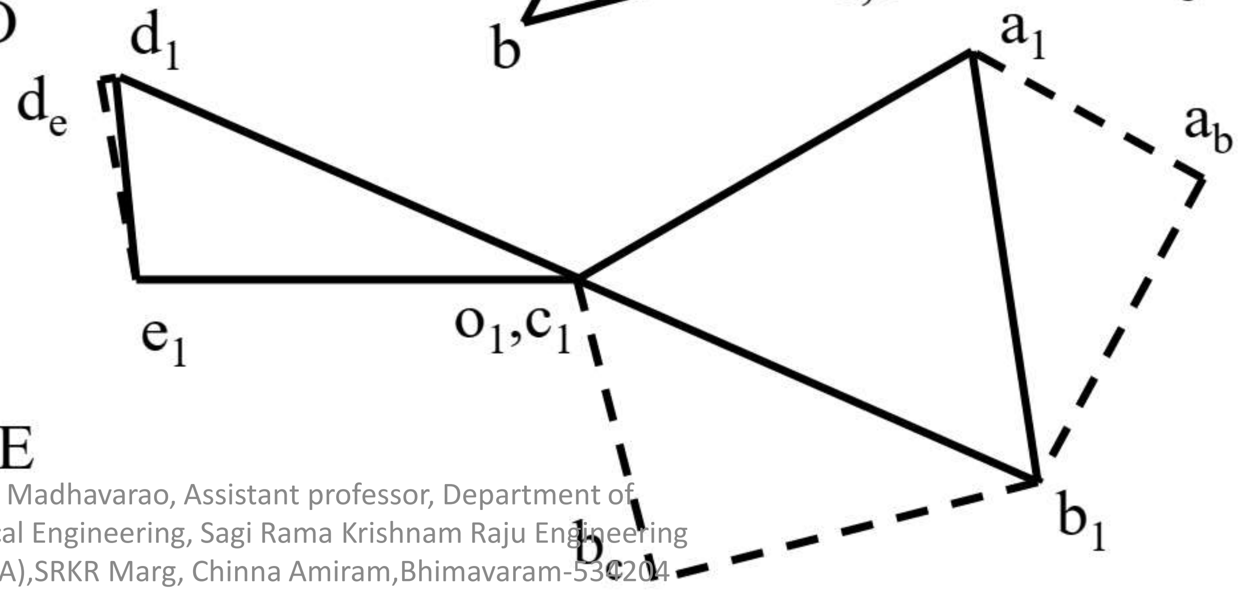
f^c_{OA}	V_{oa}^2 / OA	10.975 m/s²	II to OA	→ O
f^c_{AB}	V_{ab}^2 / AB	6.365 m/s²	II to AB	→ A
f^t_{AB}	-----	-----	L to AB	
f^c_{BC}	V_{bc}^2 / BC	7.495 m/s²	II to BC	→ C
f^t_{BC}	-----	-----	L to BC	
f^c_{DE}	V_{de}^2 / DE	0.386 m/s²	II to DE	→ D
f^t_{DE}	-----	-----	L to DE	
f^{total}_E	-----	-----	Horizontal	







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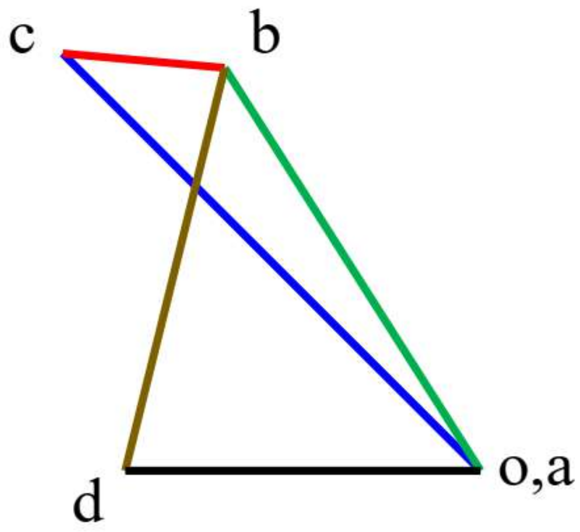
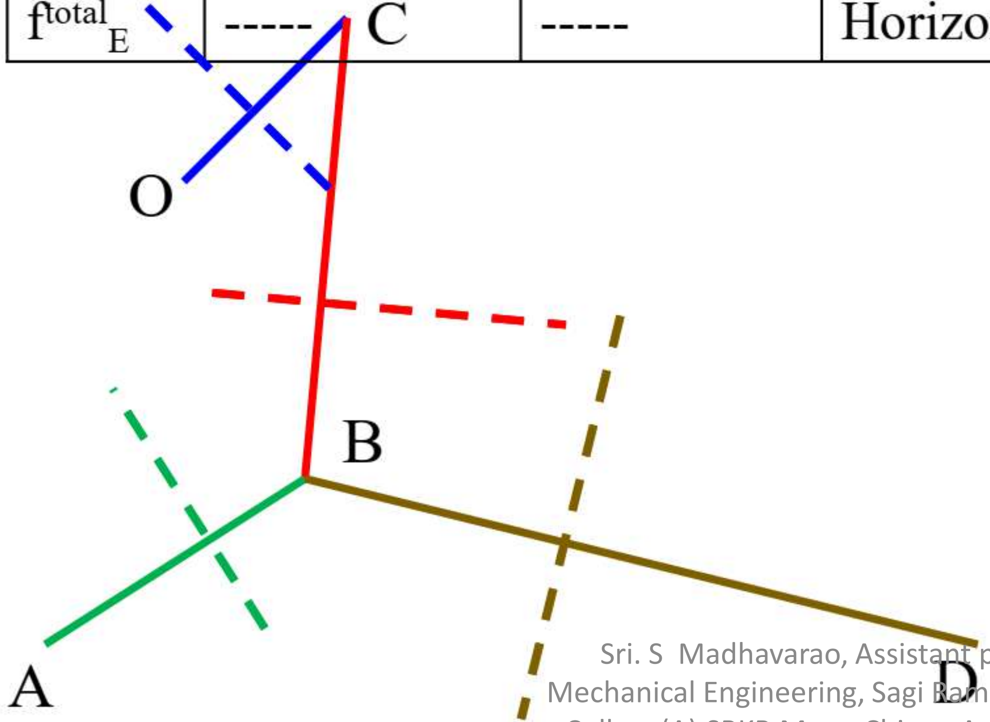
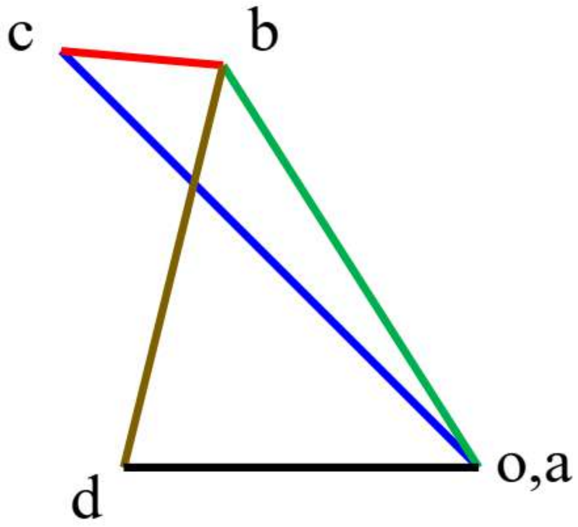
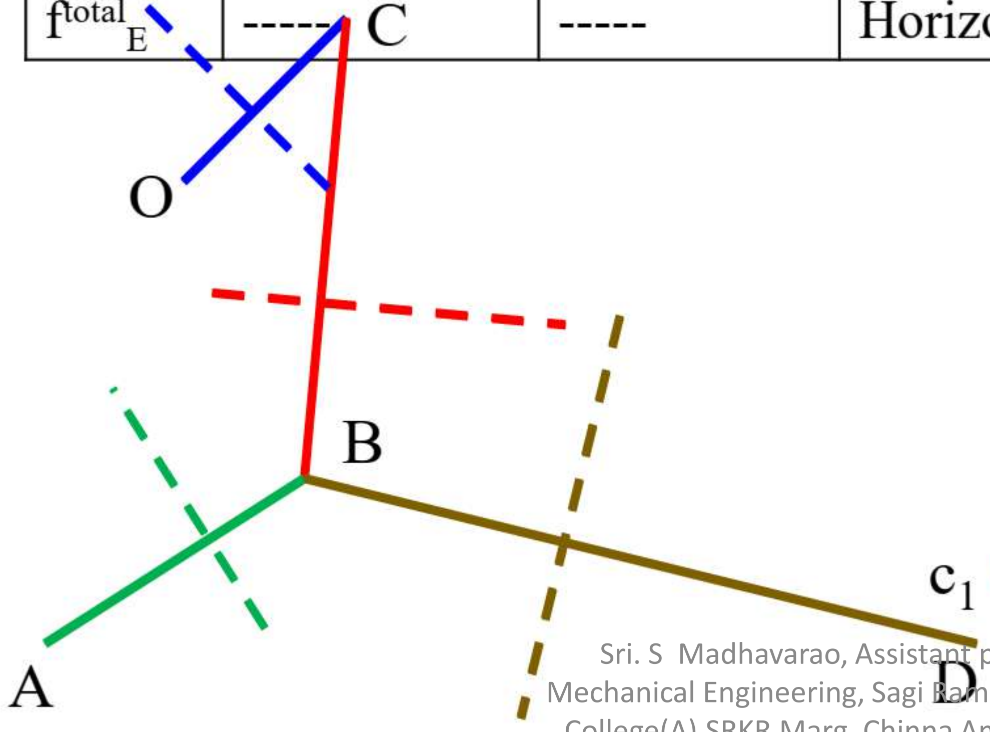
f^c_{OC}	V_{oc}^2 / OC	53.39 m/s^2	II to OC	→ O
f^t_{OC}	r α	7.5 m/s^2	L to OC	
f^c_{BC}	V_{bc}^2 / BC	2.01 m/s^2	II to BC	→ C
f^t_{BC}	-----	-----	L to BC	
f^c_{AB}	V_{ab}^2 / AB	26.17 m/s^2	II to AB	→ A
f^t_{AB}	-----	-----	L to AB	
f^c_{BD}	V_{de}^2 / DE	8.78 m/s^2	II to BD	→ D
f^t_{BD}	-----	-----	L to BD	
f^{total}_E	-----	-----	Horizontal	

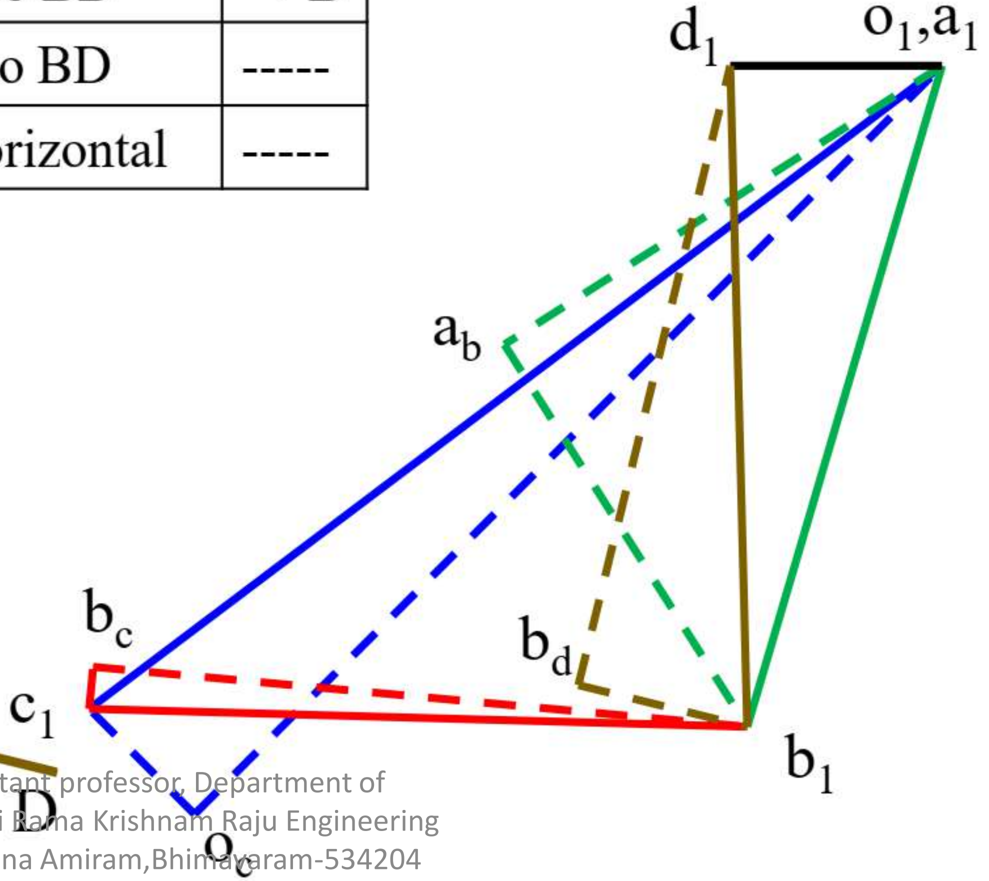




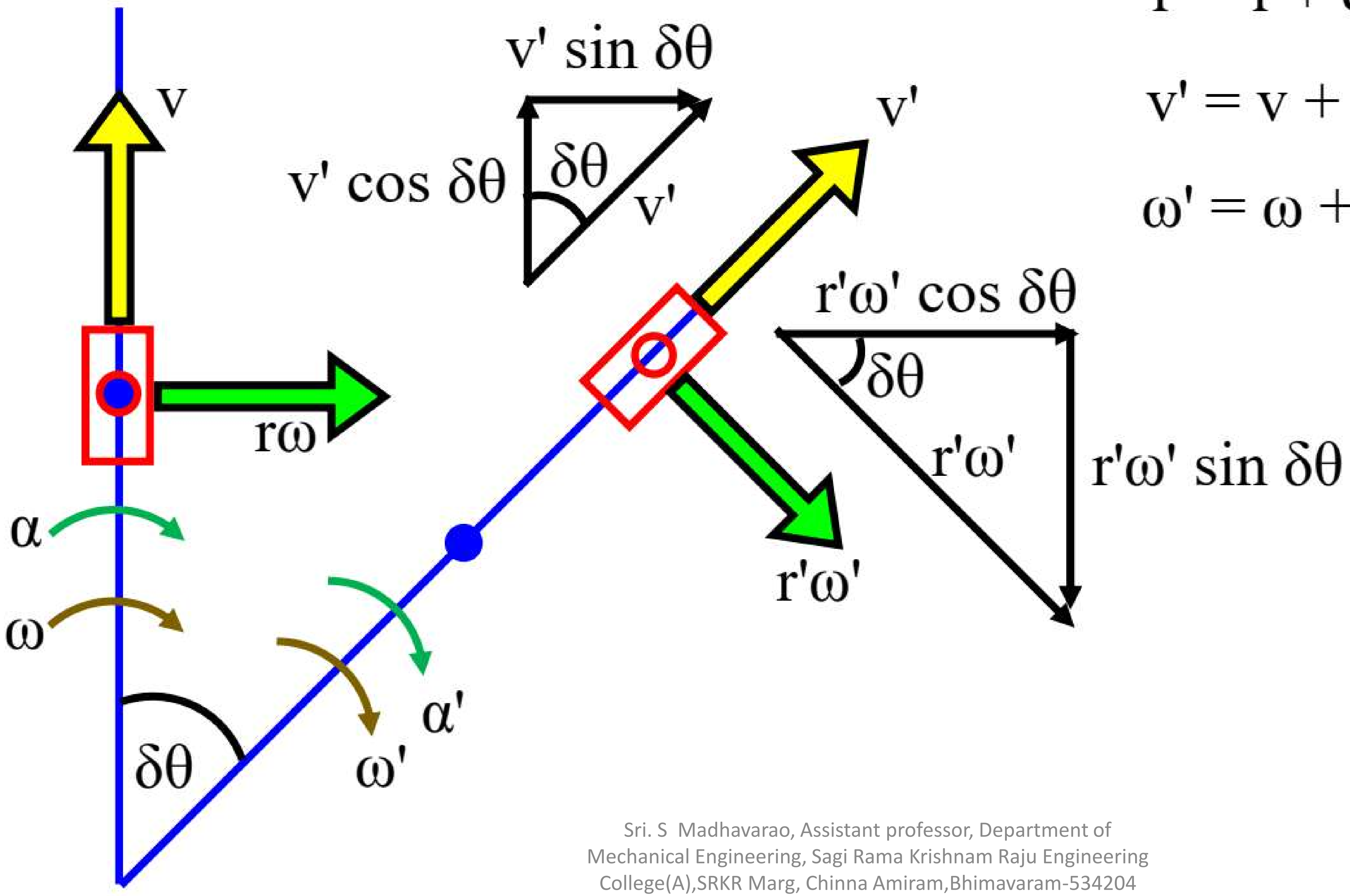
f^c_{OC}	V_{oc}^2 / OC	53.39 m/s²	II to OC	→ O
f^t_{OC}	r α	7.5 m/s²	L to OC	-----
f^c_{BC}	V_{bc}^2 / BC	2.01 m/s²	II to BC	→ C
f^t_{BC}	-----	-----	L to BC	-----
f^c_{AB}	V_{ab}^2 / AB	26.17 m/s²	II to AB	→ A
f^t_{AB}	-----	-----	L to AB	-----
f^c_{BD}	V_{de}^2 / DE	8.78 m/s²	II to BD	→ B
f^t_{BD}	-----	-----	L to BD	-----
f^{total}_E	-----	-----	Horizontal	-----







Coriolis component of acceleration:



$$r' = r + dr$$

$$v' = v + dv = v + f\,dt$$

$$\omega' = \omega + d\omega = \omega + \alpha\,dt$$

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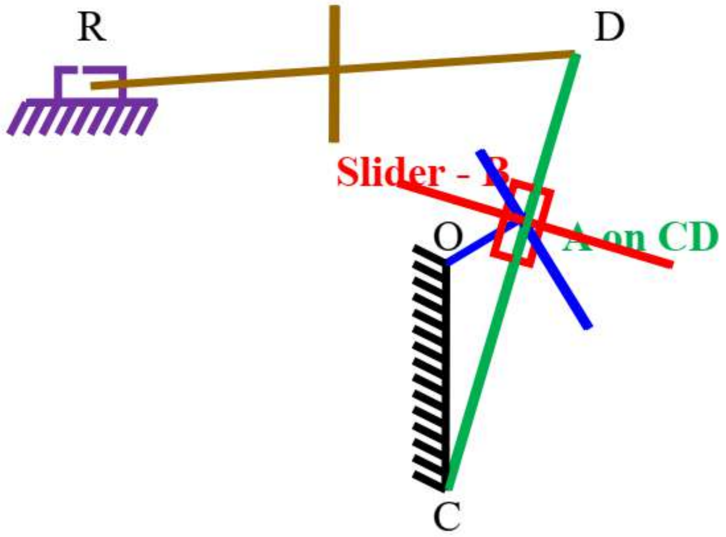
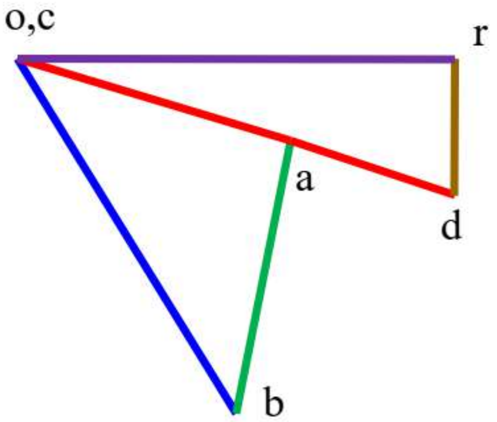
OA = 60 mm, N_{OA} = 200 RPM clockwise, CD = 300 mm, DR = 400 mm, Angle BOC = 120°, OC = 160 mm, distance between horizontal line from R & point O = 120 mm. Find out velocity & acceleration of ram R, acceleration of block A along slotted bar CD.



1	$f^{c}_{OB} = V_{ob}^2 / OB = 26.33\ m/s^2$	II to OB	→ O
2	$f^{c}_{AC} = V_{ac}^2 / AC = 4.052\ m/s^2$	II to AC	→ C
3	f^{t}_{AC} = unknown	⊥ to AC	-
4	$f^{cr}_{AB} = 2V_{ab}\ \omega_{CD} = 8.021\ m/s^2$	⊥ to CD	-
5	f^{c}_{AB} = unknown	II to CD	-
6	$f^{c}_{RD} = V_{rd}^2 / RD = 0.322\ m/s^2$	II to DR	→ D
7	f^{t}_{RD} = unknown	⊥ to DR	-
8	f^{total}_{R} = unknown	-	-



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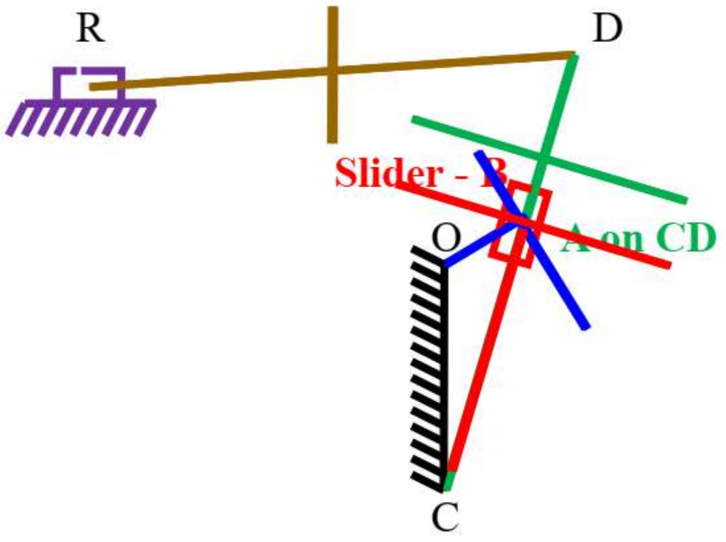
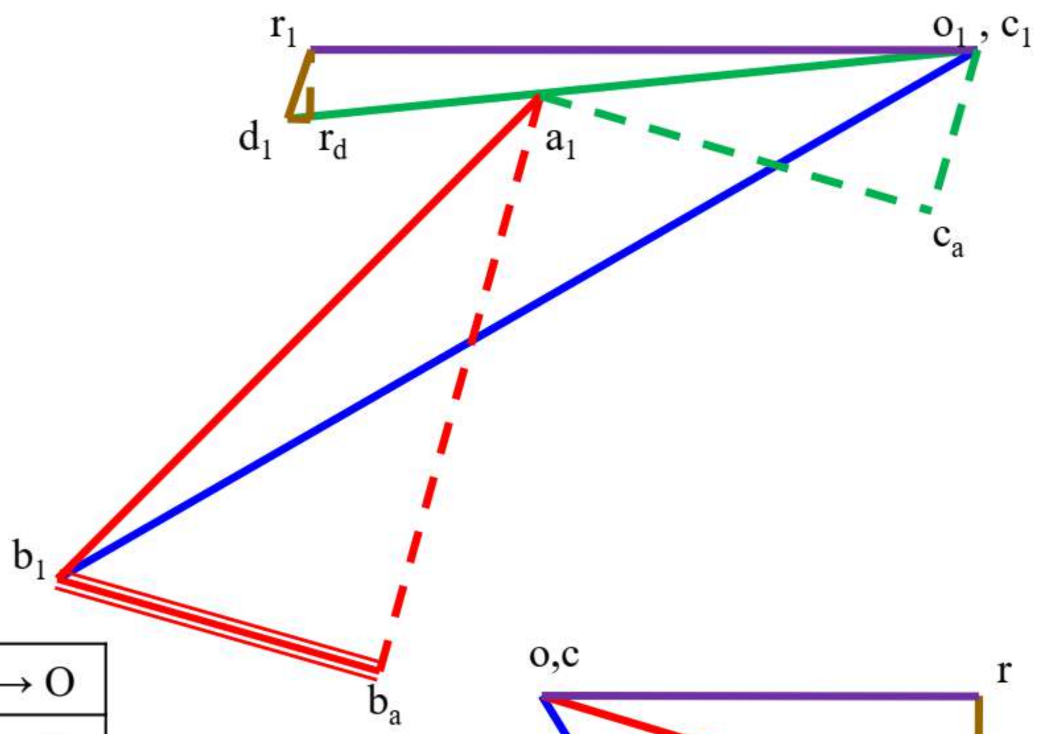
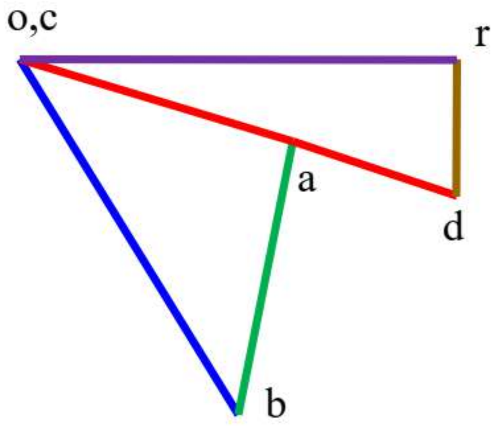
OA = 60 mm, N_{OA} = 200 RPM clockwise, CD = 300 mm, DR = 400 mm, Angle BOC = 120°, OC = 160 mm, distance between horizontal line from R & point O = 120 mm. Find out velocity & acceleration of ram R, acceleration of block A along slotted bar CD.







1	$f^{c}_{OB} = V_{ob}^2 / OB = 26.33\ m/s^2$	II to OB	→ O
2	$f^{c}_{AC} = V_{ac}^2 / AC = 4.052\ m/s^2$	II to AC	→ C
3	f^{t}_{AC} = unknown	⊥ to AC	-
4	$f^{cr}_{AB} = 2V_{ab}\ \omega_{CD} = 8.021\ m/s^2$	⊥ to CD	-
5	f^{c}_{AB} = unknown	II to CD	-
6	$f^{c}_{RD} = V_{rd}^2 / RD = 0.322\ m/s^2$	II to DR	→ D
7	f^{t}_{RD} = unknown	⊥ to DR	-
8	f^{total}_{R} = unknown	-	-

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UNIT III
Straight line Motion Mechanism

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UNIT-III

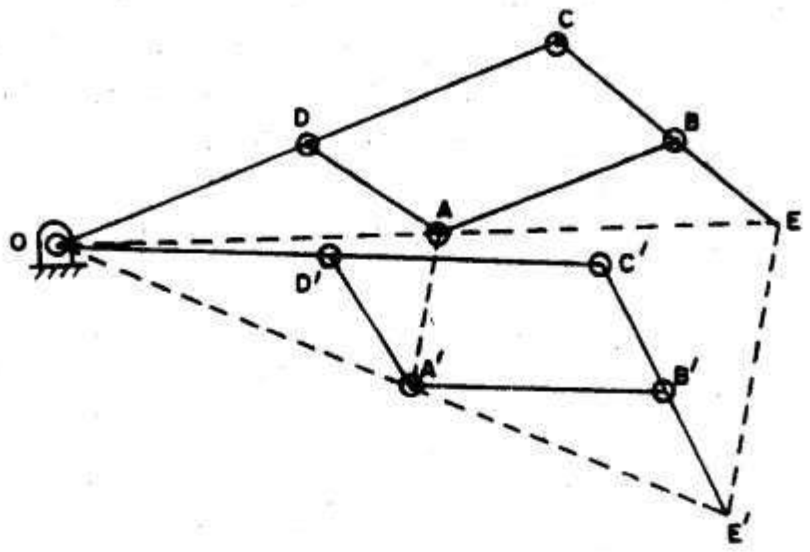
Straight-line motion mechanisms: Exact and approximate copied and generated types –Peaucellier - Hart - Scott Russel- Modified Scott Russel – Grasshopper – Watt -Tchebicheff‟s - Pantographs

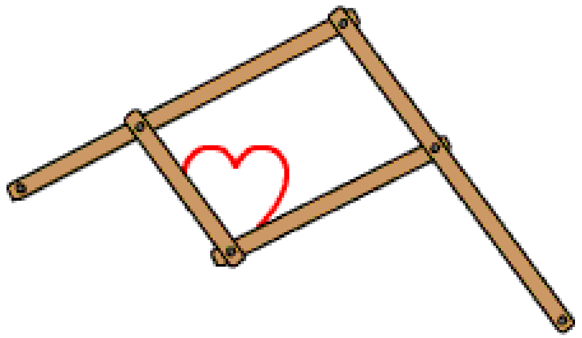
Steering gears: Conditions for correct steering – Davis Steering gear, Ackerman‟s steering gear.

Hooke's Joint: Single and double Hooke‟s joint –velocity ratio – application – problems

•Pantograph

- Pantographs are used for reducing or enlarging drawings and maps. They are also used for guiding cutting tools or torches to fabricate complicated shapes.
- In the mechanism shown in fig. path traced by point A will be magnified by point E to scale, as discussed below.
- In the mechanism shown, AB = CD; AD =BC and OAE lie on a straight line.
- When point A moves to A’ , E moves to E’ and OA’E’ also lies on a straight line.





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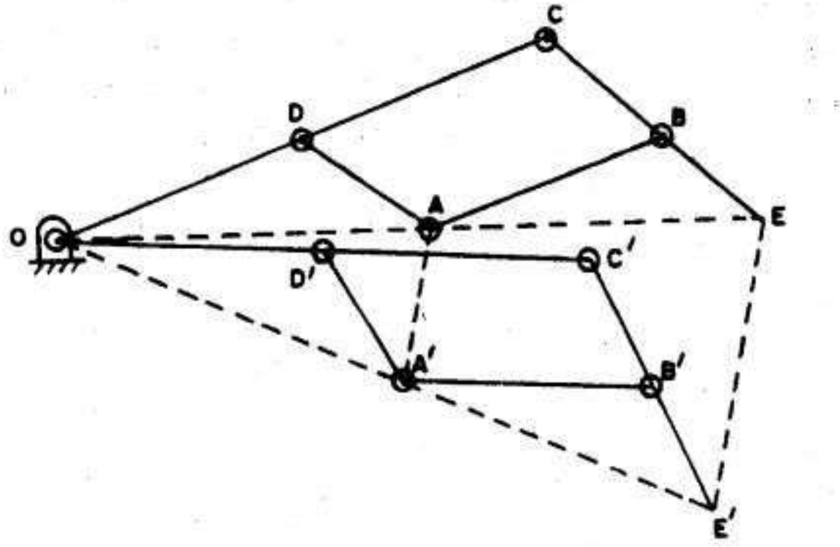
$\Delta ODA \equiv \Delta OCE$ and $\Delta OD'A' \equiv \Delta OC'E'$.

$$\therefore \frac{OD}{OC} = \frac{OA}{OE} = \frac{DA}{CE} \text{ And } \frac{OD'}{OC'} = \frac{OA'}{OE'} = \frac{D'A'}{C'E'}$$

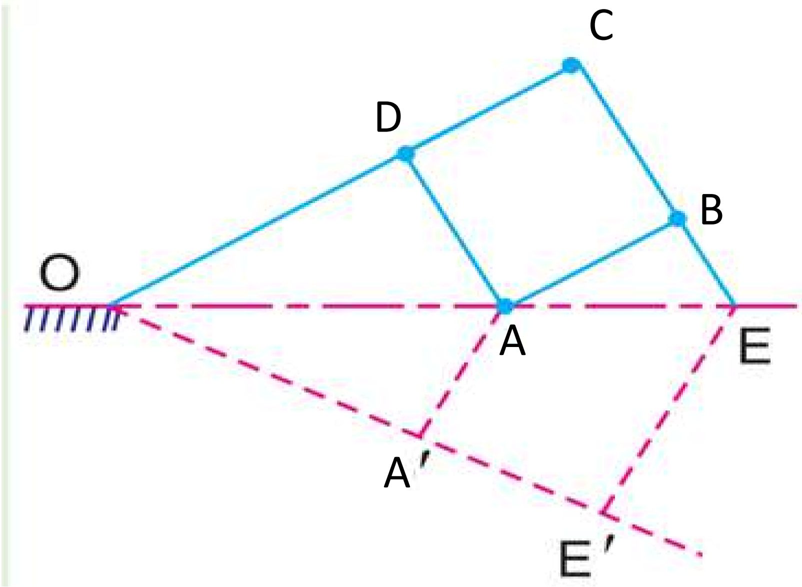
$$\text{But, } \frac{OD}{OC} = \frac{OD'}{OC'}; \therefore \frac{OA}{OE} = \frac{OA'}{OE'}; \therefore \Delta OAA' \equiv \Delta OEE'$$



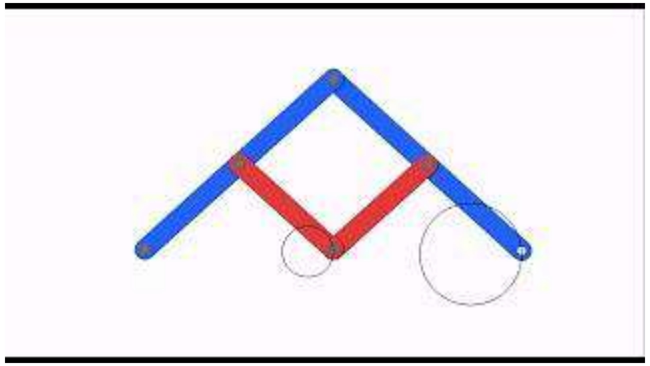
$\therefore EE' \,//\, AA'$

$$\text{And } \frac{EE'}{AA'} = \frac{OE}{OA} = \frac{OC}{OD}$$

$$\therefore EE' = AA'\left(\frac{OC}{OD}\right)$$



https://youtu.be/76WDPNMbsLc



Where $\left(\frac{OC}{OD}\right)$ is the magnification factor

Pantograph Mechanism Animation

Straight Line Mechanisms

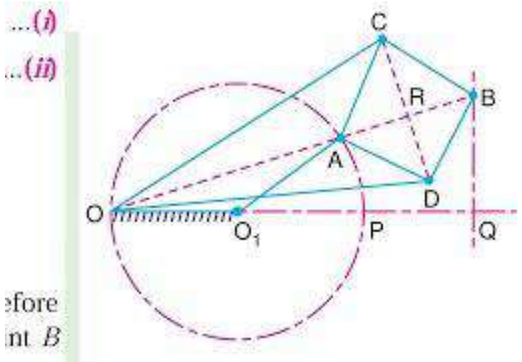
• One of the most common forms of the constraint mechanisms is that it permits only relative motion of an oscillatory nature along a straight line. The mechanisms used for this purpose are called straight line mechanisms.

• These mechanisms are of the following two types:

1. in which only turning pairs are used, and
2. in which one sliding pair is used.

• These two types of mechanisms may produce exact straight line motion or approximate straight line motion, as discussed in the following articles.

https://youtu.be/zgwEptukl6Q



Animation of Straight line mechanism-peaucellier, hart's,Scottrussel, Grasshopper, watt's,Tchiebicheff

Condition for Exact Straight Line Motion Mechanisms Made up of Turning Pairs:

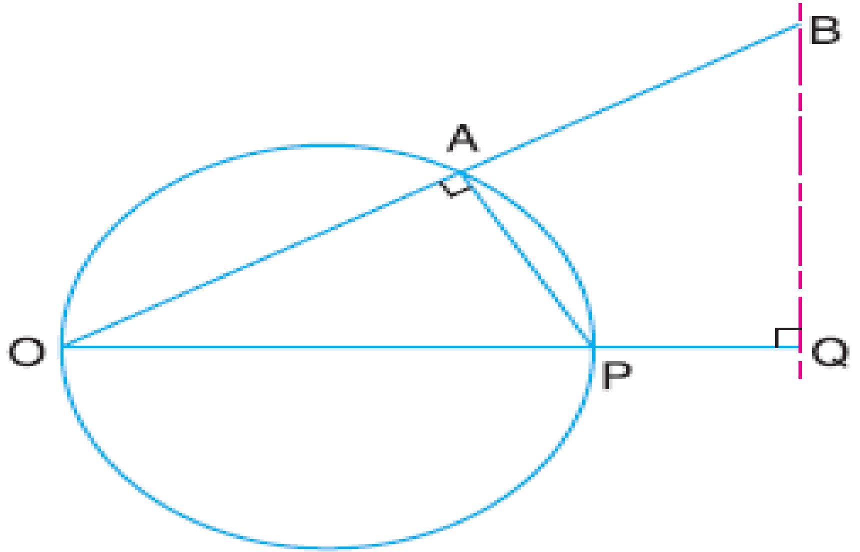
The principle adopted for a mathematically correct or exact straight line motion is described in Fig.

- Let O be a point on the circumference of a circle of diameter OP.

  Let OA be any chord and B is a point on OA produced, such that

  $OA \times OB = \text{constant}$

- Then the locus of a point B will be a straight line perpendicular to the diameter OP



Proof:

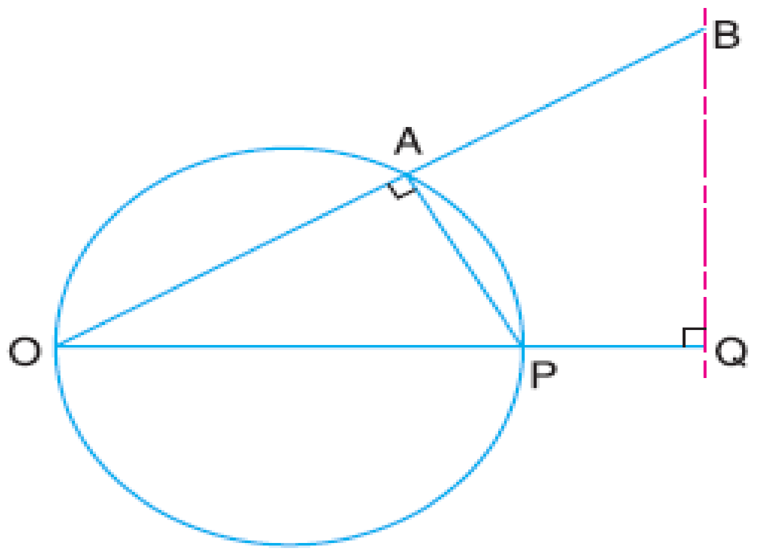
- Draw BQ perpendicular to OP produced.

  Join AP. The triangles OAP and OBQ are similar

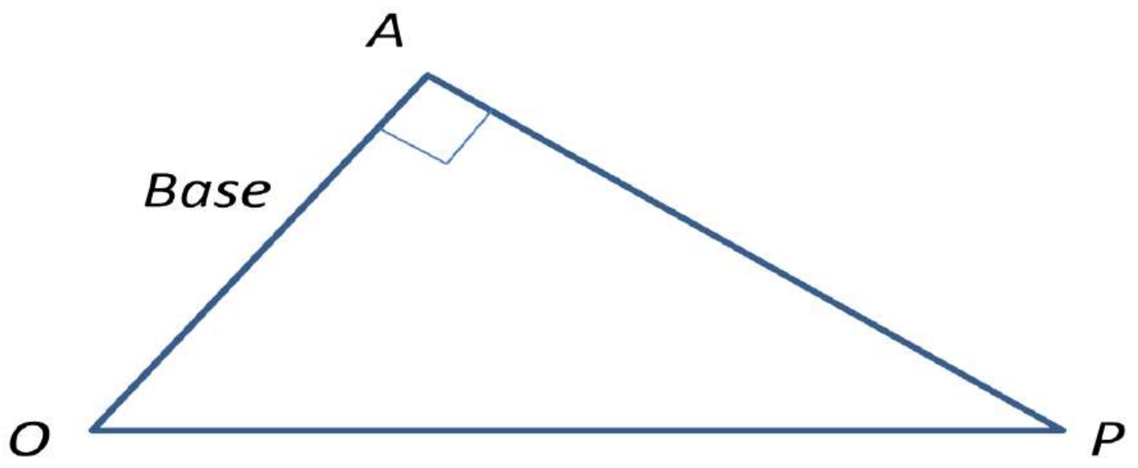
$$\therefore \qquad \frac{OA}{OP} = \frac{OQ}{OB}$$

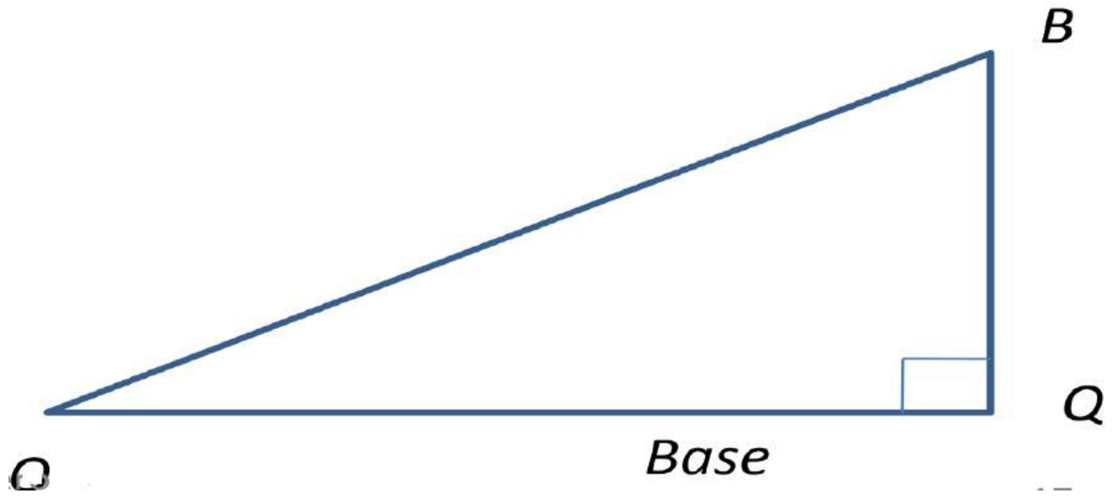
$$\text{or} \qquad OP \times OQ = OA \times OB$$

$$\text{or} \qquad OQ = \frac{OA \times OB}{OP}$$

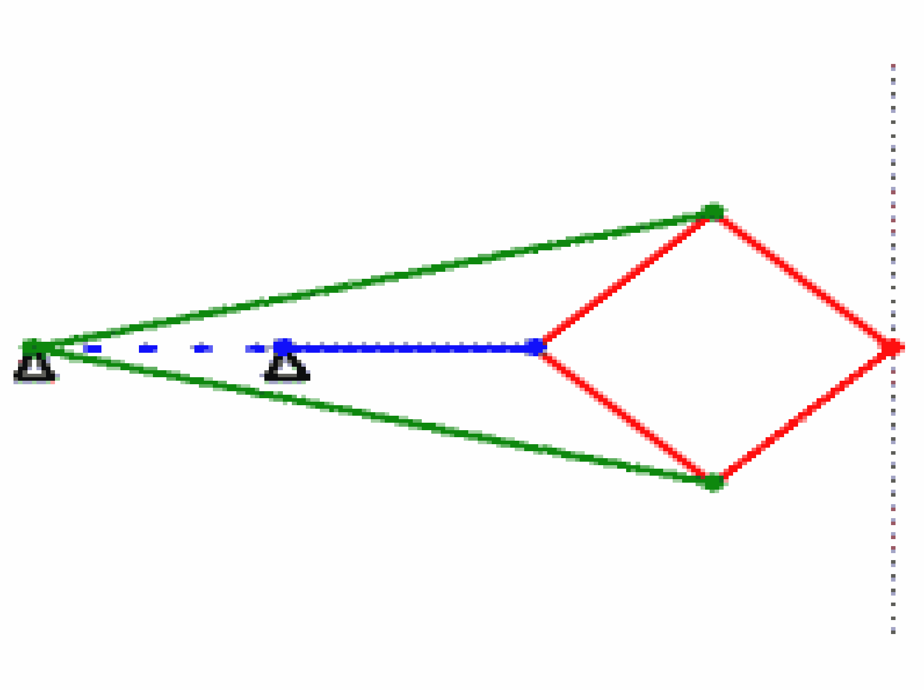


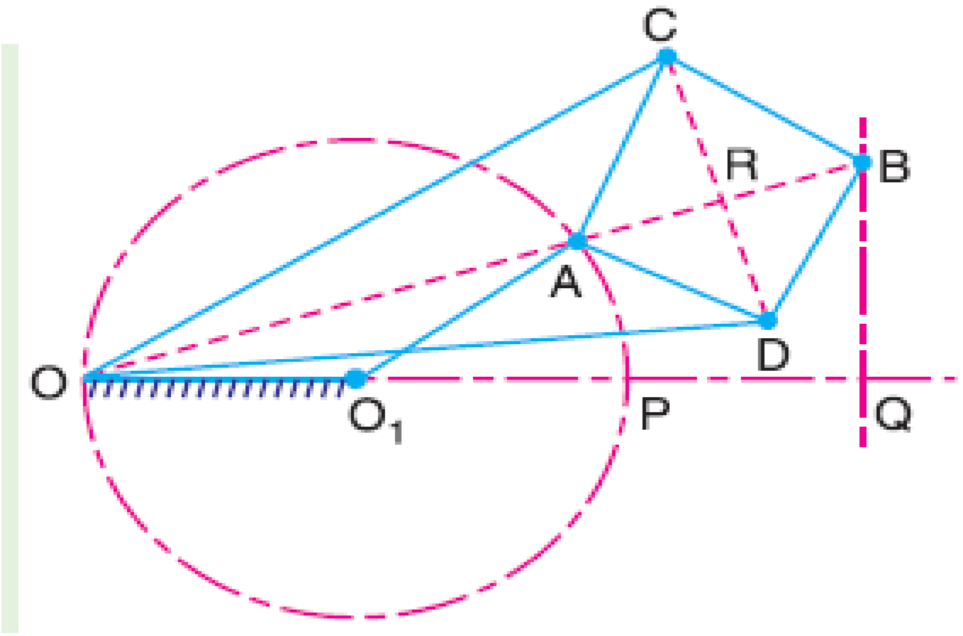
But OP is constant as it is the diameter of a circle, therefore, if OA × OB is constant, then OQ will be constant. Hence the point B moves along the straight path BQ which is perpendicular to OP





• **Peaucellier exact straight line motion mechanism:**





It consists of a fixed link OO_1 and the other straight links O_1A, OC, OD, AD, DB, BC and CA are connected by turning pairs at their intersections, as shown in Fig.

• The pin at A is constrained to move along the circumference of a circle with the fixed diameter OP, by means of the link O_1A.

• In Fig., AC = CB = BD = DA ; OC = OD ; and $OO_1 = O_1A$

- It may be proved that the product OA × OB remains constant, when the link O_1A rotates. Join CD to bisect AB at R. Now from right angled triangles ORC and BRC, we have

$$OC^2 = OR^2 + RC^2 \quad ..(i)$$

and

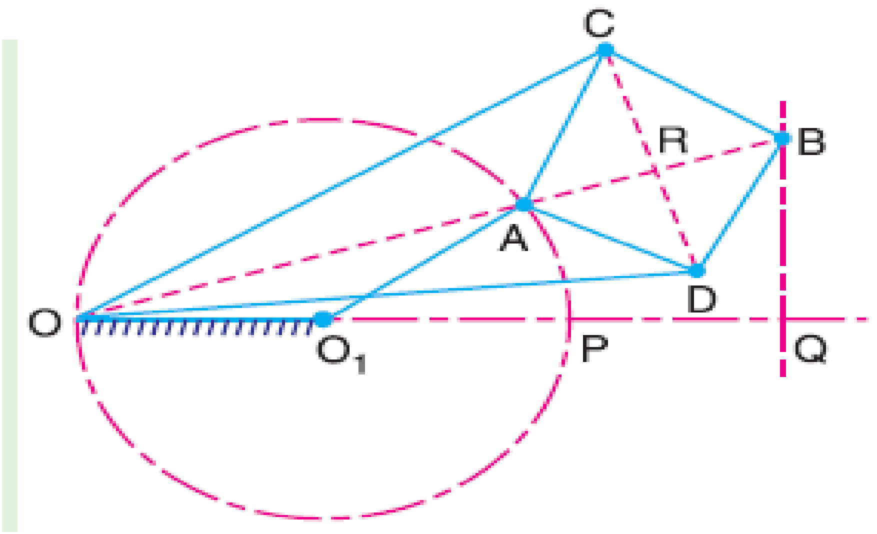
$$BC^2 = RB^2 + RC^2 \quad .(ii)$$

Subtracting equation (*ii*) from (*i*), we have

$$OC^2 - BC^2 = OR^2 - RB^2$$

$$= (OR + RB)(OR - RB)$$

$$= OB \times OA$$



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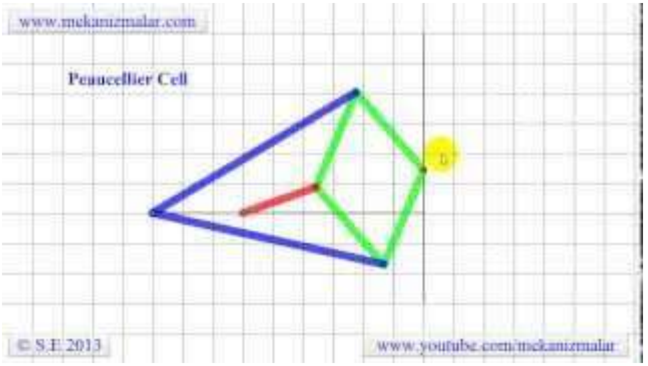

Peaucellier Cell Linkage Mechanism

Since OC and BC are of constant length, therefore the product OB × OA remains constant. Hence the point B traces a straight path perpendicular to the diameter OP

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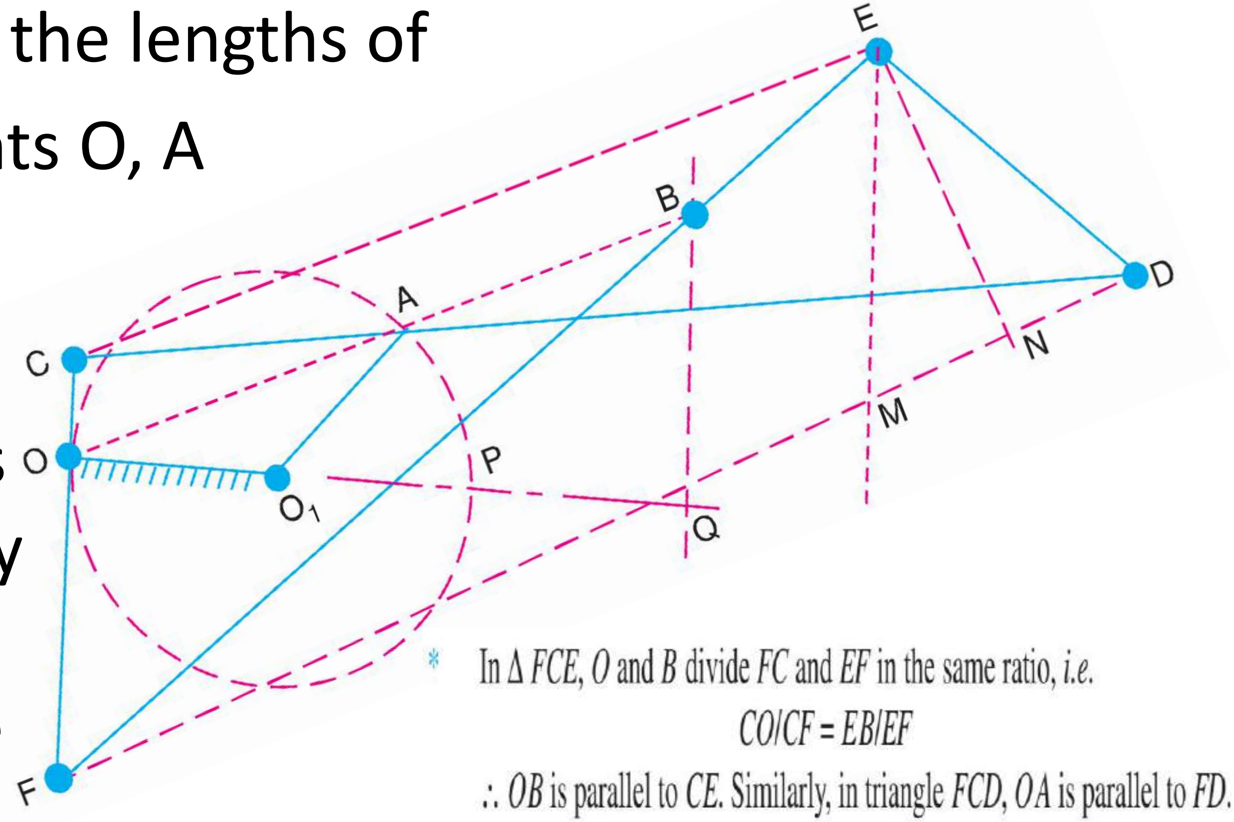
Hart's mechanism

This mechanism requires only six links as compared with the eight links required by the Peaucellier mechanism.

It consists of a fixed link OO_1 and other straight links O_1A, FC, CD, DE and EF are connected by turning pairs at their points of intersection, as shown in Fig..

The links FC and DE are equal in length and the lengths of the links CD and EF are also equal. The points O, A and B divide the links FC, CD and EF in the same ratio.

A little consideration will show that BOCE is a trapezium and OA and OB are respectively parallel to * FD and CE. Hence OAB is a straight line. It may be proved now that the product OA × OB is constant



* In Δ *FCE*, *O* and *B* divide *FC* and *EF* in the same ratio, *i.e.*

$CO/CF = EB/EF$

∴ *OB* is parallel to *CE*. Similarly, in triangle *FCD*, *OA* is parallel to *FD*.

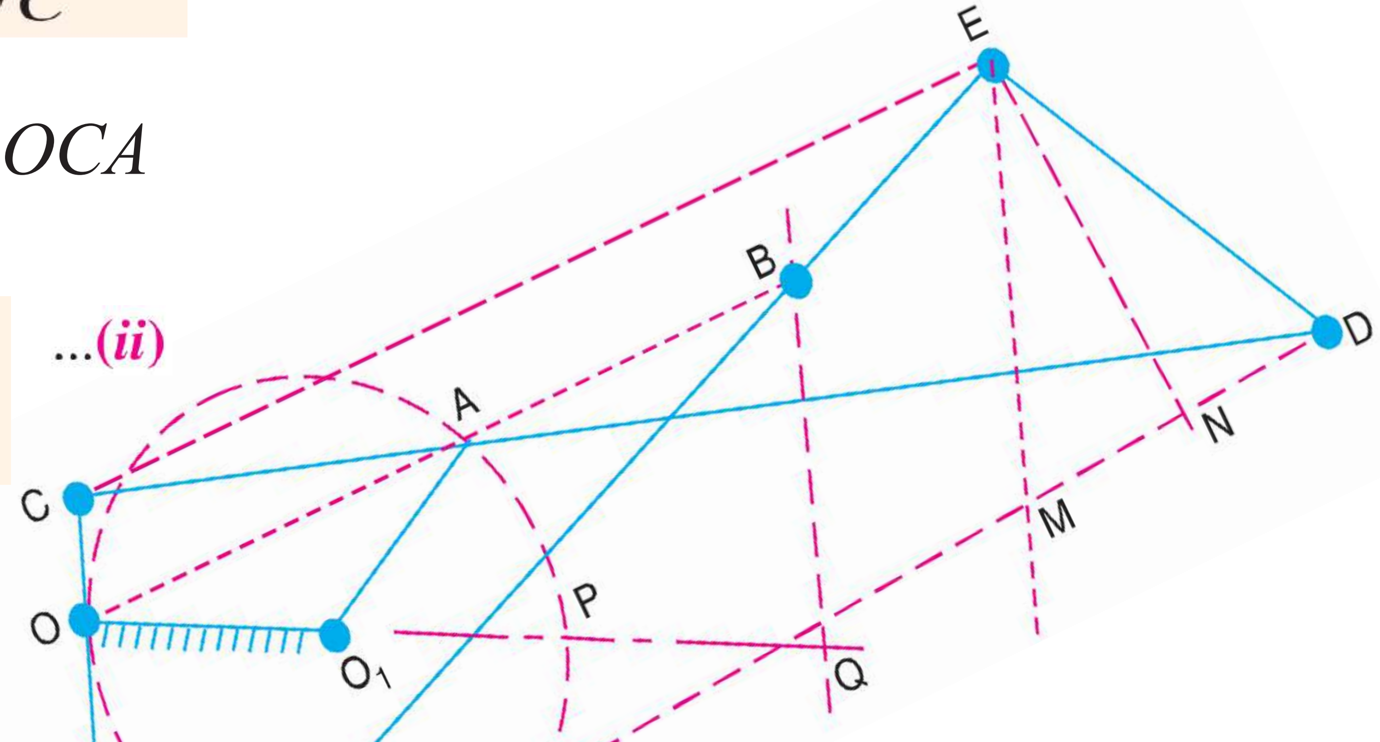
From similar triangles *CFE* and *OFB*,

$$\frac{CE}{FC} = \frac{OB}{OF} \quad \text{or} \quad OB = \frac{CE \times OF}{FC} \qquad \ldots(i)$$

https://youtu.be/IPvwz0H2Pyo

and from similar triangles *FCD* and *OCA*

$$\frac{FD}{FC} = \frac{OA}{OC} \quad \text{or} \quad OA = \frac{FD \times OC}{FC} \qquad \ldots(ii)$$



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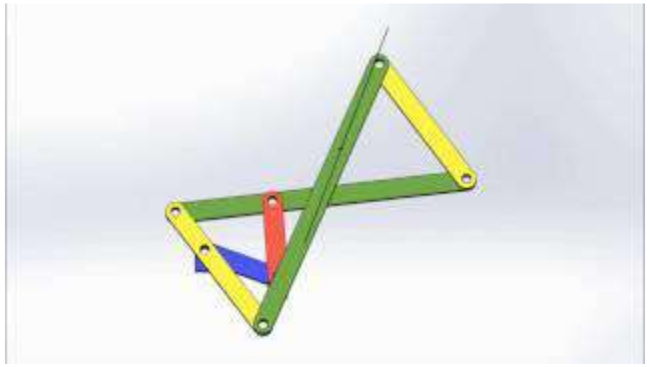


Hart straight line mechanism

Hart's straight line

Multiplying equations (*i*) and (*ii*), we have

$$OA \times OB = \frac{FD \times OC}{FC} \times \frac{CE \times OF}{FC} = FD \times CE \times \frac{OC \times OF}{FC^2}$$

Since the lengths of *OC*, *OF* and *FC* are fixed, therefore

$$OA \times OB = FD \times CE \times \text{constant} \quad \text{...(iii)}$$

$$\text{...}\left(\text{substituting } \frac{OC \times OF}{FC^2} = \text{constant}\right)$$

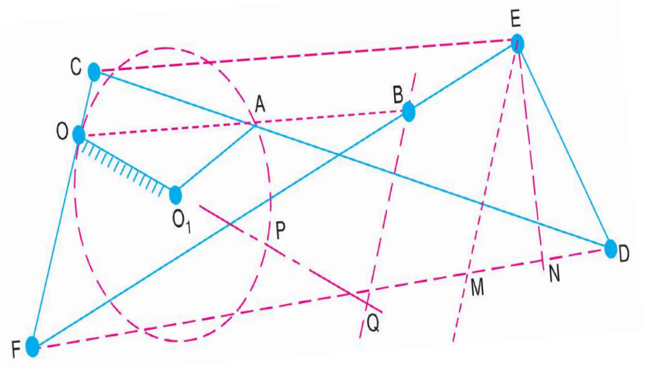
Now from point *E*, draw *EM* parallel to *CF* and *EN* perpendicular to *FD*. Therefore

$$FD \times CE = FD \times FM \quad \text{...}(\because CE = FM)$$

$$= (FN + ND)(FN - MN) = FN^2 - ND^2 \quad \text{...}(\because MN = ND)$$

$$= (FE^2 - NE^2) - (ED^2 - NE^2)$$

...(From right angled triangles *FEN* and *EDN*)



$$= FE^2 - ED^2 = \text{constant} \quad \text{...(iv)}$$

...($\because$ Length *FE* and *ED* are fixed)

From equations (*iii*) and (*iv*),

$$OA \times OB = \text{constant}$$

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Exact Straight Line Motion Consisting of One Sliding Pair-Scott Russell's Mechanism:

A Scott Russell mechanism consists of three movable links; OQ,PS and slider S which moves along OS.OQ is the Crank(as shown in fig). The links are connected in such a way that

QO=QP=QS

It can be proved that P moves in a straight line perpendicular to OS as the slider S moves along OS.

As QO=QP=QS, a circle can be drawn passing through O, P and S with PS as diameter and Q as the centre.

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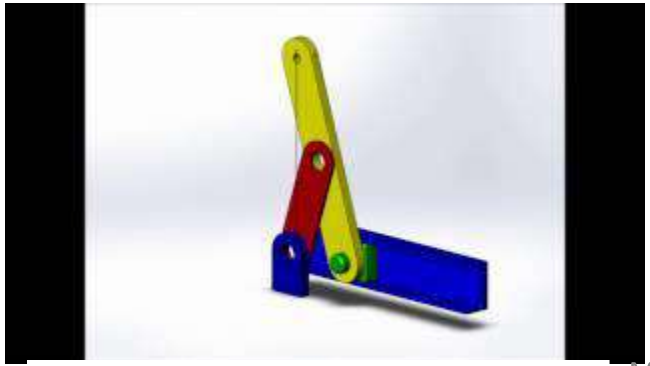
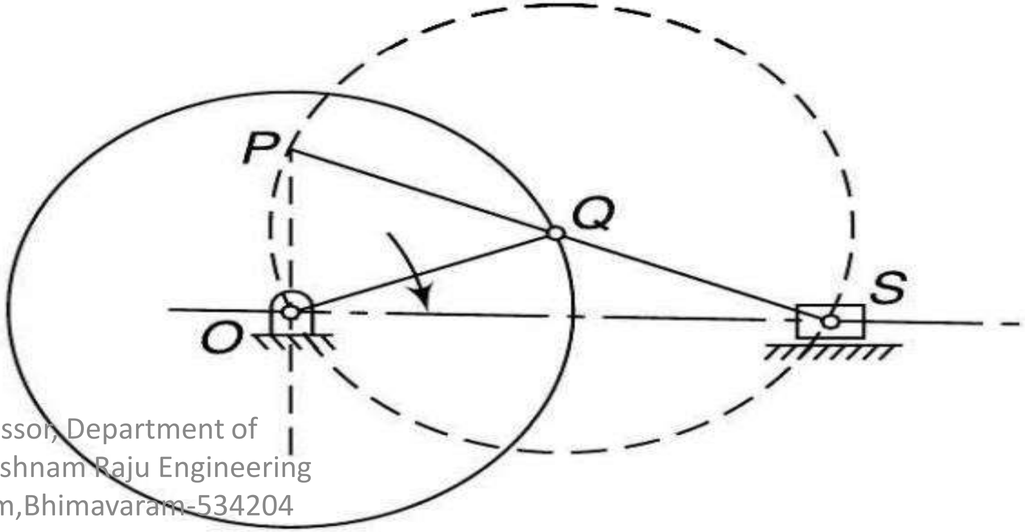
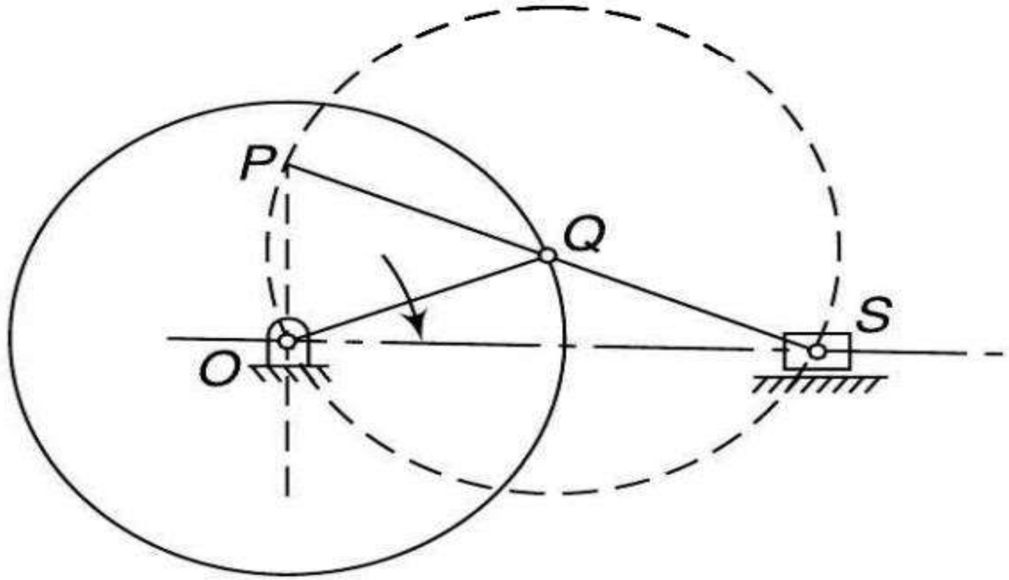


Scott Russell Mechanism



- Now, O lies on the circumference of the circle and PS is the diameter. Therefore, ∠POS is a right angle. This is true for all positions of S and is possible only P moves in a straight line perpendicular to OS at O

Note: Since the friction and wear of a sliding pair is much more than those of turning pair, therefore this mechanism is not of much practical value.



Approximate Straight Line Motion Mechanisms

Modified Scott-Russel mechanism:

Grasshopper mechanism.

Watt's mechanism

Tchebicheff's Mechanism.

Modified Scott-Russel mechanism:

Note that in such a mechanism, the path of P through the joint O which is not desirable. This can be avoided if the links are proportioned in a way that QS is the mean proportional between OQ and QP, i.e.,

$$\frac{OQ}{QS} = \frac{QS}{QP}$$

However, in this case P will approximately traverse a straight line perpendicular to OS and that also for small movement of S or for small values of the angle Θ (as shown in fig.). A mathematical proof of this, being not simple, is omitted here. However, by drawing the mechanism in a number of positions, the fact can be verified.

Usually this is known as modified Scott Russell mechanism

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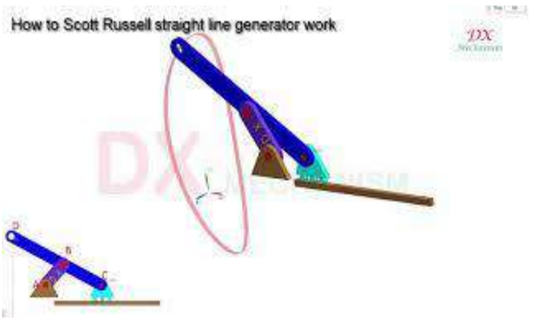

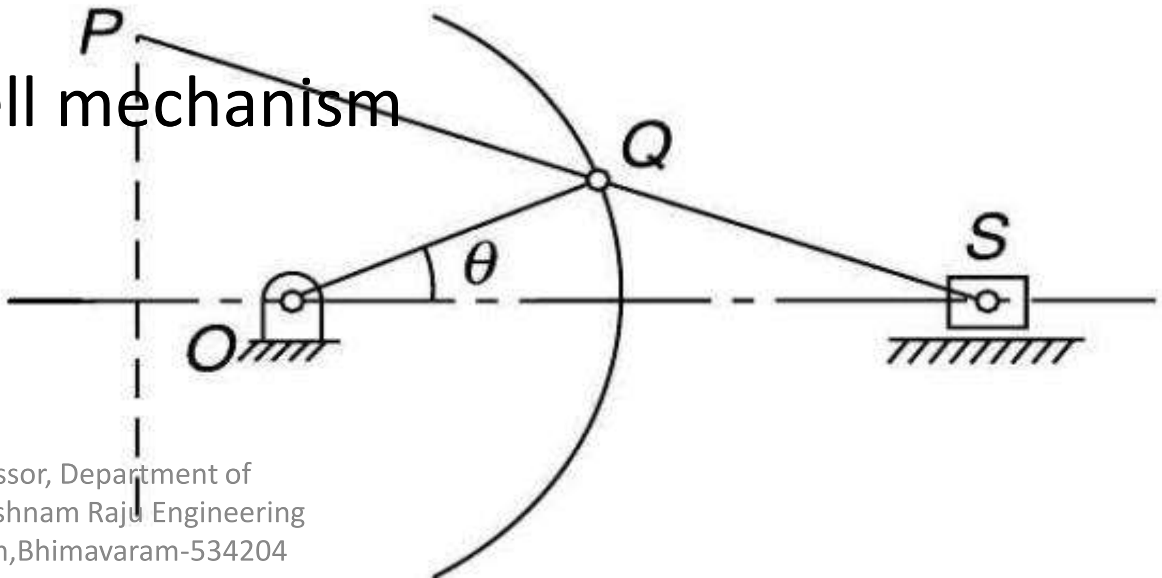

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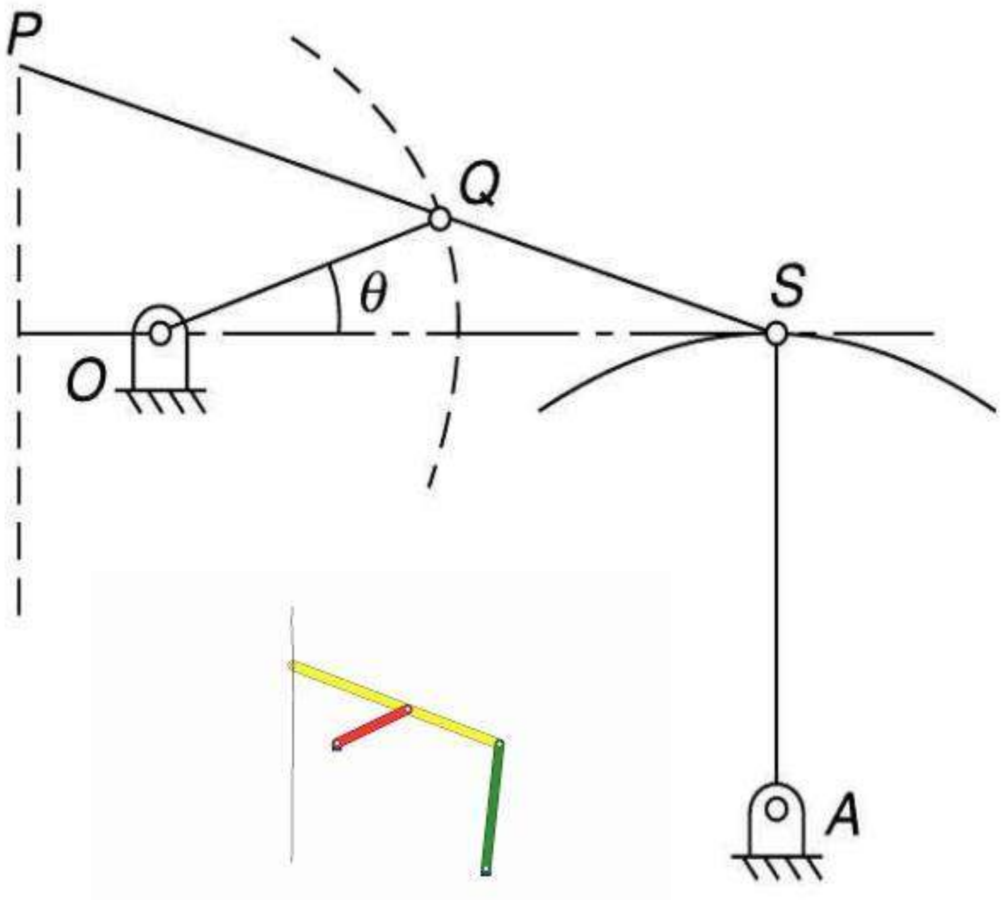
- Grasshopper mechanism:
- This mechanism is a modification of modified Scott Russell mechanism in which the sliding pair at S is replaced by a turning pair. This is achieved by replacing the slider with a link AS perpendicular to OS in the mean position. AS is pin- jointed at A.(as shown in fig.)

https://youtu.be/dYZVBPJ1dqE

If the length AS is larger enough, S moves in an approximate straight line perpendicular to AS (or in line with OS) for small angular movements. P again will move in an approximate straight line if QS is the mean proportional between OQ and QP, i.e.,

$$\frac{OQ}{QS} = \frac{QS}{QP}$$



Grasshopper straight line mechanism

Watt's mechanism

- It is a very simple mechanism. it has four links OQ,OA,QB and AB. OQ is the fixed. Link OA and QB can oscillate about centres O and Q respectively. It is seen that if P is a point on the link AB such that (PA/PB) = (QB/OA), then for small oscillation of OA and QB, P will trace an approximate straight line. This has been shown in fig. for three positions.
- In earlier times, the mechanism was used by watt to guide the piston, as it was difficult to machine plane surfaces

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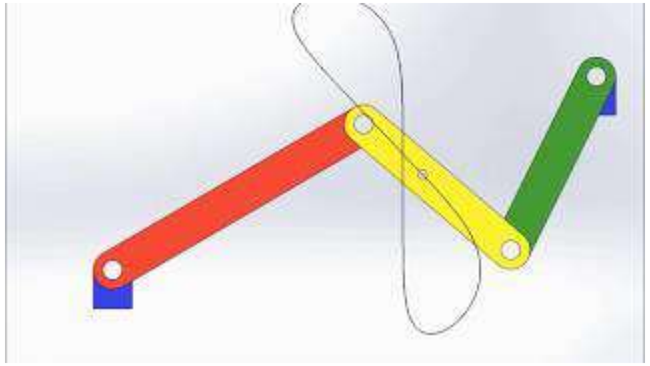

Watt straight line mechanism

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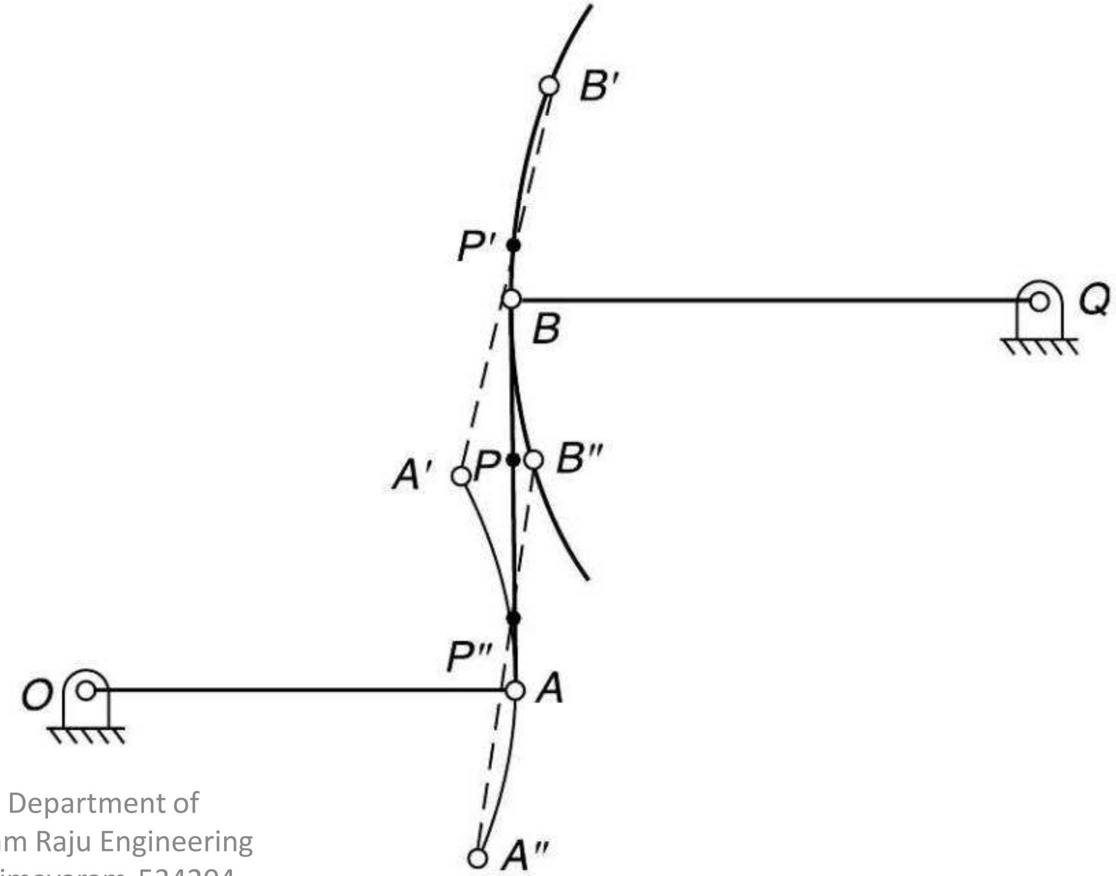
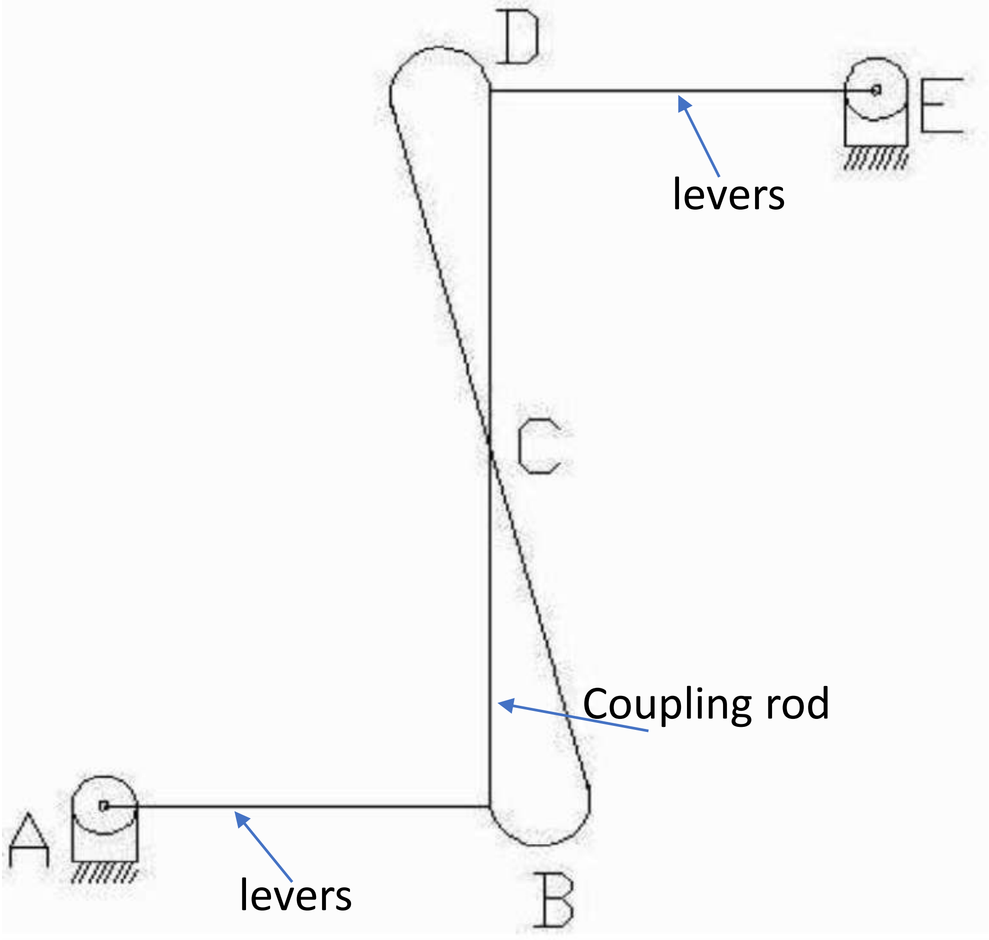


Watt's mechanism.



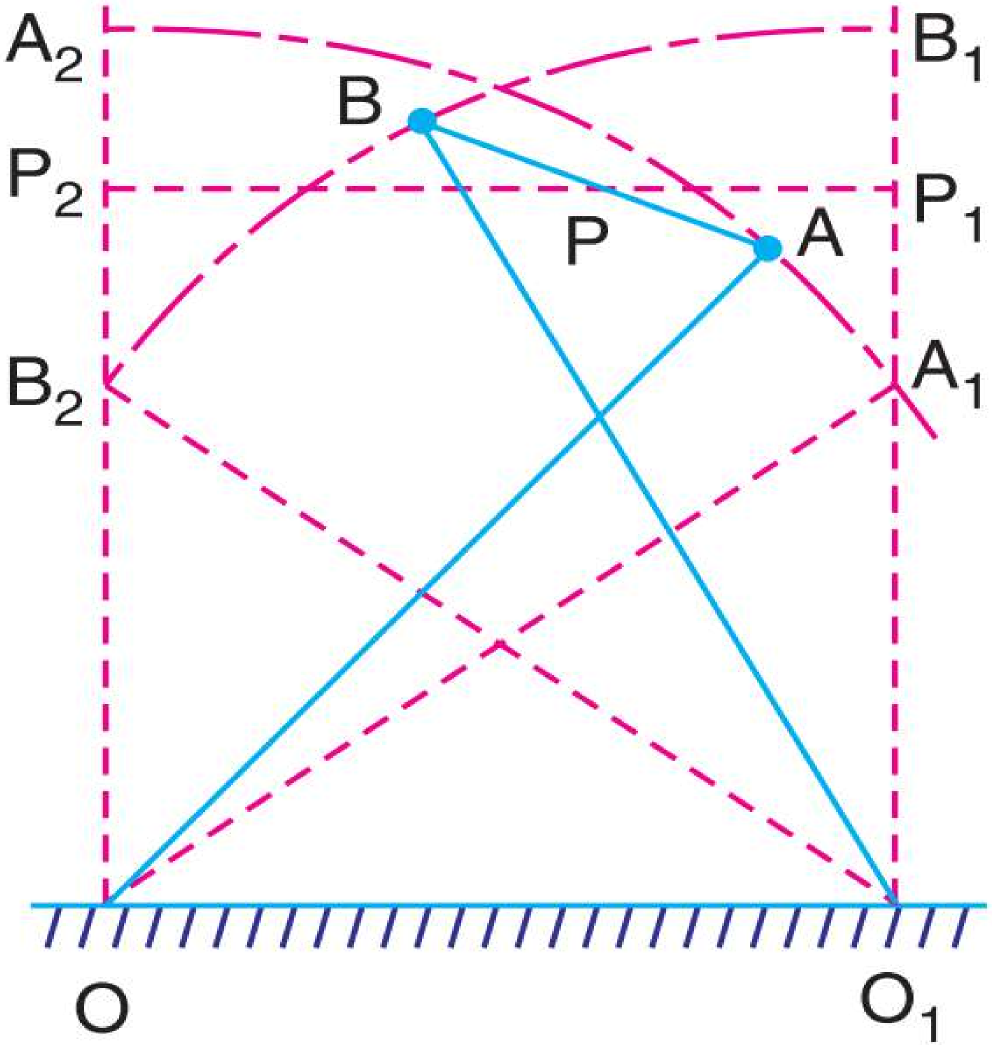
- The links AB & DE act as levers.
- Ends A & E of these levers are fixed.
- The AB & DE are parallel in the mean position.
- Coupling rod BD is perpendicular to the levers AB & DE.

On displacement of the mechanism, the tracing point '**C**' traces the shape of number '**8**', a portion of which will be approximately straight line.

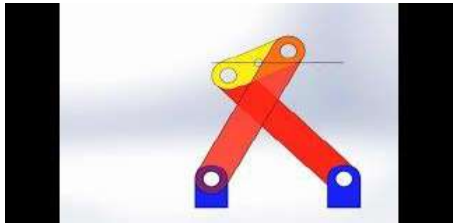
Tchebicheff's Mechanism.

It is a four bar mechanism in which the crossed links OA and $O1B$ are of equal length, as shown in Fig. The point P, which is the mid-point of $A\ B$ traces out an approximately straight line parallel to $OO1$.

The proportions of the links are, usually, such that point P is exactly above O or $O1$ in the extreme positions of the mechanism *i.e.* when $B\ A$ lies along OA or when AB lies along $BO1$.



It may be noted that the point P will lie on a straight line parallel to $OO1$, in the two extreme positions and in the mid position, if the lengths of the links are in proportions $A\ B : OO1 : OA = 1 : 2 : 2.5$.



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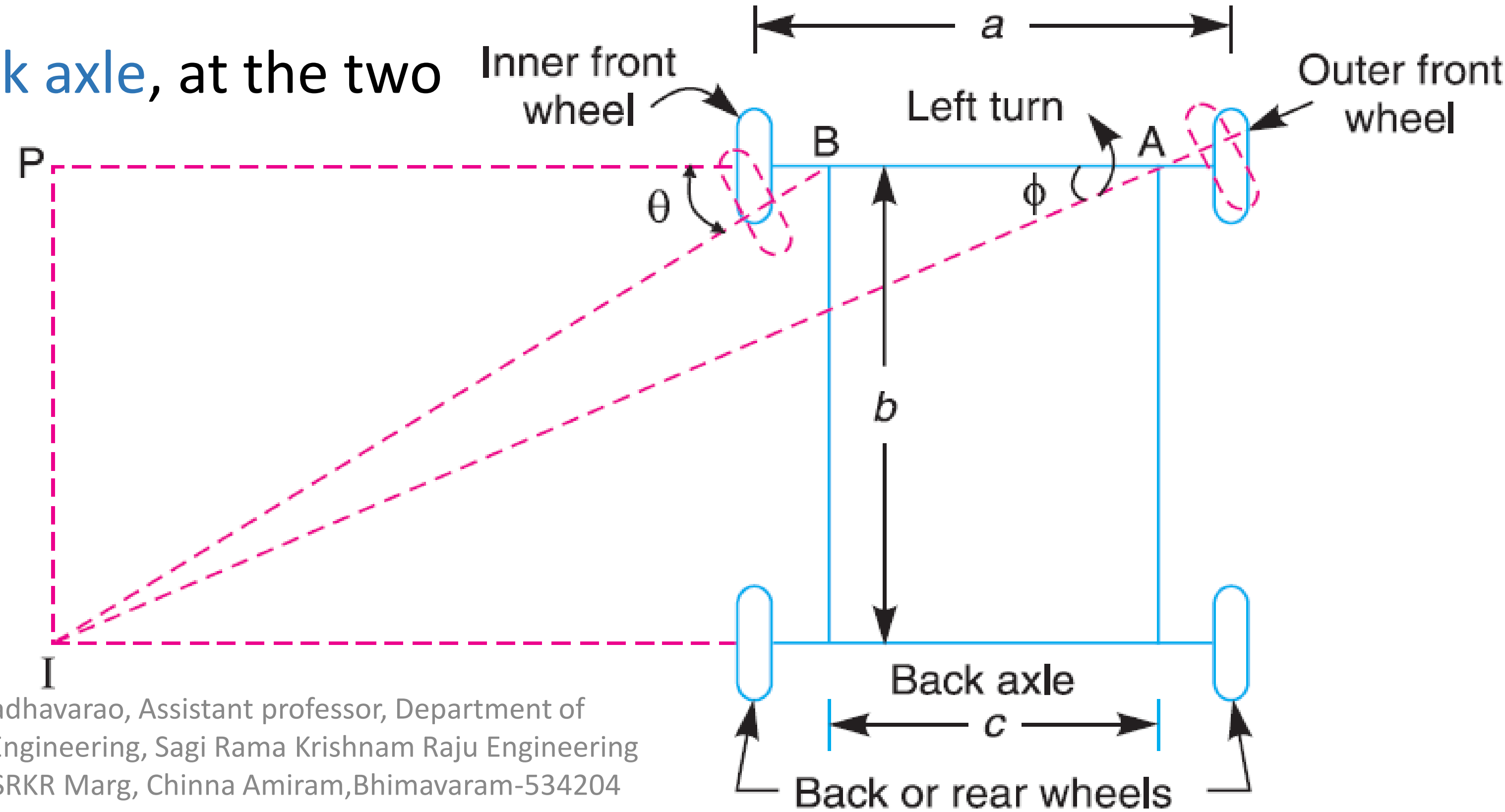
• STEERING GEAR MECHANISM

- In automobile, the front wheels are mounted over the front axles.
- The wheels are pivoted at the points A and B. These points are fixed to the chassis.
- Rear wheels are placed over the back axle, at the two ends of the differential tube.
- **When the vehicle makes a turn**, the front wheels along with the respective axles turn about the respective pivoted points.



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STEERING GEAR MECHANISM

The **rear wheels remain straight and do not turn**. Therefore, steering is done by means of front wheels only.

In order to avoid skidding (i.e. slipping), the two front wheels must turn about the same instantaneous center **"I"** which lies on the axis of the back wheels

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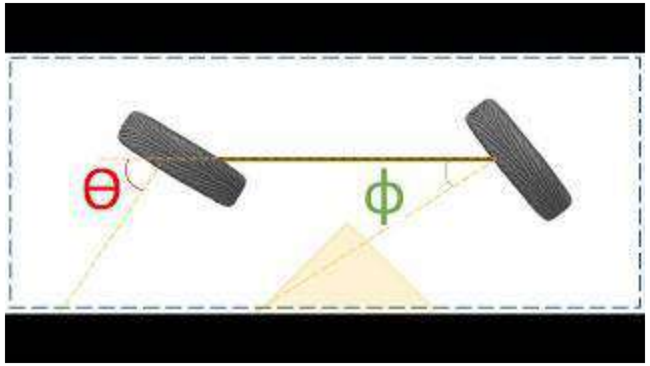
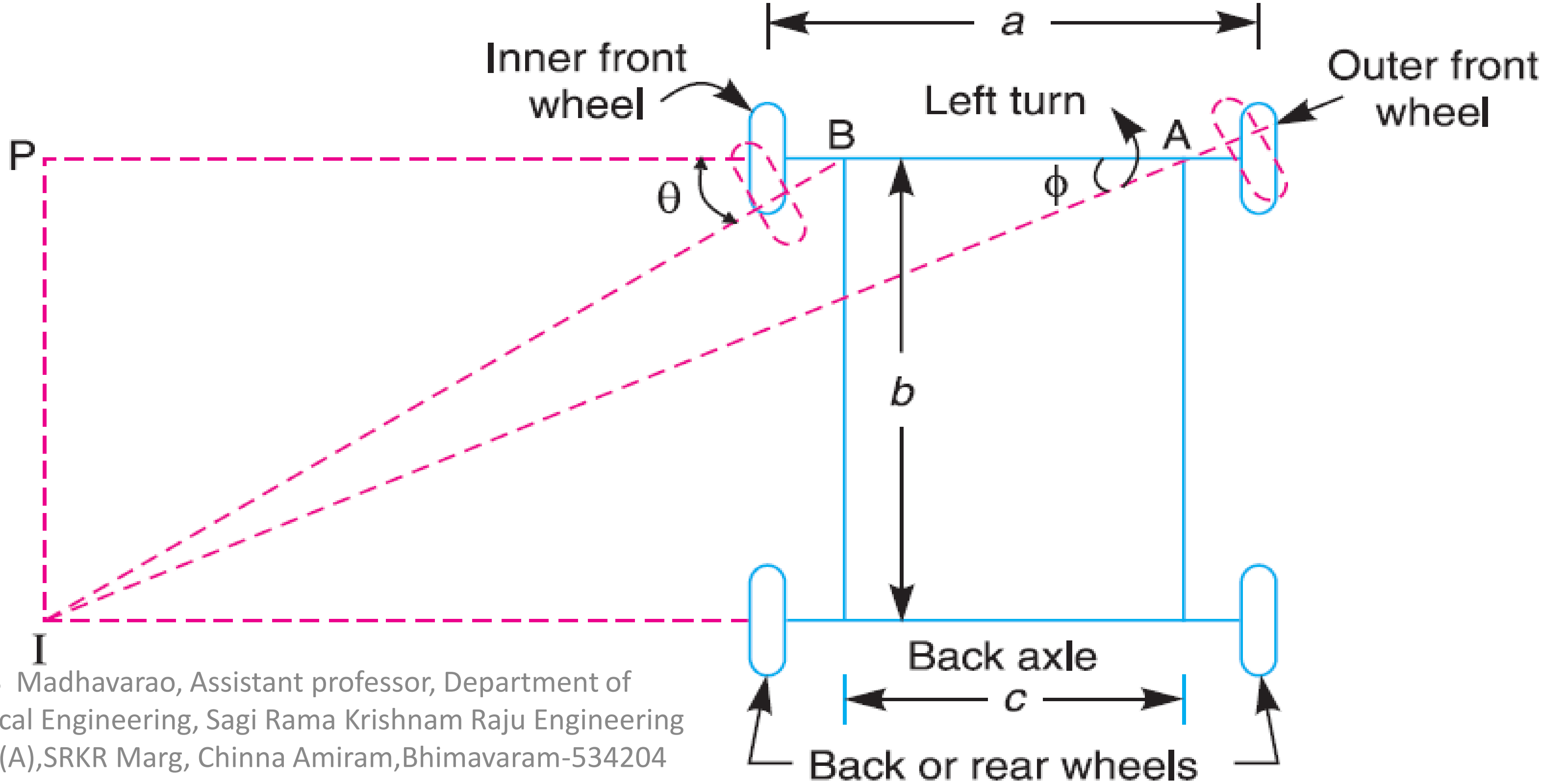
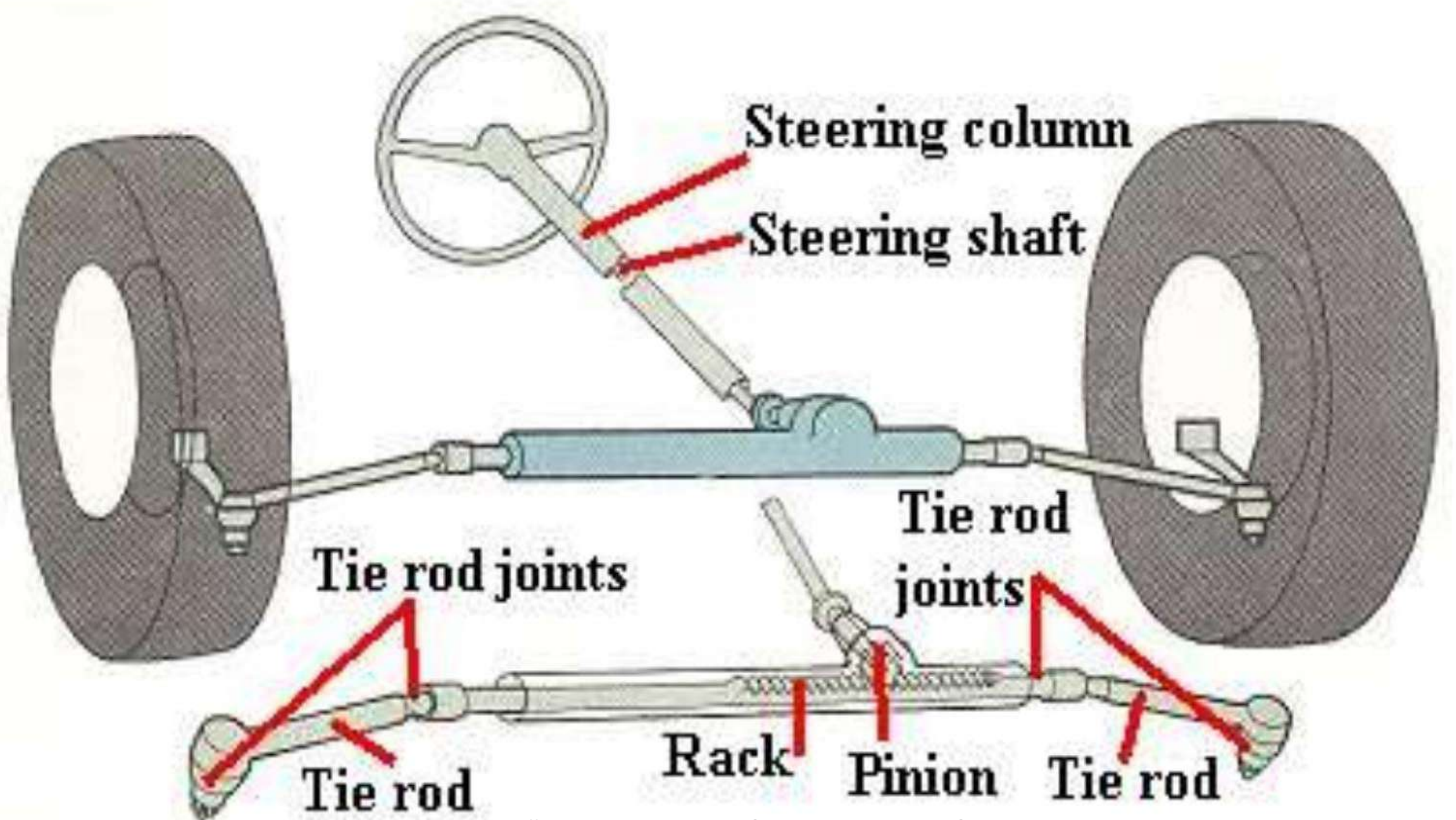




What is the condition for perfect steering?

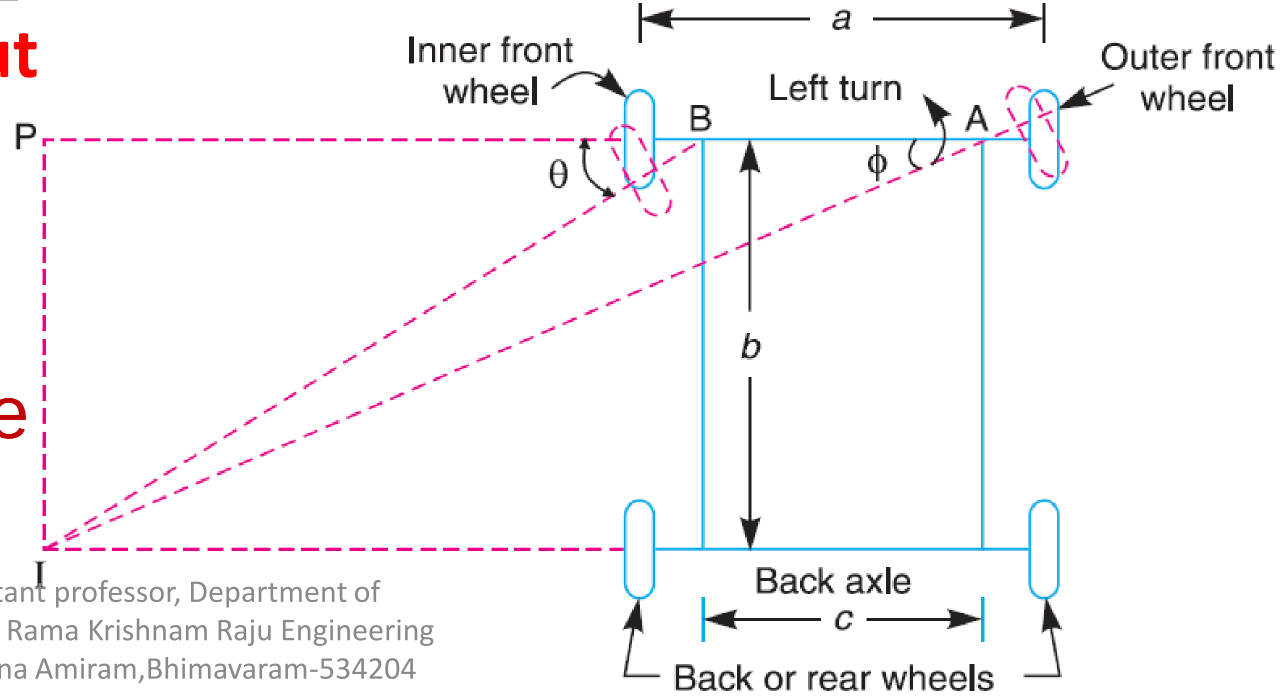
STEERING GEAR MECHANISM



- **Condition for perfect steering**
  - In order to avoid skidding (i.e. slipping of the wheels sideways), the two front wheels must turn about the same instantaneous centre I which lies on the axis of the back wheels.
  - If the instantaneous centre of the two front wheels do not coincide with the instantaneous centre of the back wheels, the skidding on the front or back wheels will definitely take place, which will cause more wear and tear of the tyres.

Thus, the condition for correct steering is that all the four wheels must turn about the same instantaneous centre.

The axis of the inner wheel makes a larger turning angle '**θ**' than the angle '**φ**' subtended by the axis of outer wheel



- **Condition for perfect steering**

Let a = Wheel track,

b = Wheel base, and

c = Distance between the pivots A and B of the front axle.

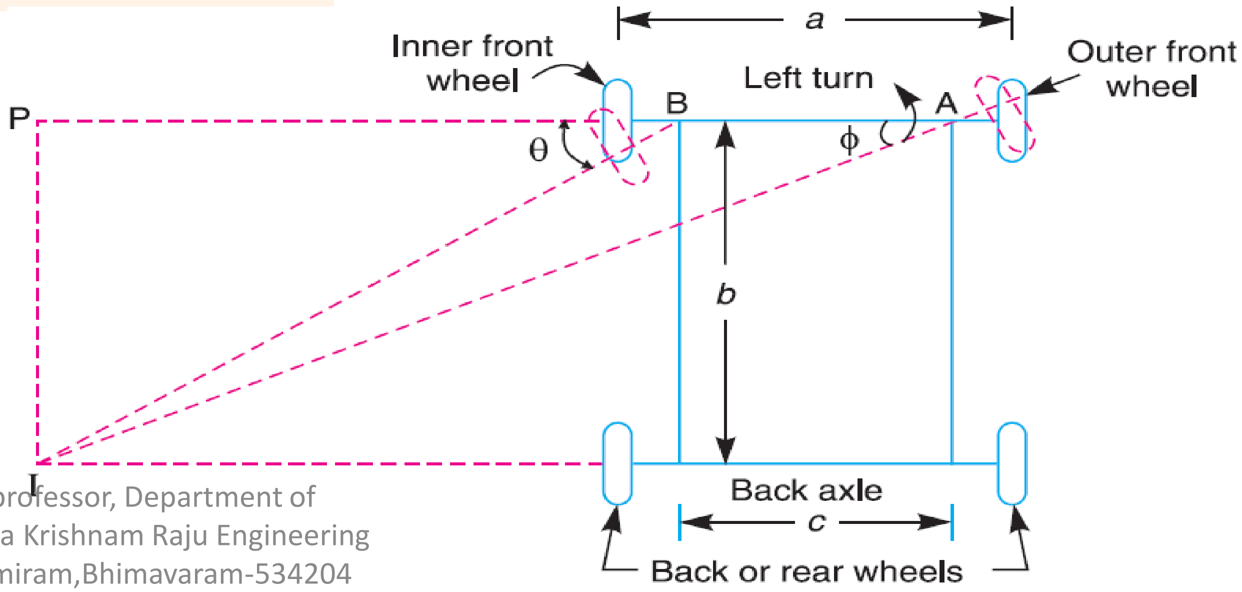
Now from triangle IBP,

$$\cot\theta = \frac{BP}{IP}$$

and from triangle IAP,

$$\cot\phi = \frac{AP}{IP} = \frac{AB + BP}{IP} = \frac{AB}{IP} + \frac{BP}{IP} = \frac{c}{b} + \cot\theta \qquad ...(\because IP = b)$$

$$\therefore \cot\phi - \cot\theta = c / b$$



This is the fundamental equation for correct steering. If this condition is satisfied, there will be no skidding of the wheels, when the vehicle takes a turn.

Davis Steering Gear

The Davis steering gear is shown in Fig. It is an exact steering gear mechanism. The slotted links *A M* and *BH* are attached to the front wheel axle, which turn on pivots *A* and *B* respectively. The rod *CD* is constrained to move in the direction of its length, by the sliding members at *P* and *Q*.

These constraints are connected to the slotted link *A M* and *BH* by a sliding and a turning pair at each end.

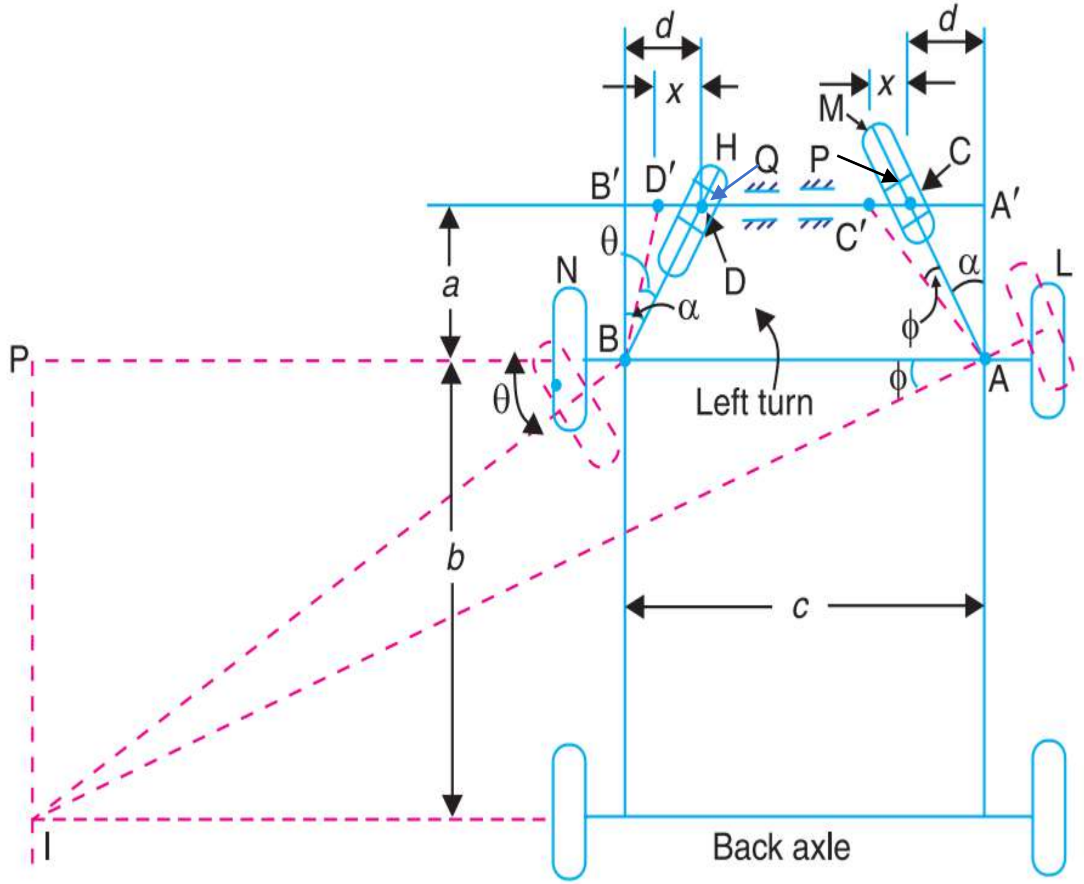
The steering is affected by moving *CD* to the right or left of its normal position. *C′D′* shows the position of *CD* for turning to the left.

Let a = Vertical distance between A B and CD,

b = Wheel base,

d = Horizontal distance between AC and BD,

c = Distance between the pivots A and B of the front axle.



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x = Distance moved by AC to $AC' = CC' = DD'$, and

α = Angle of inclination of the links AC and BD, to the vertical.

From triangle $AA'C'$,

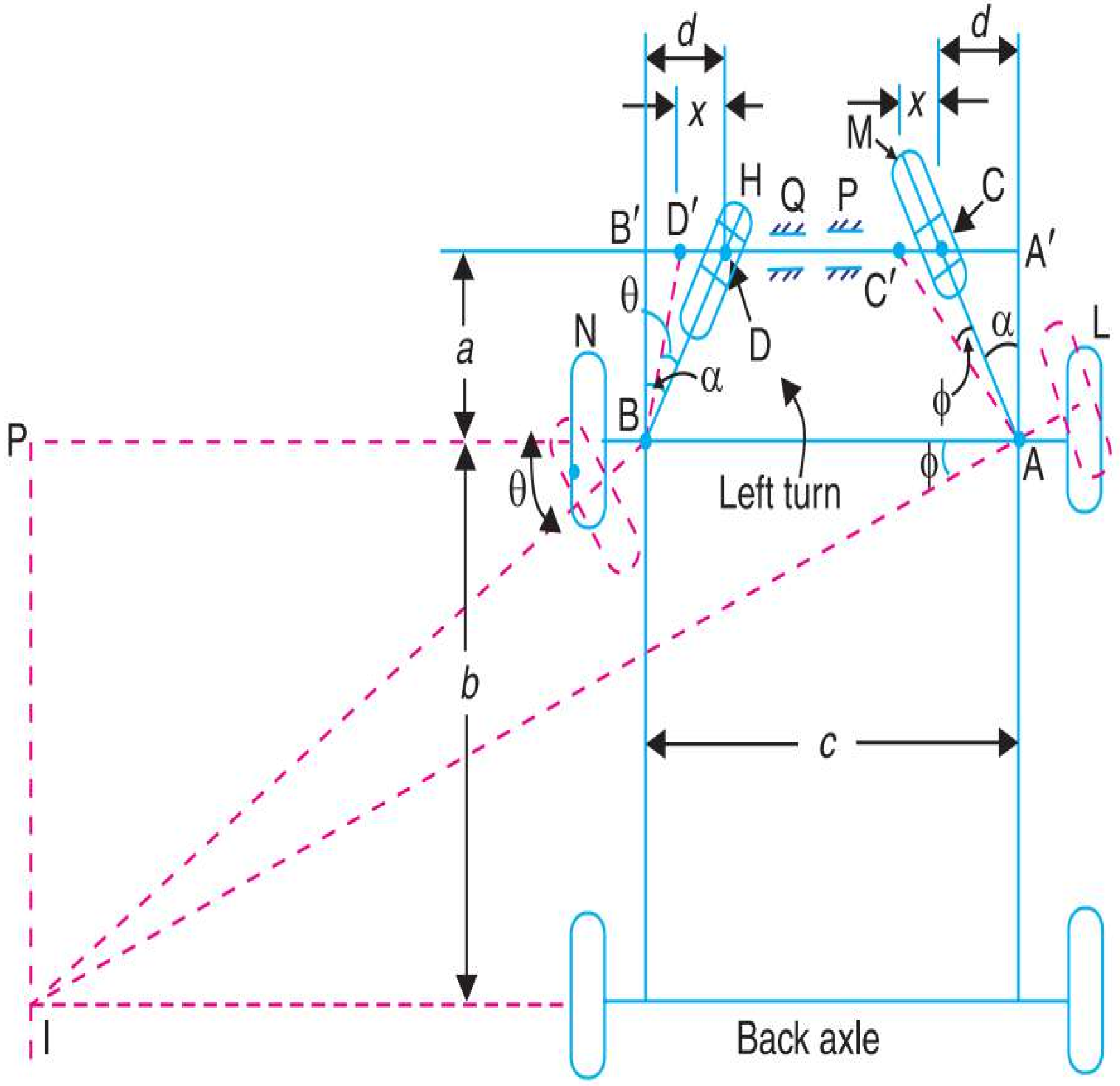
$$\tan(\alpha + \phi) = \frac{A'C'}{AA'} = \frac{d + x}{a} \quad \ldots(i)$$

From triangle $AA'C$,

$$\tan \alpha = \frac{A'C}{AA'} = \frac{d}{a} \quad \ldots(ii)$$

From triangle $BB'D'$,

$$\tan(\alpha - \theta) = \frac{B'D'}{BB'} = \frac{d - x}{a} \quad \ldots(iii)$$



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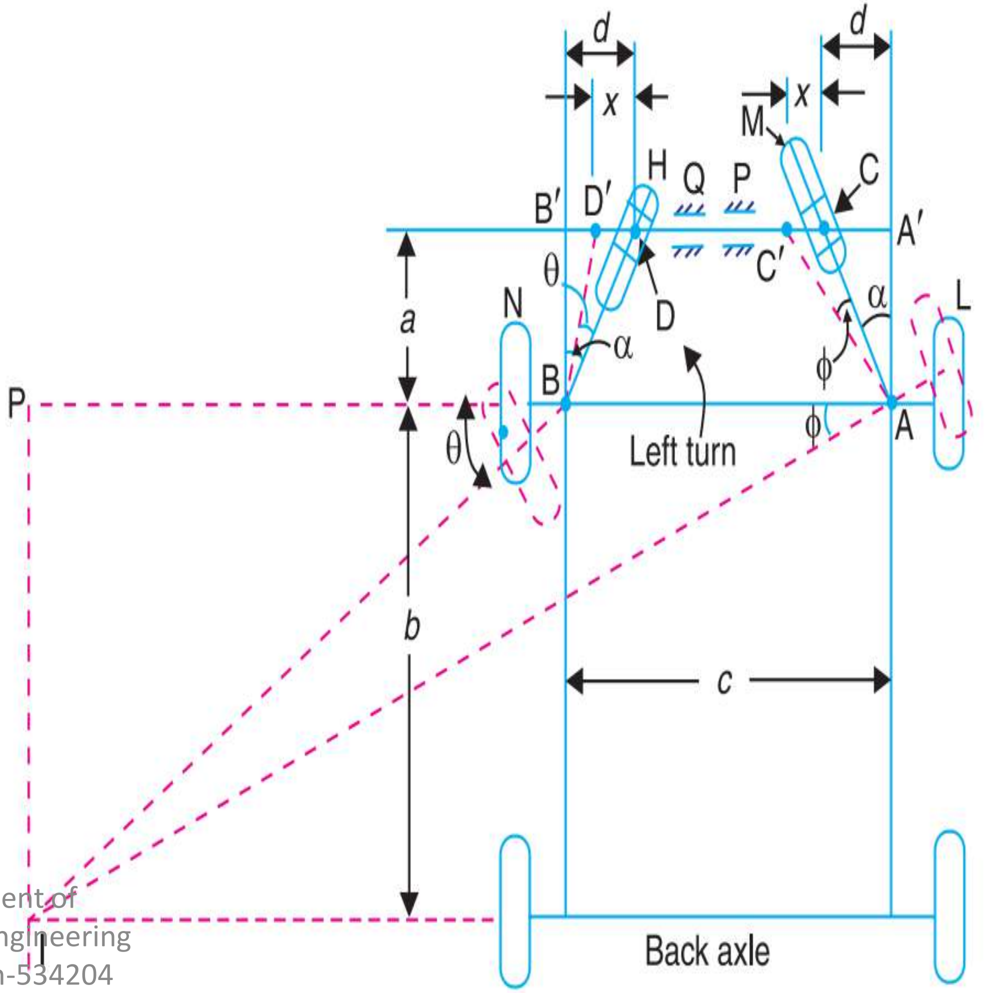
We know that

$$\tan(\alpha + \phi) = \frac{\tan\alpha + \tan\phi}{1 - \tan\alpha.\tan\phi}$$

$$\frac{d + x}{a} = \frac{d/a + \tan\phi}{1 - d/a \times \tan\phi} = \frac{d + a\tan\phi}{a - d\tan\phi}$$

[From equations ***(i)*** and ***(ii)***]



$$(d + x)(a - d\tan\phi) = a(d + a\tan\phi)$$

$$a.d - d^2\tan\phi + a.x - d.x\tan\phi = a.d + a^2\tan\phi$$

$$\tan\phi\,(a^2 + d^2 + d.x) = ax \quad \text{or} \quad \tan\phi = \frac{a.x}{a^2 + d^2 + d.x}$$

..***(iv)***

Similarly, from $\tan(\alpha - \theta) = \frac{d - x}{a}$, we get

$$\tan\theta = \frac{ax}{a^2 + d^2 - d.x}$$

..***(v)***

We know that for correct steering,

$$\cot\phi - \cot\theta = \frac{c}{b} \quad \text{or} \quad \frac{1}{\tan\phi} - \frac{1}{\tan\theta} = \frac{c}{b}$$

$$\frac{a^2 + d^2 + d.x}{a.x} - \frac{a^2 + d^2 - d.x}{a.x} = \frac{c}{b}$$

..[From equations (*iv*) and (*v*)]

$$\frac{2d.x}{a.x} = \frac{c}{b} \quad \text{or} \quad \frac{2d}{a} = \frac{c}{b}$$

$$2\tan\alpha = \frac{c}{b} \quad \text{or} \quad \tan\alpha = \frac{c}{2b} \qquad .(\because d/a = \tan\alpha)$$

Note: Though the gear is theoretically correct, but due to the presence of more sliding members, the wear will be increased which produces slackness between the sliding surfaces, thus eliminating the original accuracy.
Hence Davis steering gear is not in common use.

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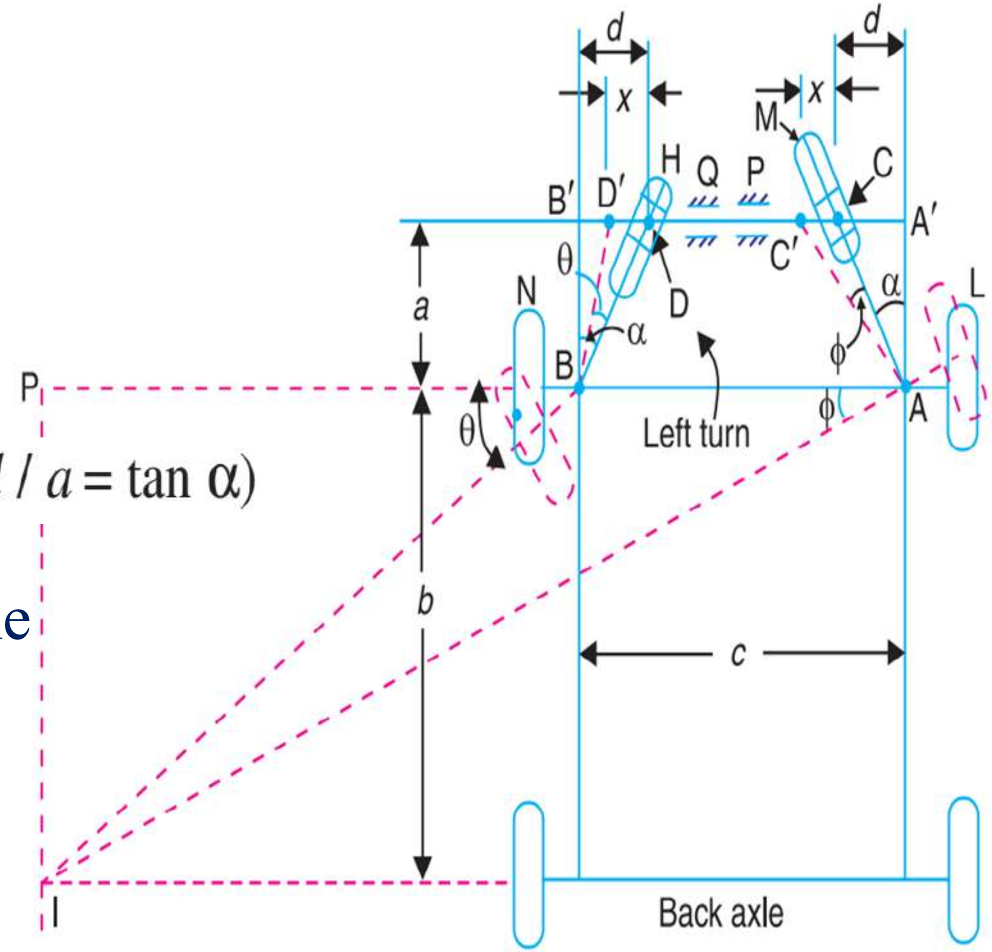
Davis Steering Mechanism

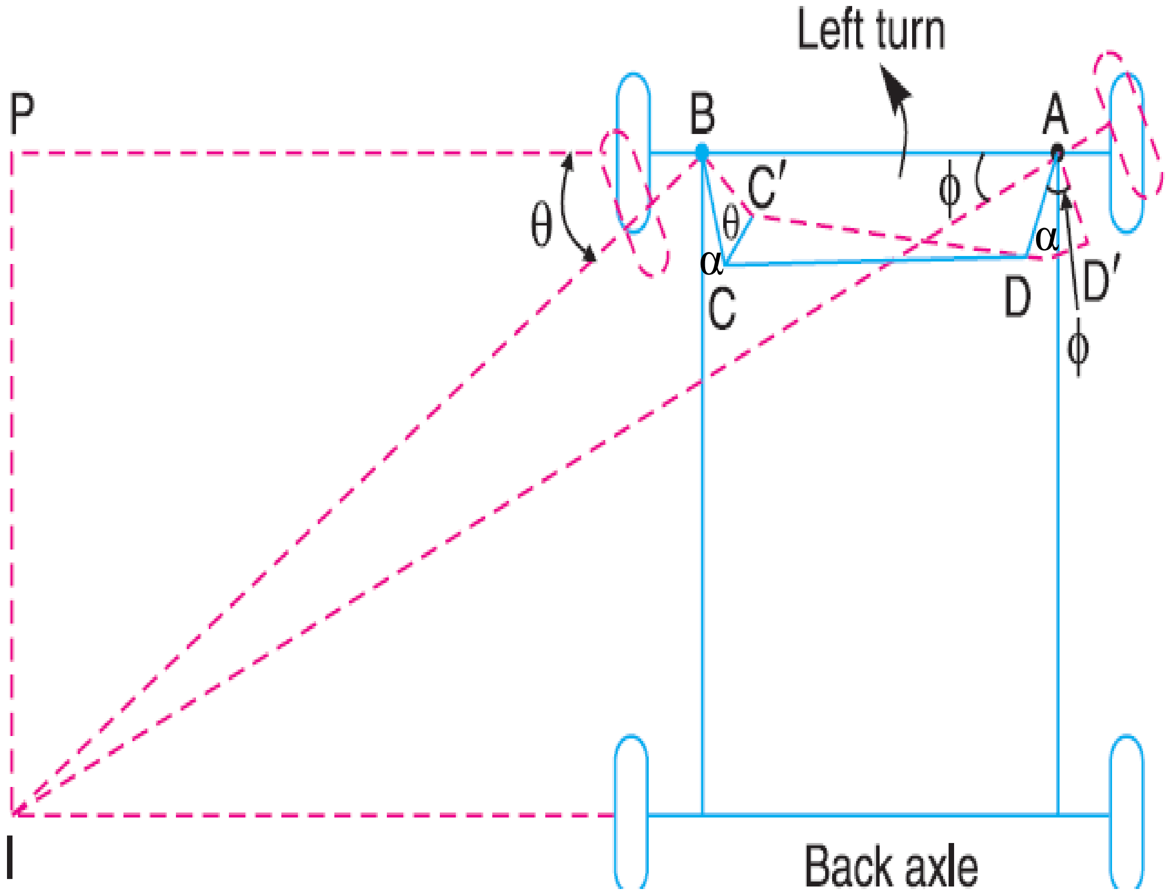


Ackermann steering gear mechanism

- The Ackerman steering gear mechanism is much simpler than Davis gear. The difference between the Ackerman and Davis steering gears are :
  1. The whole mechanism of the Ackerman steering gear is on back of the front wheels; whereas in Davis steering gear, it is in front of the wheels.
  2. The Ackerman steering gear consists of turning pairs, whereas Davis steering gear consists of sliding members

- In Ackerman steering gear, the mechanism ABCD is a four bar crank chain, as shown in Fig. The shorter links BC and AD are of equal length and are connected by hinge joints with front wheel axles.

The longer links AB and CD are of unequal length.



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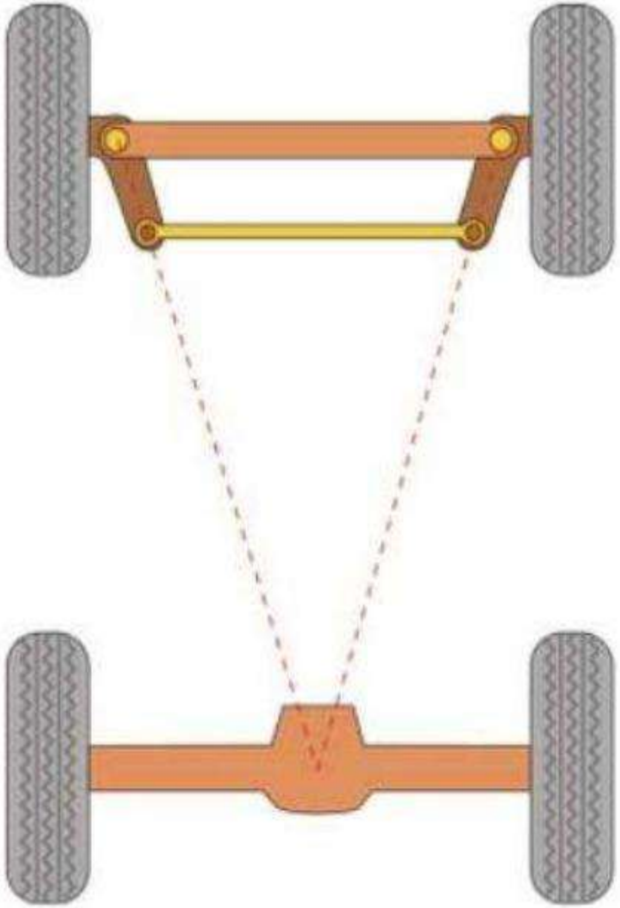
Ackermann steering gear mechanism



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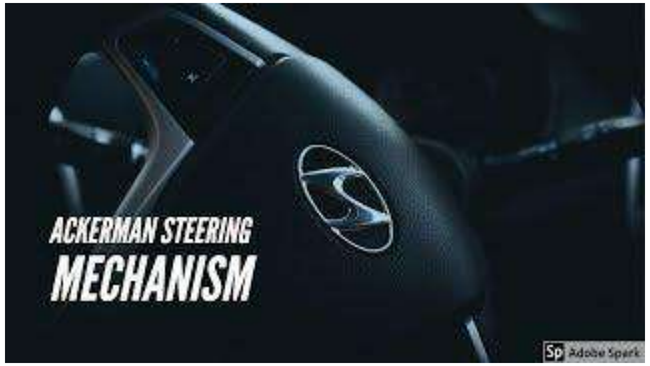

Ackerman Steering Mechanism

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Ackerman Steering | Davis Steering | Principle

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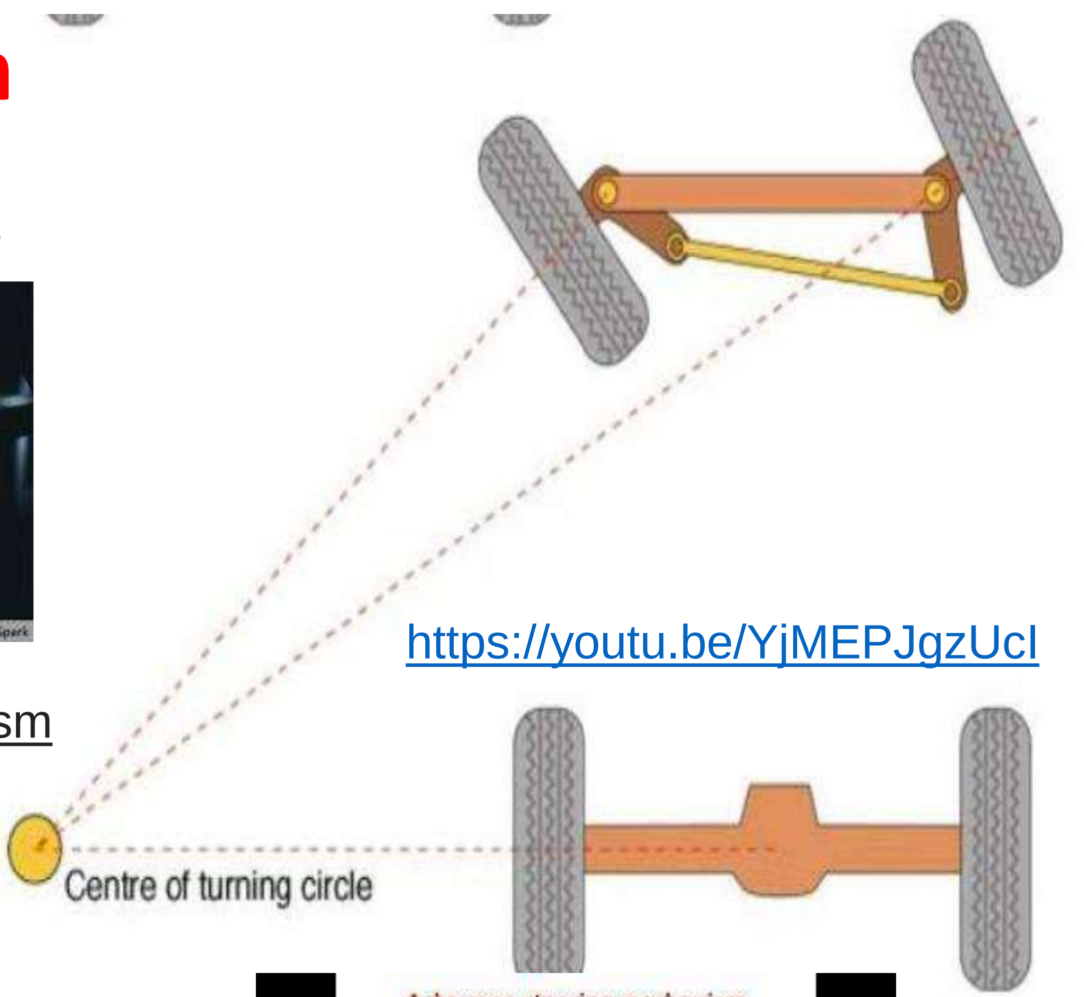
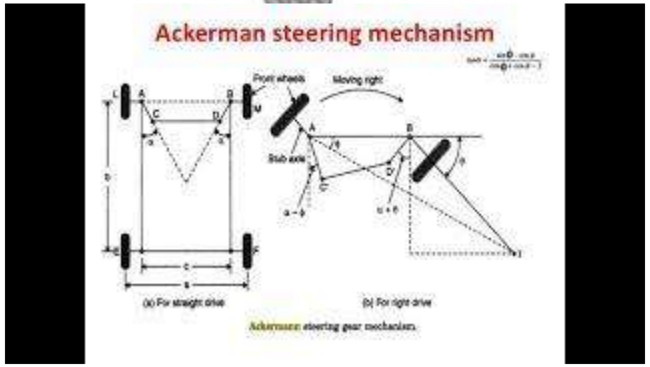




ACKERMANN STEERING MECHANISM

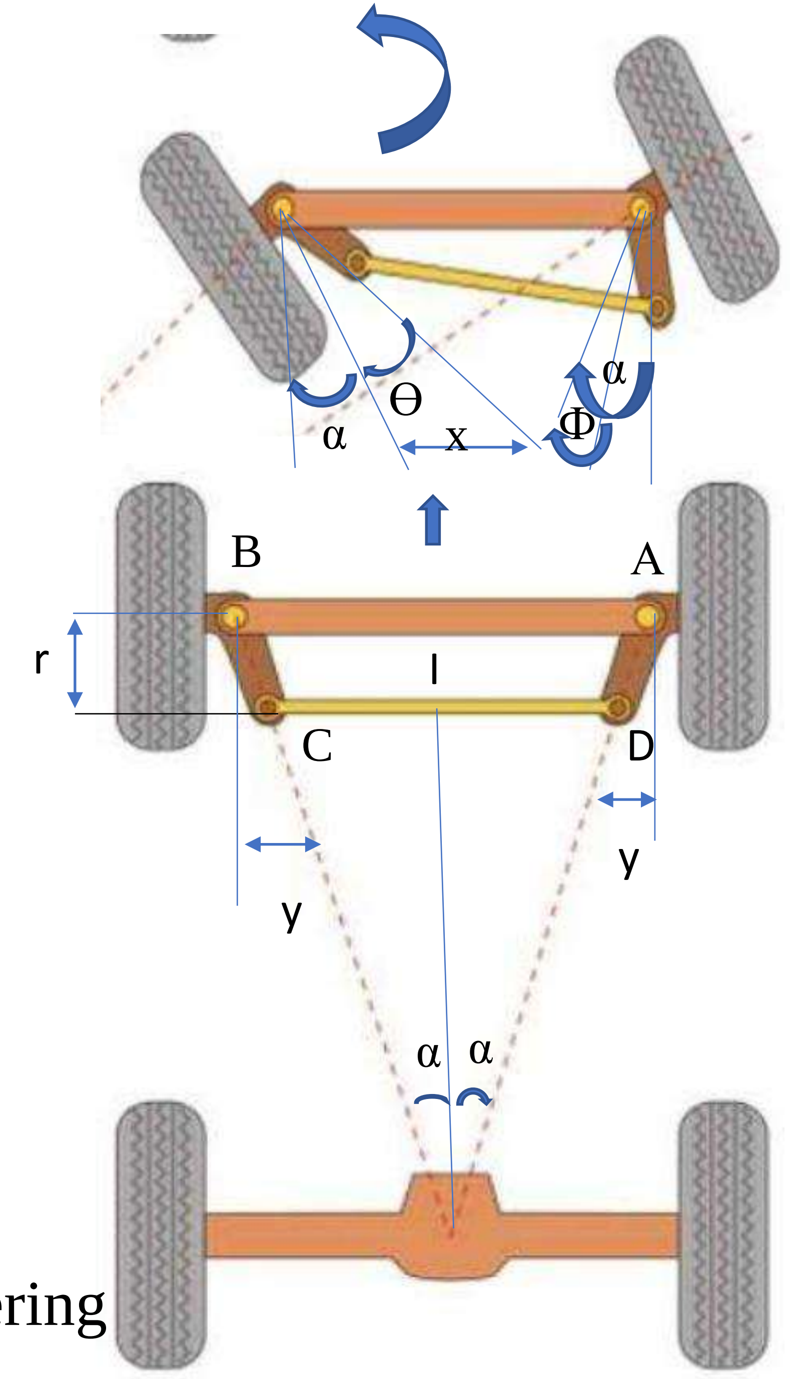
Ө=Angle by which arm BC moves

Φ=Angle by which arm AD moves

l = length of track rod CD

r = length of links BC and AD

x= Hoizantal movement of points C and D



Now, For link BC, the equation can be given as

$$\text{Sin}(\alpha + \Theta) = (y+x)/\,r$$

link AD, the equation can be given as

$$\text{Sin}(\alpha - \Phi) = (y - x)/\,r$$

adding both equation we get

$$\text{Sin}(\alpha + \Theta) + \text{Sin}(\alpha - \Phi) = 2y/\,r$$

$$\text{Sin}(\alpha + \Theta) + \text{Sin}(\alpha - \Phi) = 2\,\text{Sin}\,\alpha$$ Equation for perfect steering

The following are the only three positions for correct steering.

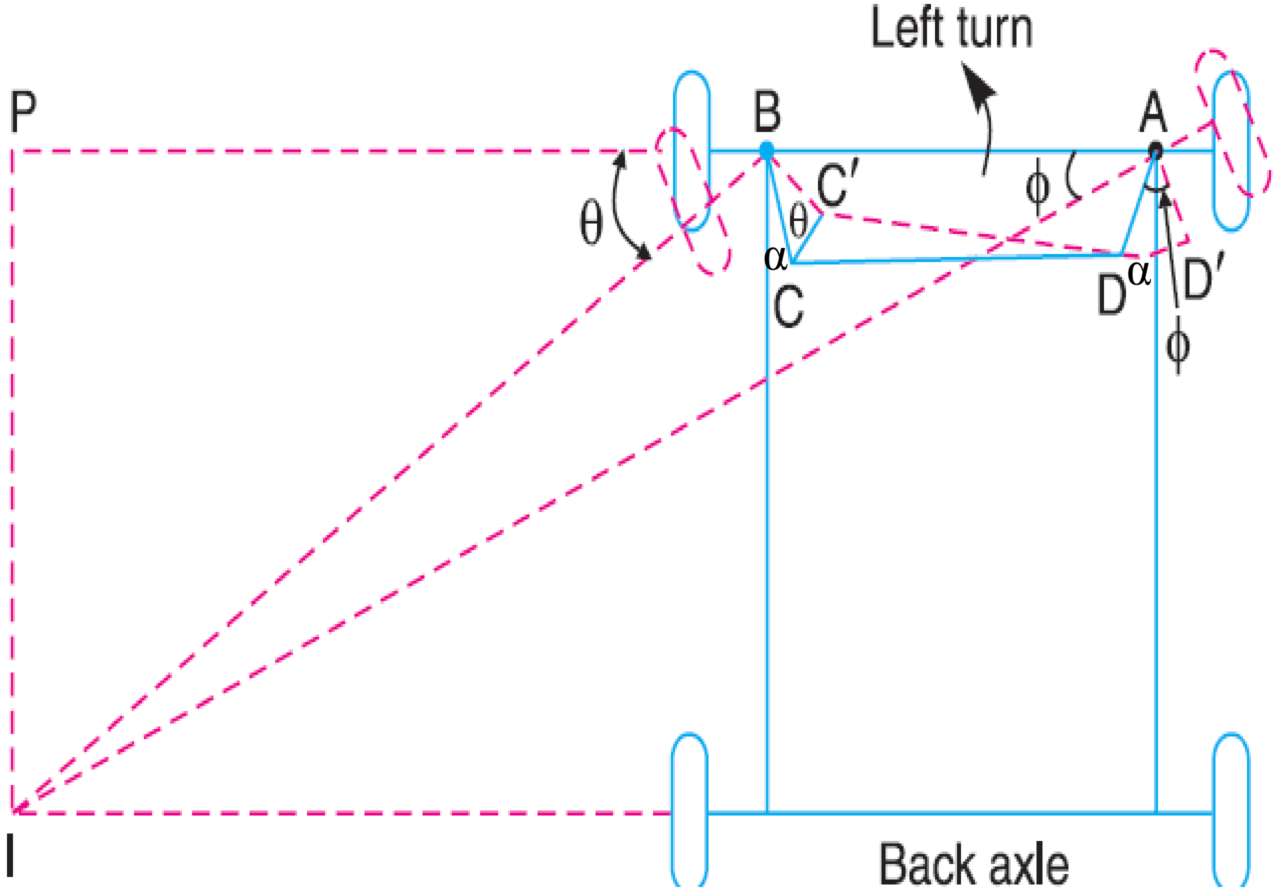
1. When the vehicle moves along a straight path, the longer links AB and CD are parallel and the shorter links BC and AD are equally inclined to the longitudinal axis of the vehicle, as shown by firm lines in Fig..

2. When the vehicle is steering to the left, the position of the gear is shown by dotted lines in Fig. In this position, the lines of the front wheel axle intersect on the back wheel axle at I, for correct steering.

3. When the vehicle is steering to the right,

the similar position may be obtained.

In order to satisfy the fundamental equation for correct steering, as discussed,
The links *AD* and *DC* are suitably proportioned. The value of Θ and Φ may be obtained either graphically or by calculations.



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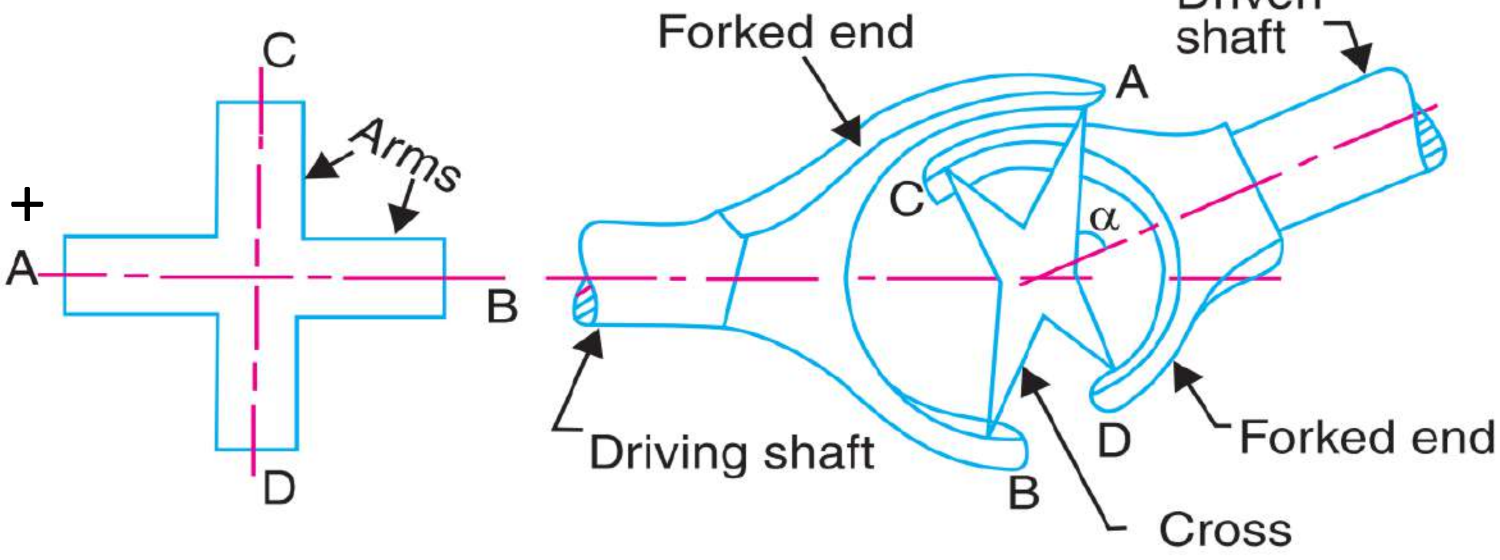
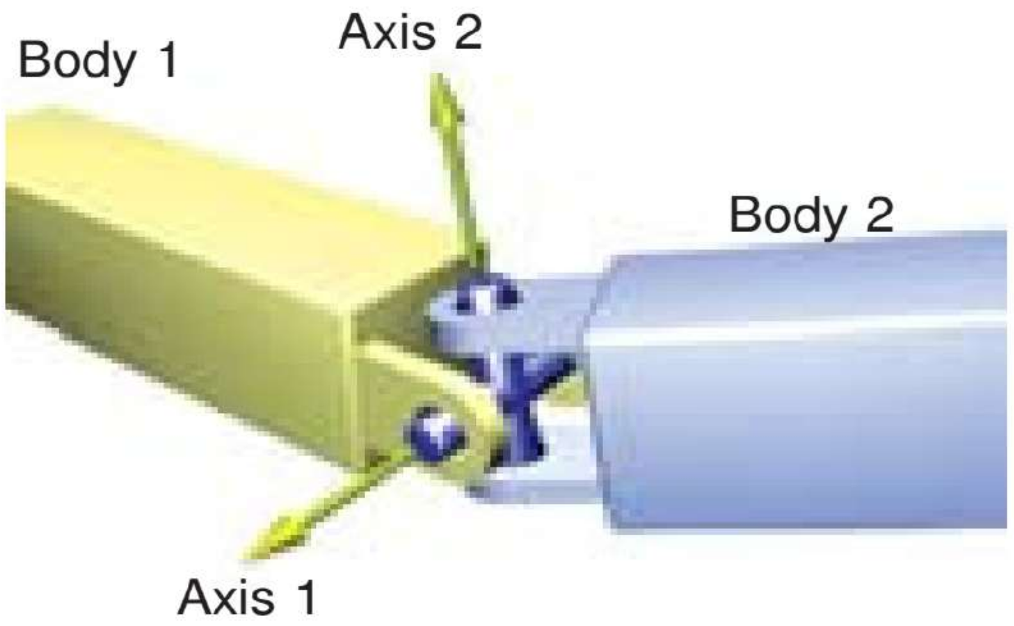
Universal or Hooke's Joint

A *Hooke's joint is used to connect two non-parallel and intersecting shafts. It is also used for shafts with angular misalignment as shown in Fig. The end of each shaft is forked to U-type and each fork provides two bearings for the arms of a cross. The arms of the cross are perpendicular to each other. The motion is transmitted from the driving shaft to driven shaft through a cross. The driving shaft rotates at a uniform angular speed where as driven shaft rotates at a continuously varying angular speed. The inclination of the two shafts may be constant, but in actual practice it varies, when the motion is transmitted



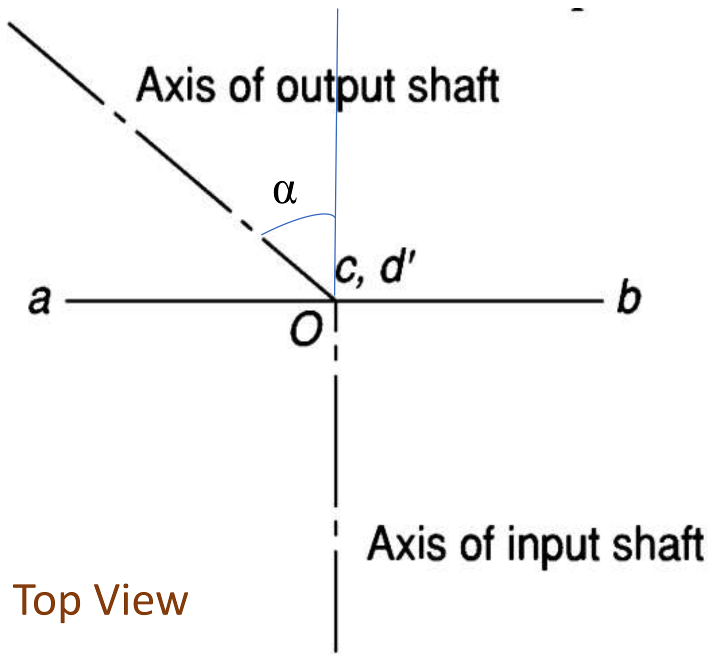


- The main application of the Universal or Hooke's joint is found in the transmission from the **gear box to the differential or back axle of the automobiles. It is also used for transmission of power to different spindles of multiple drilling machine. It is also used as a knee joint in milling machines
- ** In case of automobiles, we use two Hooke's joints one at each end of the propeller shaft, connecting the gear box on one end and the differential on the other end.

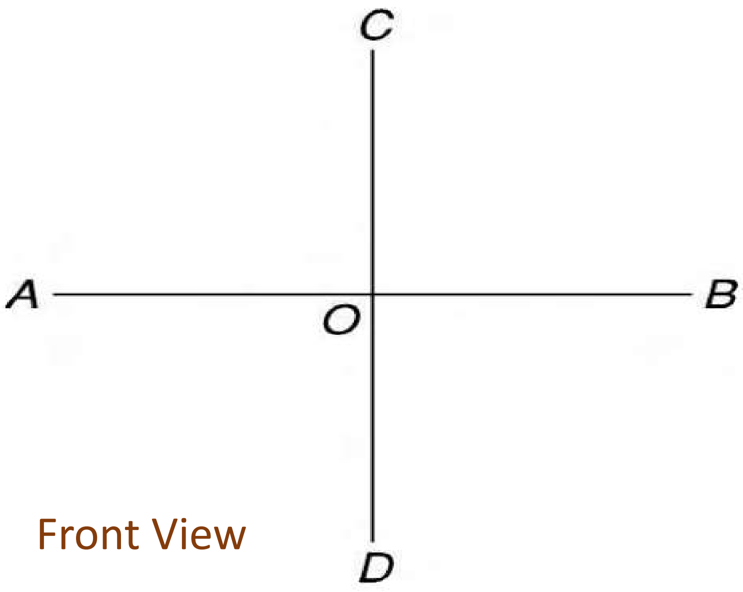
https://youtu.be/y8QaD8NJLxM

universal joint and propeller shaft

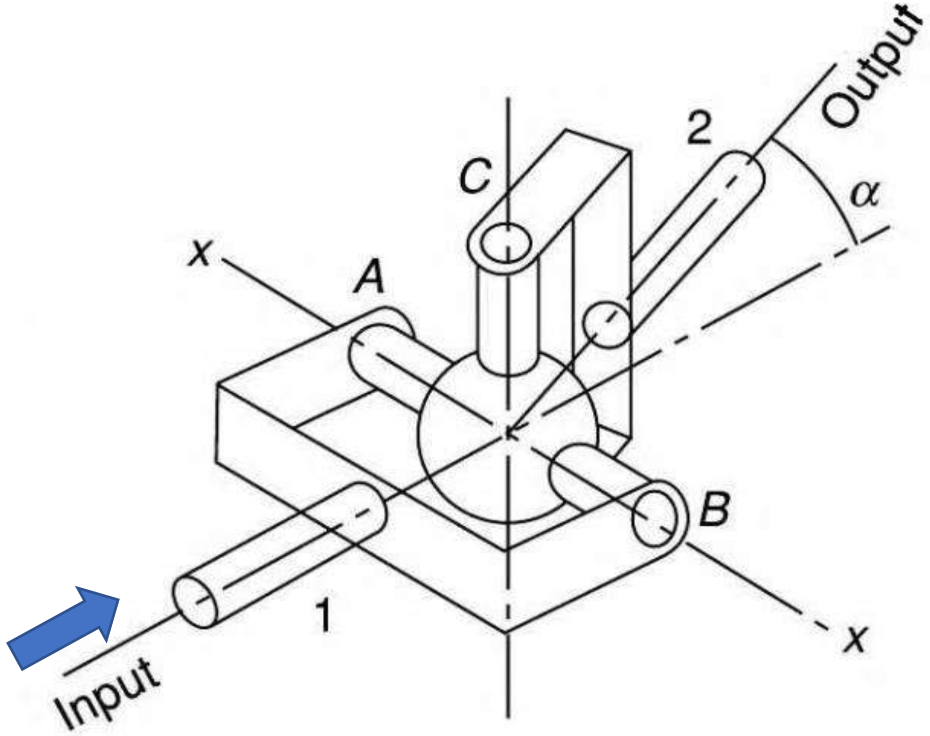
Universal or Hooke's Joint



Top View



Front View



Let two horizontal shafts, the axes of which are at an angle α, be connected by Hooke's joint. If the joint is viewed along the axis of the shaft 1, the fork ends this shaft will be A and B as shown in Fig. C and D are the positions assumed by the fork ends of shaft 2.

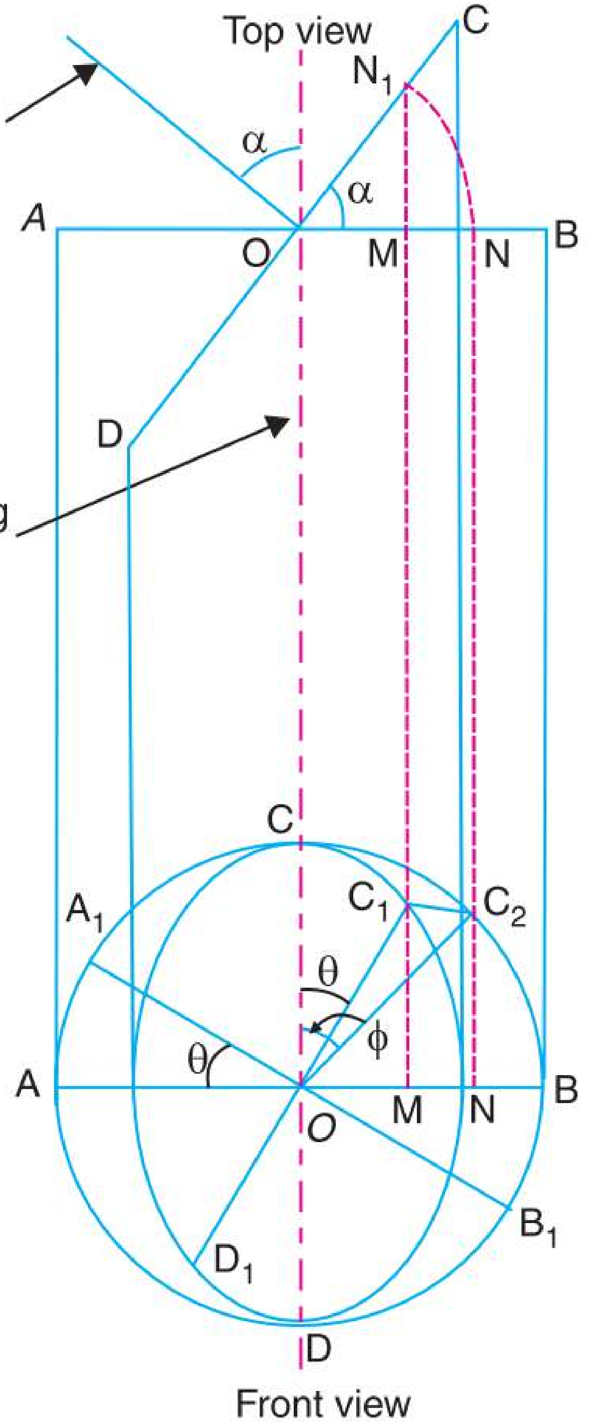
The axis of the shaft 1 is along the perpendicular to the plane of paper at O and that of the shaft 2 along OA. When viewed from top, c and d , projections of C and D coincide with that of O whereas a and b remain unchanged

Ratio of the Shafts Velocities:

The top and front views connecting the two shafts by a universal joint are shown in Fig. 9.19. Let the initial position of the cross be such that both arms lie in the plane of the paper in front view, while the arm A B attached to the driving shaft lies in the plane containing the axes of the two shafts.

Let the driving shaft rotates through an angle θ, so that the arm A B moves in a circle to a new position A1 B1 as shown in front view. A little consideration will show that the arm CD will also move in a circle of the same size. This circle when projected in the plane of paper appears to be an ellipse. Therefore the arm CD takes new position C1D1 on the ellipse, at an angle θ. But the true angle must be on the circular path.

To find the true angle, project the point C1 horizontally to intersect the circle at C2. Therefore the angle COC2 (equal to ϕ) is the true angle turned by the driven shaft. Thus when the driving shaft turns through an angle θ, the driven shaft turns through an angle ϕ. It may be noted that it is not necessary that ϕ may be greater than θ or less than θ. At a particular point, it may be equal to θ.



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In triangle OC_1M, $\angle OC_1M = \theta$

$$\therefore \quad \tan\theta = \frac{OM}{MC_1} \quad \ldots(i)$$

and in triangle OC_2N, $\angle OC_2N = \phi$

$$\therefore \quad \tan\phi = \frac{ON}{NC_2} = \frac{ON}{MC_1} \quad \ldots(\because NC_2 = MC_1) \quad \ldots(ii)$$

Dividing equation (*i*) by (*ii*),

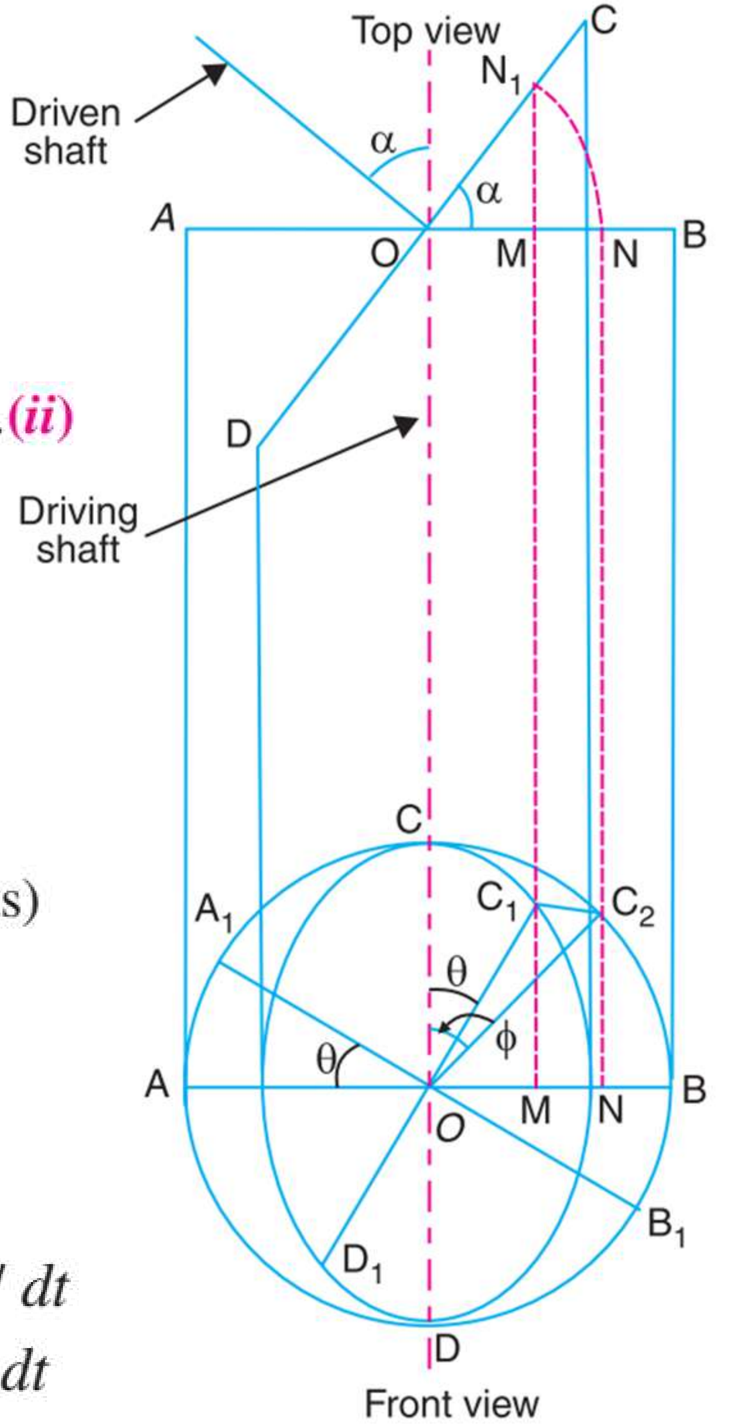
$$\frac{\tan\theta}{\tan\phi} = \frac{OM}{MC_1} \times \frac{MC_1}{ON} = \frac{OM}{ON}$$

But $\quad OM = ON_1 \cos\alpha = ON \cos\alpha$

..(where α = Angle of inclination of the driving and driven shafts)

$$\frac{\tan\theta}{\tan\phi} = \frac{ON\cos\alpha}{ON} = \cos\alpha$$

$$\tan\theta = \tan\phi \cdot \cos\alpha \quad \ldots(iii)$$



Angular Velocity Ratio :

Let $\quad \omega$ = Angular velocity of the driving shaft = $d\theta / dt$

ω_1 = Angular velocity of the driven shaft = $d\phi / dt$

Differentiating both sides of equation (*iii*),

$$\sec^2\theta \times d\theta / dt = \cos\alpha \,.\, \sec^2\phi \times d\phi / dt$$

$$\sec^2\theta \times \omega = \cos\alpha \,.\, \sec^2\phi \times \omega_1$$

$$\frac{\omega_1}{\omega} = \frac{\sec^2\theta}{\cos\alpha.\sec^2\phi} = \frac{1}{\cos^2\theta.\cos\alpha.\sec^2\phi} \qquad ...(iv)$$

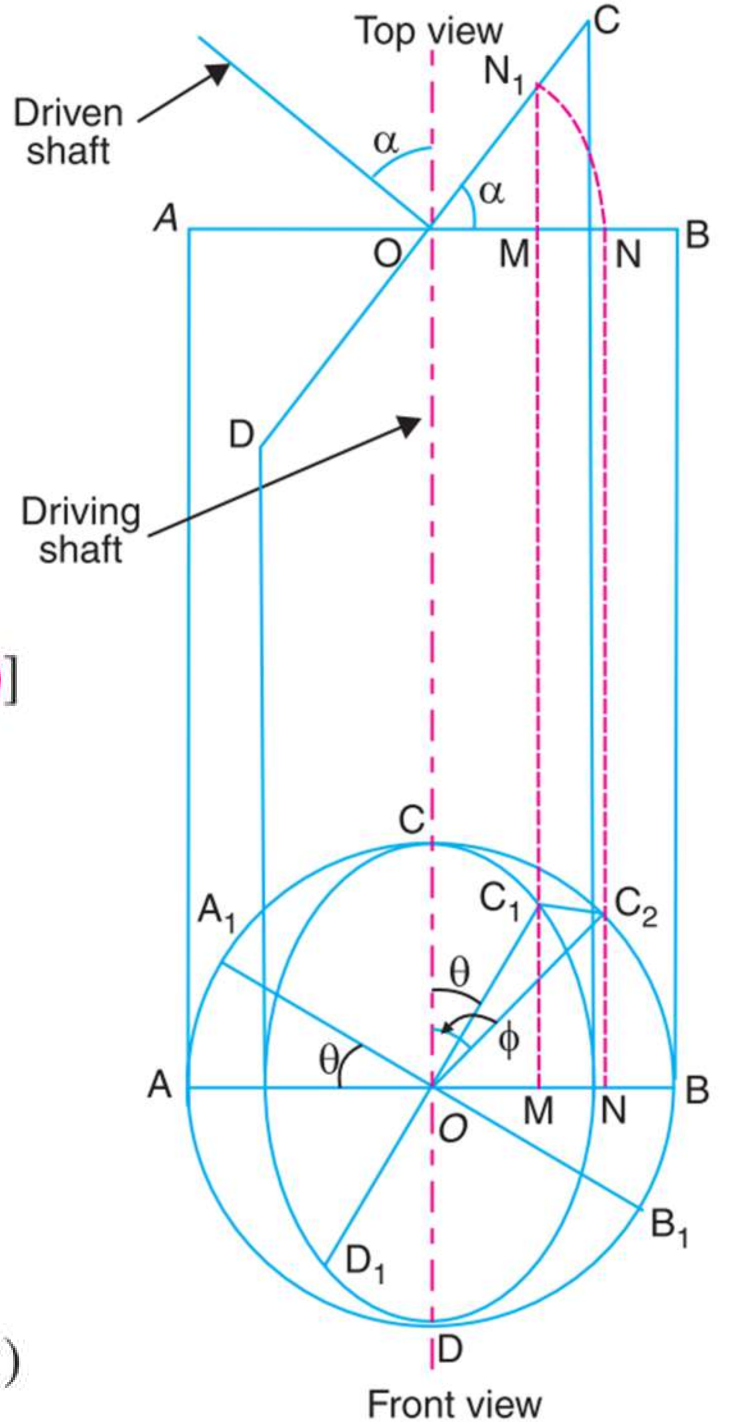
We know that

$$\sec^2\phi = 1 + \tan^2\phi = 1 + \frac{\tan^2\theta}{\cos^2\alpha} \qquad ...[\text{From equation } (iii)]$$

$$= 1 + \frac{\sin^2\theta}{\cos^2\theta.\cos^2\alpha} = \frac{\cos^2\theta.\cos^2\alpha + \sin^2\theta}{\cos^2\theta.\cos^2\alpha}$$

$$= \frac{\cos^2\theta(1 - \sin^2\alpha) + \sin^2\theta}{\cos^2\theta.\cos^2\alpha} = \frac{\cos^2\theta - \cos^2\theta.\sin^2\alpha + \sin^2\theta}{\cos^2\theta.\cos^2\alpha}$$

$$= \frac{1 - \cos^2\theta.\sin^2\alpha}{\cos^2\theta.\cos^2\alpha} \qquad ...(\because \cos^2\theta + \sin^2\theta = 1)$$



Substituting this value of $\sec^2\phi$ in equation (*iv*), we have veloity ratio,

$$\frac{\omega_1}{\omega} = \frac{1}{\cos^2\theta.\cos\alpha} \times \frac{\cos^2\theta.\cos^2\alpha}{1-\cos^2\theta.\sin^2\alpha} = \frac{\cos\alpha}{1-\cos^2\theta.\sin^2\alpha} \quad ...(v)$$

Note: If N = Speed of the driving shaft in r.p.m., and

N_1 = Speed of the driven shaft in r.p.m.

Then the equation (*v*) may also be written as

$$\frac{N_1}{N} = \frac{\cos\alpha}{1-\cos^2\theta.\sin^2\alpha}.$$

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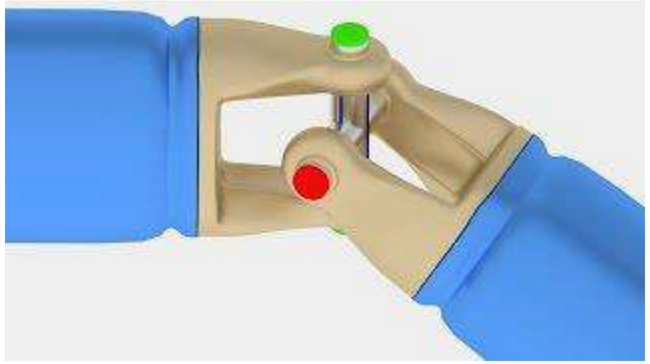
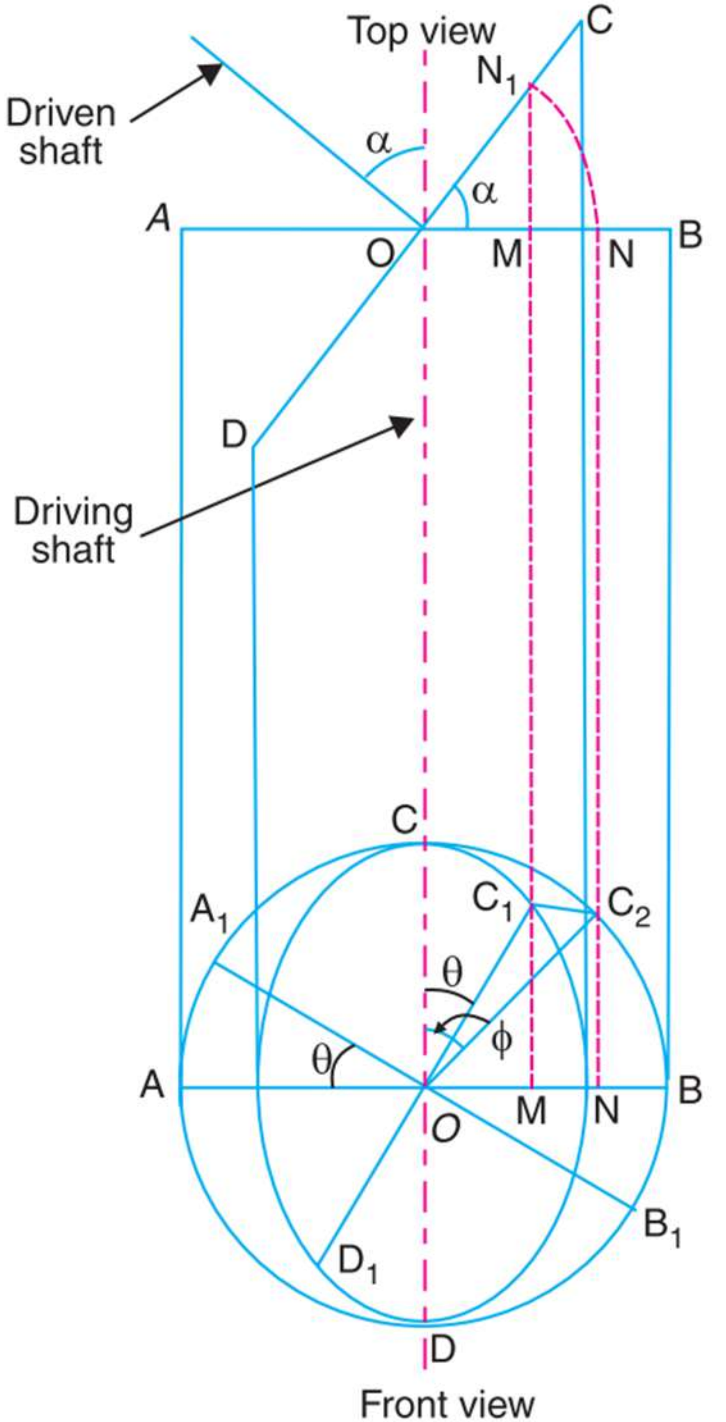


Understanding Universal Joint



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Maximum and Minimum Speeds of Driven Shaft

We have discussed in the previous article that velocity ratio,

$$\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1 - \cos^2\theta.\sin^2\alpha} \quad \text{or} \quad \omega_1 = \frac{\omega.\cos\alpha}{1 - \cos^2\theta.\sin^2\alpha} \qquad ...(i)$$

The value of ω_1 will be maximum for a given value of α, if the denominator of equation (*i*) is minimum. This will happen, when

$$\cos^2\theta = 1, \quad i.e. \text{ when } \theta = 0°, 180°, 360° \text{ etc.}$$

$\therefore$ Maximum speed of the driven shaft,

$$\omega_{1(max)} = \frac{\omega\cos\alpha}{1 - \sin^2\alpha} = \frac{\omega\cos\alpha}{\cos^2\alpha} = \frac{\omega}{\cos\alpha} \qquad ...(ii)$$

$$N_{1(max)} = \frac{N}{\cos\alpha} \qquad \text{...(where } N \text{ and } N_1 \text{ are in r.p.m.)}$$

Similarly, the value of ω_1 is minimum, if the denominator of equation (*i*) is maximum. This will happen, when $(\cos^2\theta \,.\, \sin^2\alpha)$ is maximum, or

$$\cos^2\theta = 0, \; i.e. \text{ when } \theta = 90°, 270° \text{ etc.}$$

∴ Minimum speed of the driven shaft,

$$\omega_{1\,(min)} = \omega \cos \alpha$$

$$N_{1\,(min)} = N \cos \alpha \qquad \text{...(where } N \text{ and } N_1 \text{ are in r.p.m.)}$$

Condition for Equal Speeds of the Driving and Driven Shafts

We have already discussed that the ratio of the speeds of the driven and driving shafts is

$$\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1 - \cos^2\theta.\sin^2\alpha} \quad \text{or} \quad \omega = \frac{\omega_1 (1 - \cos^2\theta.\sin^2\alpha)}{\cos\alpha}$$

For equal speeds, $\omega = \omega_1$, therefore

$$\cos\alpha = 1 - \cos^2\theta\,.\,\sin^2\alpha \quad \text{or} \quad \cos^2\theta\,.\,\sin^2\alpha = 1 - \cos\alpha$$

$$\cos^2\theta = \frac{1 - \cos\alpha}{\sin^2\alpha} \qquad \text{...}(i)$$

Condition for Equal Speeds of the Driving and Driven Shafts

We know that $\sin^2\theta = 1 - \cos^2\theta = 1 - \dfrac{1-\cos\alpha}{\sin^2\alpha} = 1 - \dfrac{1-\cos\alpha}{1-\cos^2\alpha}$

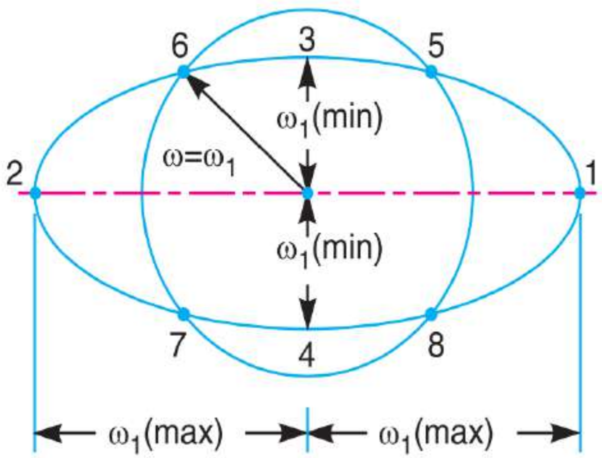
$$= 1 - \frac{1-\cos\alpha}{(1+\cos\alpha)(1-\cos\alpha)} = 1 - \frac{1}{1+\cos\alpha} = \frac{\cos\alpha}{1+\cos\alpha} \quad \ldots(ii)$$

Dividing equation (*ii*) by equation (*i*),

$$\frac{\sin^2\theta}{\cos^2\theta} = \frac{\cos\alpha}{1+\cos\alpha} \times \frac{\sin^2\alpha}{1-\cos\alpha}$$

$$\tan^2\theta = \frac{\cos\alpha\,\sin^2\alpha}{1-\cos^2\alpha} = \frac{\cos\alpha.\sin^2\alpha}{\sin^2\alpha} = \cos\alpha$$

$$\tan\theta = \pm\sqrt{\cos\alpha}$$



There are two values of Θ corresponding to positive sign and two values corresponding to negative sign. Hence, there are four values of Θ, at which the speeds of the driving and driven shafts are same.

Angular Acceleration of the Driven Shaft

We know that $\omega_1 = \dfrac{\omega \cos\alpha}{1 - \cos^2\theta . \sin^2\alpha} = \omega . \cos\alpha (1 - \cos^2\theta . \sin^2\alpha)^{-1}$

Differentiating the above expression, we have the angular acceleration of the driven shaft,

$$\frac{d\omega_1}{dt} = \omega\cos\alpha\left[-1(1-\cos^2\theta\sin^2\alpha)^{-2}\times(2\cos\theta\sin\theta\sin^2\alpha)\right]\frac{d\theta}{dt}$$

$$= \frac{-\omega^2\cos\alpha\times\sin 2\theta . \sin^2\alpha}{(1-\cos^2\theta\sin^2\alpha)^2} \quad \ldots(i)$$

...(2 cos θ sin θ = sin 2 θ, and $d\theta/dt = \omega$)

The negative sign does not show that there is always retardation. The angular acceleration may be positive or negative depending upon the value of sin 2 θ. It means that during one complete revolution of the driven shaft, there is an angular acceleration corresponding to increase in speed of ω1 and retardation due to decrease in speed of ω1.

For angular acceleration to be maximum or minimum when $\dfrac{d\,(acc)}{d\theta} = 0$

The resulting expression being very cumbersome, the result can be approximated to

$$\cos 2\theta = \frac{2\sin^2\alpha}{2 - \sin^2\alpha}$$

Maximum Fluctuation of Speed

We know that the maximum speed of the driven shaft,

$$\omega_{1\,(max)} = \omega/\cos\alpha$$

and minimum speed of the driven shaft,

$$\omega_{1\,(min)} = \omega\cos\alpha$$

∴ Maximum fluctuation of speed of the driven shaft,

$$q = \omega_{1(max)} - \omega_{1(min)} = \frac{\omega}{\cos\alpha} - \omega\cos\alpha$$

$$= \omega\left(\frac{1}{\cos\alpha} - \cos\alpha\right) = \omega\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right) = \frac{\omega\sin^2\alpha}{\cos\alpha}$$

$$= \omega\tan\alpha\,.\,\sin\alpha$$

Since α is a small angle, therefore substituting $\cos\alpha = 1$, and $\sin\alpha = \alpha$ radians.

∴ Maximum fluctuation of speed

$$= \omega\,.\,\alpha^2$$

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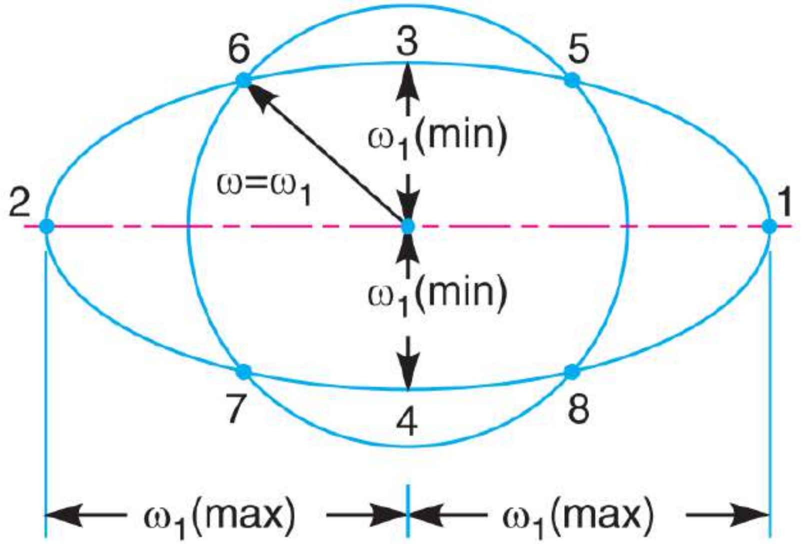
Maximum variation of velocity of the driven shaft of its mean velocity

$$= \frac{\omega_{1\,max} - \omega_{1\,min}}{\omega_{mean}}$$



But ω_{mean} of the driven shaft is equal to the angular velocity ω of the driving shaft as both the shafts complete one revolution in the same period of time

Maximum variation $= \dfrac{\omega / \cos\alpha - \omega \cos\alpha}{\omega}$

$$= \frac{1 - \cos^2\alpha}{\cos\alpha}$$

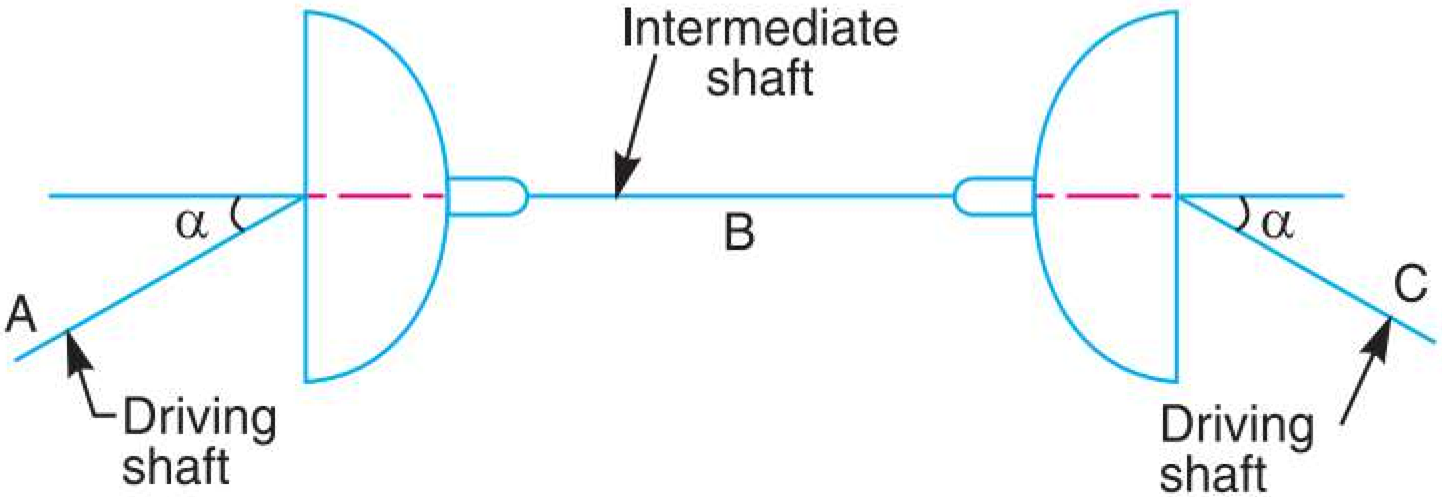
$$= \frac{\sin^2\alpha}{\cos\alpha}$$

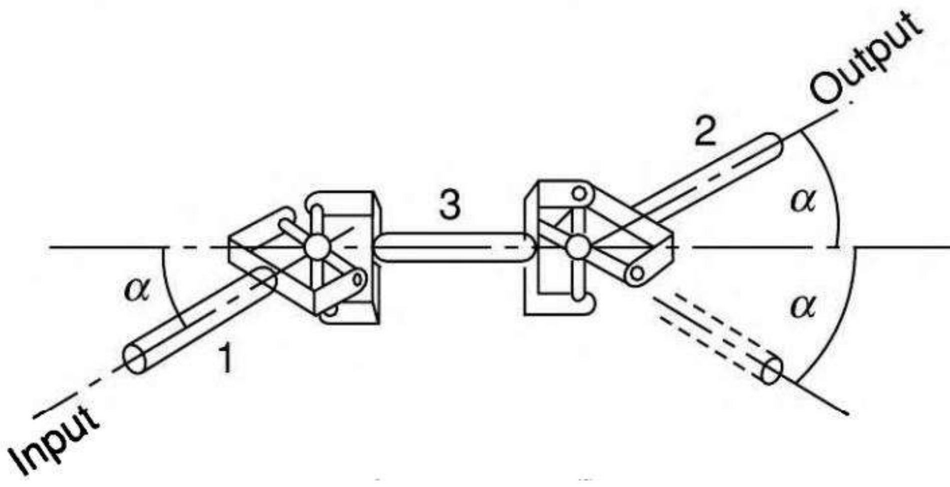
$$= \tan\alpha \sin\alpha$$

(The mean speed ω is not equal to $\dfrac{\omega_{1\,max} + \omega_{1\,min}}{2}$ as the variation of speed is not linear throughout the rotation of the driven shaft.)

Double Hooke's Joint

In a single Hooke's joint, the speed of the driven shaft is not uniform although the driving shaft rotates at a uniform speed. To get uniform velocity ratio, a double Hooke's joint has to be used .In a double Hooke's joint , two universal joints and an intermediate shaft are used . If the angular misalignment between each shaft and intermediate shaft is equal, the driving and driven shafts remain in exact angular alignment , through the intermediate shaft rotates with varying speed.





Double Hooke's Joint

A single Hooke's joint was analysed assuming the axes of the two shafts and the fork of the driving shaft to be horizontal. The results showed that the speed of the driven shaft is the same after an angular displacement of 180 . Therefore, it is immaterial whether the driven shaft makes the angle α with the axis of the driving shaft to its left or right

Thus, to have a constant velocity ratio

1) The driving and driven shafts should make equal angles with the intermediate shaft, and

2) The fork of the intermediate shaft should lie in the same plane

Let γ be the angle turned by the intermediate shaft 3 while the angle turned by the driving shaft 1 and the driven shaft 2 to be Θ and Φ respectively as before

Then,

$$\tan\Theta = \cos\alpha \tan\gamma \qquad \text{(fork of shaft 1 horizontal)}$$

and

$$\tan\Phi = \cos\alpha \tan\gamma \qquad \text{(fork of shaft 2 horizontal)}$$

Therefore $\Theta = \Phi$

This type of joint can be used for two intersecting as well as for two parallel shafts. However, if somehow the forks of the intermediate shafts lie in planes perpendicular to each other, the variation of speed of the driven shaft will be there

$$\left(\frac{\omega_3}{\omega_1}\right)_{min} = \cos\alpha$$ (fork of shaft 1 horizontal)

$$\left(\frac{\omega_2}{\omega_3}\right)_{min} = \cos\alpha$$ (fork of shaft 1 horizontal)

https://youtu.be/y8QaD8NJLxM

$$\left(\frac{\omega_2}{\omega_1}\right)_{min} = \cos^2\alpha$$

Similarly $$\left(\frac{\omega_2}{\omega_1}\right)_{max} = \frac{1}{\cos^2\alpha}$$

Therefore, the maximum variation (fluctuation) of speed of the driven shaft is from $\cos^2\alpha$ to $1/\cos^2\alpha$.

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Q1.Two shafts with an included angle of 160° are connected by a Hooke's joint. The driving shaft runs at a uniform speed of 1500 r.p.m. The driven shaft carries a flywheel of mass12 kg and 100 mm radius of gyration. Find the maximum angular acceleration of the driven shaft and the maximum torque required

Solution. Given : $\alpha = 180° - 160° = 20°$; $N = 1500$ r.p.m.; $m = 12$ kg ; $k = 100$ mm $= 0.1$ m

We know that angular speed of the driving shaft,

$$\omega = 2\,\pi \times 1500 / 60 = 157 \text{ rad/s}$$

and mass moment of inertia of the driven shaft,

$$I = m.k^2 = 12\,(0.1)^2 = 0.12 \text{ kg - m}^2$$

Maximum angular acceleration of the driven shaft

Let $d\omega_1 / dt$ = Maximum angular acceleration of the driven shaft, and

θ = Angle through which the driving shaft turns.

We know that, for maximum angular acceleration of the driven shaft,

$$\cos 2\theta = \frac{2\sin^2\alpha}{2 - \sin^2\alpha} = \frac{2\sin^2 20°}{2 - \sin^2 20°} = 0.124 \qquad 2\theta = 82.9° \quad \text{or} \quad \theta = 41.45°$$

$$\frac{d\omega_1}{dt} = \frac{\omega^2 \cos\alpha . \sin 2\theta . \sin^2 \alpha}{(1 - \cos^2 \theta . \sin^2 \alpha)^2}$$

$$= \frac{(157)^2 \cos 20° \times \sin 82.9° \times \sin^2 20°}{(1 - \cos^2 41.45° \times \sin^2 20°)^2} = 3090 \text{ rad/s}^2$$ **Ans.**

Maximum torque required

We know that maximum torque required

$$= I \times d\omega_1 / dt = 0.12 \times 3090 = 371 \text{ N-m}$$ **Ans.**

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Q2.The angle between the axes of two shafts connected by Hooke's joint is 18°. Determine the angle turned through by the driving shaft when the velocity ratio is maximum and unity

Solution. Given : $\alpha = 18^\circ$

Let θ = Angle turned through by the driving shaft.

When the velocity ratio is maximum

We know that velocity ratio,

$$\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1 - \cos^2\theta.\sin^2\alpha}$$

The velocity ratio will be maximum when $\cos^2\theta$ is minimum, *i.e.* when

$$\cos^2\theta = 1 \quad \text{or} \quad \text{when } \theta = 0^\circ \quad \text{or} \quad 180^\circ$$ **Ans.**

When the velocity ratio is unity

The velocity ratio (ω / ω_1) will be unity, when

$$1 - \cos^2 \theta \,.\, \sin^2 \alpha = \cos \alpha \quad \text{or} \quad \cos^2 \theta = \frac{1 - \cos \alpha}{\sin^2 \alpha}$$

$$\therefore \quad \cos \theta = \pm \sqrt{\frac{1 - \cos \alpha}{\sin^2 \alpha}} = \pm \sqrt{\frac{1 - \cos \alpha}{1 - \cos^2 \alpha}} = \pm \sqrt{\frac{1}{1 + \cos \alpha}}$$

$$= \pm \sqrt{\frac{1}{1 + \cos 18^\circ}} = \pm \sqrt{\frac{1}{1 + 0.9510}} = \pm 0.7159$$

$$\therefore \quad \theta = 44.3^\circ \quad \text{or} \quad 135.7^\circ$$ **Ans.**

Q3. Two shafts are connected by a universal joint. The driving shaft rotates at a uniform speed of 1200 r.p.m. Determine the greatest permissible angle between the shaft axes so that the total fluctuation of speed does not exceed 100 r.p.m. Also calculate the maximum and minimum speeds of the driven shaft.

Solution. Given : $N = 1200$ r.p.m.; $q = 100$ r.p.m.

Greatest permissible angle between the shaft axes

Let α = Greatest permissible angle between the shaft axes.

We know that total fluctuation of speed (q),

$$100 = N\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right) = 1200\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$$

$$\frac{1-\cos^2\alpha}{\cos\alpha} = \frac{100}{1200} = 0.083$$

$$\cos^2\alpha + 0.083\cos\alpha - 1 = 0$$

$$\cos\alpha = \frac{-0.083 \pm \sqrt{(0.083)^2 + 4}}{2} = 0.9593 \qquad \text{...(Taking + sign)}$$

$$\alpha = 16.4° \quad \textbf{Ans.}$$

Maximum and minimum speed of the driven shaft

We know that maximum speed of the driven shaft,

$$N_{1\,(max)} = N / \cos\alpha = 1200 / 0.9593 = 1251 \text{ r.p.m.} \quad \textbf{Ans.}$$

and minimum speed of the driven shaft,

$$N_{1\,(min)} = N \cos\alpha = 1200 \times 0.9593 = 1151 \text{ r.p.m.} \quad \textbf{Ans.}$$

UNIT III BIT BANK

1. In a pantograph, all the pairs are (a)

(a) turning pairs (b) sliding pairs (c) spherical pairs (d) self-closed pairs

2. Which of the following mechanism is made up of turning pairs ? (b)

(a) Scott Russel's mechanism (b) Peaucellier's mechanism

(c) Modified Scott Russel's mechanism (d) none of these

3. Which of the following mechanism is used to enlarge or reduce the size of a drawing ? (c)

(a) Grasshopper mechanism (b) Watt mechanism (c) Pantograph (d) none of these

4. The Ackerman steering gear mechanism is preferred to the Davis steering gear mechanism, because (d)

(a) whole of the mechanism in the Ackerman steering gear is on the back of the front wheels.

(b) the Ackerman steering gear consists of turning pairs

(c) the Ackerman steering gear is most economical (d) both (a) and (b)

5. The driving and driven shafts connected by a Hooke's joint will have equal speeds, if (c)

(a) $\cos\theta = \sin\alpha$ (b) $\sin\theta = \pm\sqrt{\tan\alpha}$ (c) $\tan\theta = \pm\sqrt{\cos\alpha}$ (d) $\cot\theta = \cos\alpha$

where θ = Angle through which the driving shaft turns, and
α = Angle of inclination of the driving and driven shafts.

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6. Which of these is an approximate straight line motion mechanism? (d)

(a) Scott Russell's mechanism (b) Hart's mechanism

(c) Peaucellier's mechanism (d) Watt's mechanism

7. Which of the given mechanisms is an exact straight line motion mechanism? (c)

(a) Grasshopper mechanism (b) Robert's mechanism

(c) Peaucellier's mechanism (d) Tchebicheff's mechanism

8. _______mechanism is a four bar chain mechanism in early steam engines to guide the piston rod in a cylinder to have an approximate straight line motion. (d)

(a) Peaucellier's mechanism (b) Tchebicheff's mechanism

(c) Grasshopper mechanism (d) Watt's mechanism

9. Hart's mechanism requires only eight links whereas the Peaucellier mechanism requires six links. True or false? (b)

(a)True (b) False

10. Davis steering gear consists of (a)

(a) Sliding pairs & Turning pairs (b) Turning pairs (c)Rolling pairs (d)Higher pairs

11. The coupling used to connect two shafts with large angular misalignment is (d)

(a)a Flange coupling (b) an Oldham's coupling (c) a Flexible bush coupling (d) Hooke's Joint

12. Assertion (A): The Ackermann Steering gear is commonly used in all automobiles. (C)

Reason(R) : It has the correct inner turning angle for all positions.

(a)Both A and R are individually true and R is the correct explanations of A

(b)Both A and R are individually true but R is not the correct explanations of A

(c)A is true but R is false (d) Ais false but R is true

13. Ackermann steering gear consists of (b)

(a)Sliding pairs (b) Turning pairs (c) Rolling pairs (d) Higher pairs

14. The Ackerman steering gear mechanism is preferred to the Davis steering gear mechanism, because (d)

(a)Whole of the mechanism in the Ackerman steering gear is on the back of the front wheels

(b) The Ackerman steering gear consists of turning pairs

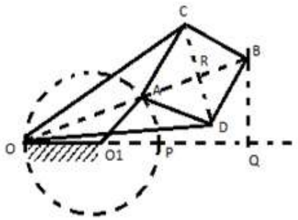
(c)The Ackerman steering gear is most economical

(d) Both (A) and (B)

15. Identify the given straight line mechanism. (b)

(a) Hart's mechanism (b) Peaucellier mechanism

(c) Tchebicheff's mechanism (d) Watt's mechanism



16. In automobiles, Hooke's joint is used between which of the following? (b)

(a) Clutch and gear box (b) Gear box and differential

(c) Differential and wheels (d) Flywheel and clutch

17. Which of the following statements is not correct? (a)

a) Hooke's joint is used to connect two rotating co-planar, non-intersecting shafts

b) Hooke's joint is used to connect two rotating co-planar, intersecting shafts

c) Oldham's coupling is used to connect two parallel rotating shafts

d) Hooke's joint is used in the steering mechanism for automobiles

18.The Hooke's joint consists of (a)

(a)Two forks (b)One fork (c)Three forks (d)Four forks

19.The Double Hooke's joint consists of (d)

(a)Two forks (b)One fork (c)Three forks (d)Four forks

20. What is the purpose of double hooke's joint? (c)

a) Have constant linear velocity ratio of driver and driven shafts

b) Have constant acceleration ratio of driver and driven shafts

c) Have constant angular velocity ratio of driver and driven shafts

d) Have constant angular acceleration ratio of driver and driven shafts

21.Two shafts connected by a Hooke's joint have an angle of 18 degrees between the axes. Find the angle through which it should be turned to get the velocity ratio maximum. (a)
(a) 180 (b) 30 (c) 45 (d) 90

- Explanation: Velocity ratio is $\omega_1/\omega = \cos\alpha/(1 - \cos^2\theta\sin^2\alpha)$
  now this to be maximum $\cos^2\theta = 1$
  therefore $\theta = 0$ or 180 degrees.

22. Two shafts having an included angle of 150° are connected by a Hooke's joint. The driving shaft runs at a uniform speed of 1500 r.p.m. The driven shaft carries a flywheel of mass 12 kg and 100 mm radius of gyration. Using the above data, calculate the maximum angular acceleration of the driven shaft in rad/s². (a)
(a) 6853 (b) 6090 (c) 6100 (d) 6500

Explanation: $\alpha = 180 - 150 = 30^o$
$\cos 2\theta = 2\sin^2 \alpha/1-\sin^2 \alpha = 0.66$
angular acc = $d\omega/dt$
= 6853.0 rad/s².

23. Two shafts having an included angle of 150° are connected by a Hooke’s joint. The driving shaft runs at a uniform speed of 1500 r.p.m. The driven shaft carries a flywheel of mass 12 kg and 100 mm radius of gyration. Using the above data, calculate the maximum torque required in N-m. (a)

(a) 822 (b) 888 (c) 890 (d) 867

Explanation: $\alpha = 180 - 160 = 30^o$

$\cos 2\theta = 2\sin^2 \alpha / 1 - \sin^2 \alpha = 0.66$

angular acc = $d\omega/dt$ = 6853 rad/s^2

I = 0.12 Kg-m^2

Therefore max torque = I.ang acc. = 822 N-m.

24. Two shafts connected by a Hooke’s joint have an angle of 18 degrees between the axes. Find the angle through which it should be turned to get the velocity ratio equal to 1. (c)

(a) 30.6 (b) 30.3 (c) 44.3 (d) 91.2

Explanation: Velocity ratio is $\omega_1/\omega = \cos\alpha/(1 - \cos^2\theta \sin^2\alpha)$

now this to be 1

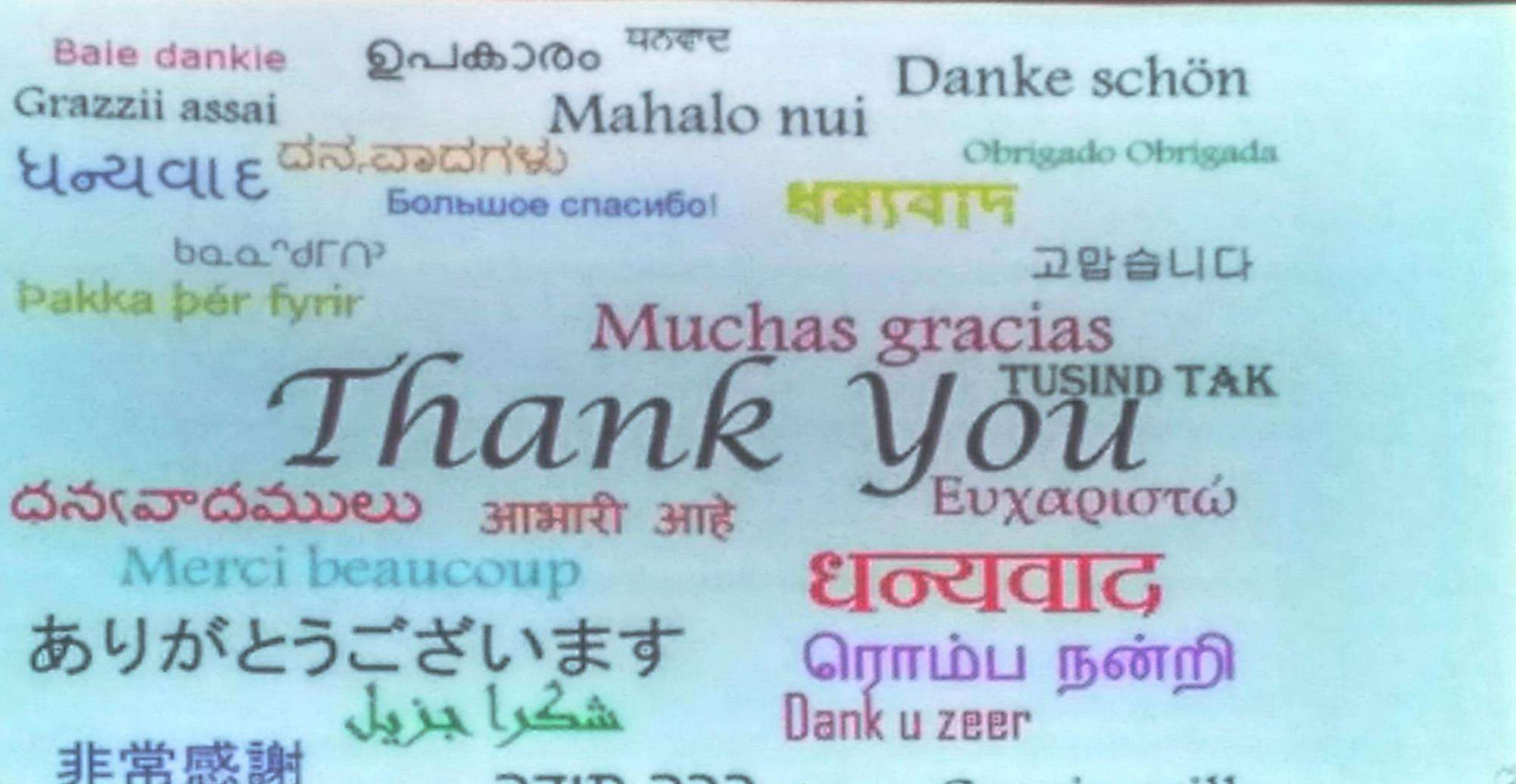
we get, $\cos\alpha = 1 - \cos^2\theta \sin^2\alpha$

solving this equation we get

θ = 44.3 or 135.7 degrees.

References:

1. Theory of Machines, Rattan, Tata McGraw-Hill Education, 2009.
2. Theory of Machines, R S Kurmi, Eurasia Publishing House, 2005



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UNIT IV
CAMS

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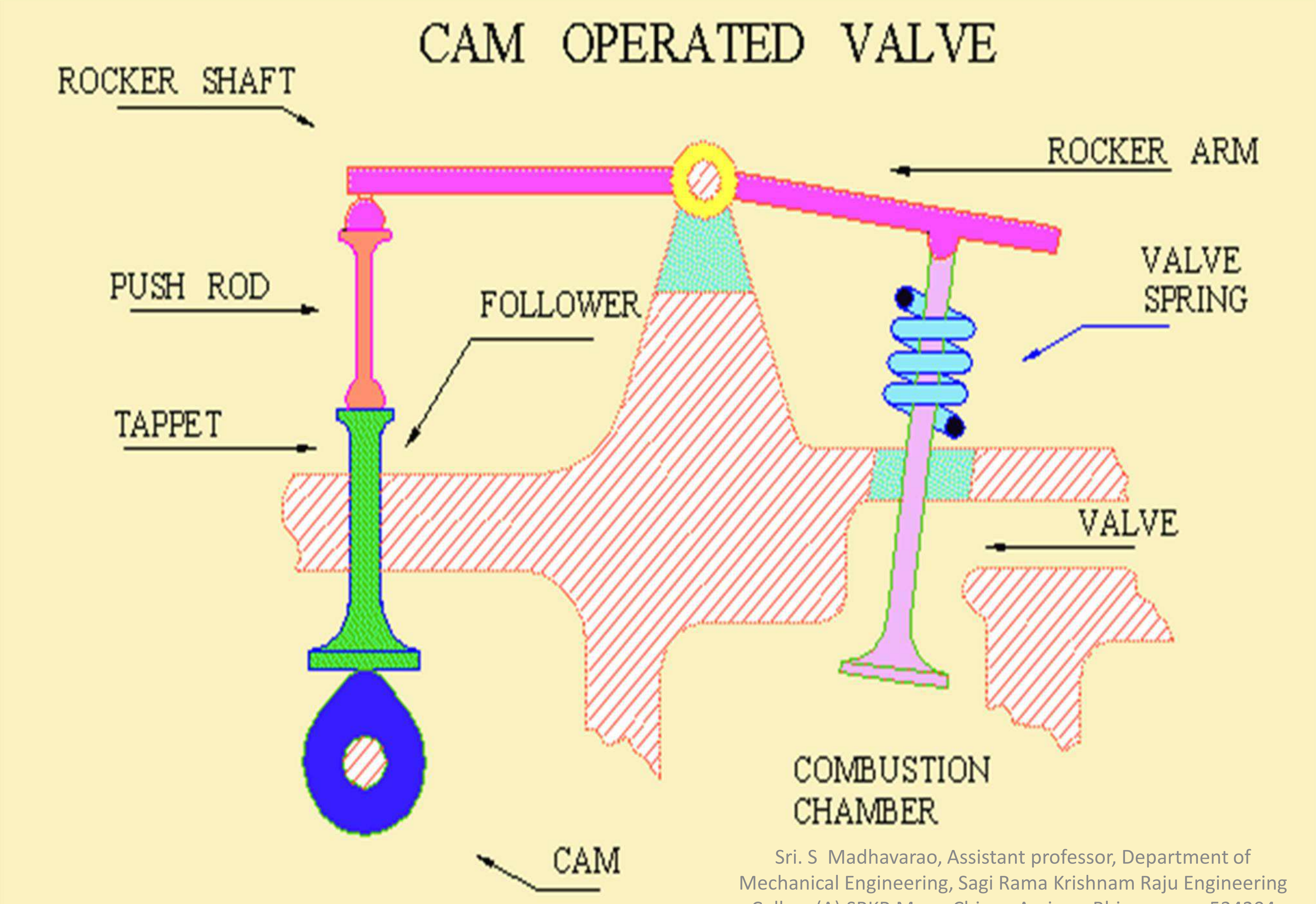
Cams: Definitions of cam and followers – their uses – Types of followers and cams – Terminology – Types of follower motion - Uniform velocity, Simple harmonic motion and uniform acceleration and retardation. Construction of cam profiles- Cam with knife edged follower and roller follower Maximum velocity and maximum acceleration during outward and return strokes.



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Introduction

- A rotating machine element, which gives reciprocating or oscillating motion to a second element(i.e follower) is know as a cam
- A cam is a mechanical member used to impart desired motion to a follower by direct contact.
- The cam may be rotating or reciprocating whereas the follower may be rotating, reciprocating or oscillating.
- Complicated output motions which are otherwise difficult to achieve can easily be produced with the help of cams.
- Cams are widely used in automatic machines, internal combustion engines, machine tools, printing control mechanisms, and so on.
- They are manufactured usually by die-casting, milling or by punch-presses.

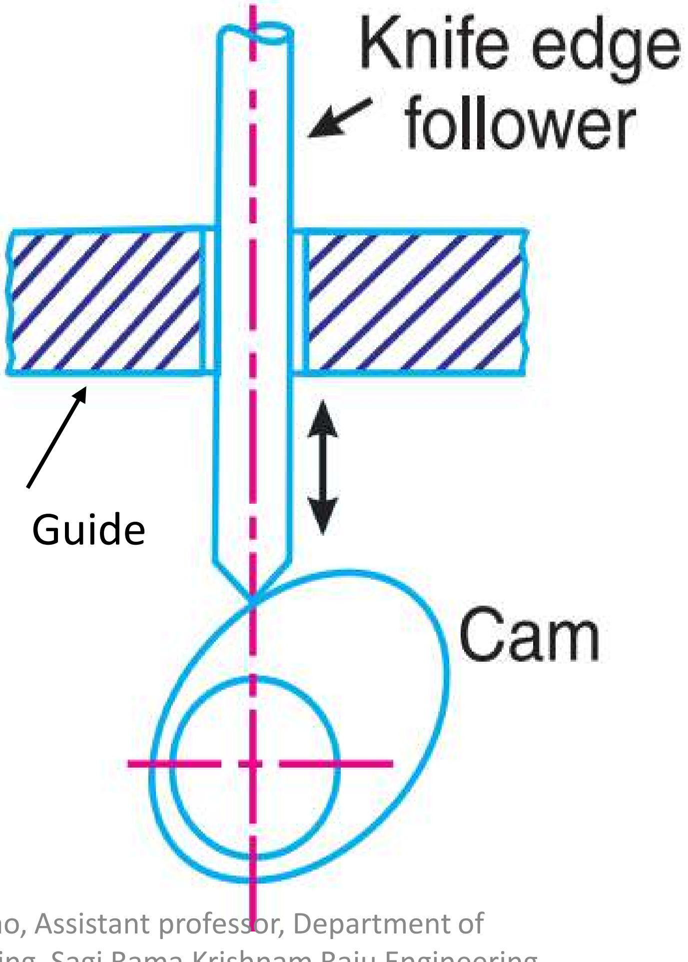
Introduction

- A cam and the follower combination belong to the category of higher pairs
- Necessary elements of a cam mechanism are

– A driver member known as the cam

– A driven member called the follower

– A frame which supports the cam and guides the follower



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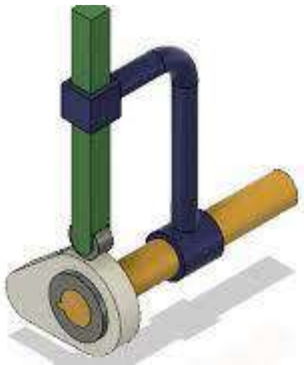

cam and follower mechanism animation

Cam Analysis Machine

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TYPES OF CAMS

Cams are classified according to

1. shape,
2. follower movement, and
3. manner of constraint of the follower

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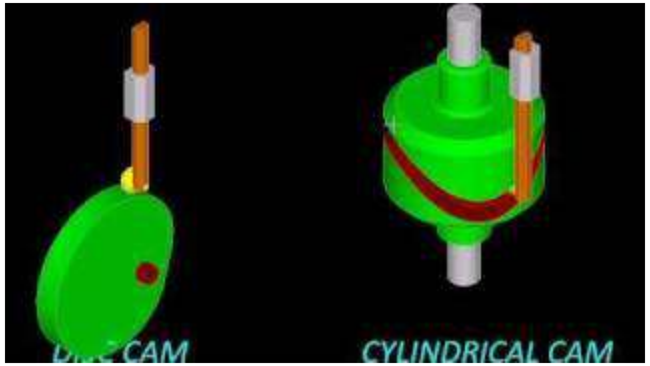

Kinematics Cams Concept & Types

Classification Of Cams

- According to Shape
  - 1. Wedge & Flat Cam
  - 2. Radial or Disc Cam
  - 3. Spiral Cam
  - 4. Cylindrical Cam
  - 5. Conjugate Cam
  - 6. Globoidal Cam
  - 7. Spherical Cam
- According to Follower Movement
  - Rise-Return-Rise
  - Dwell-Rise-Return-Dwell
  - Dwell-Rise-Dwell-Return-Dwell
- According to Manner of constraint of the Follower
  - Pre-Loaded Spring Cam
  - Positive-drive Cam
  - Gravity Cam

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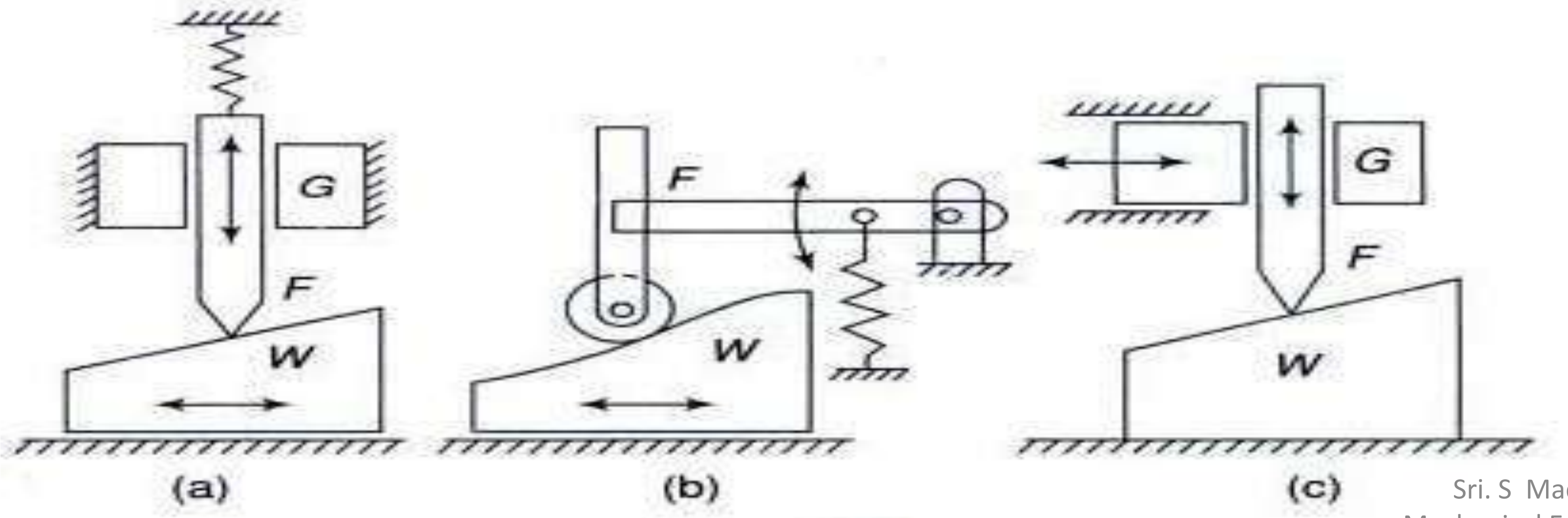
I. According to Shape

1) Wedge and Flat Cams

- A wedge cam has a wedge W which, in general, has a translational motion.
- The follower F can either translate [Fig.1.(a)] or oscillate [Fig.1.(b)].
- A spring is, usually, used to maintain the contact between the cam and the follower.

In Fig.1.(c), the cam is stationary and the follower constraint or guide G causes the relative motion of the cam and the follower.



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2. Radial or Disc Cams

- A cam in which the follower moves radially from the centre of rotation of the cam is known as a radial or a disc cam (Fig.2. (a) and (b)].
- Radial cams are very popular due to their simplicity and compactness.

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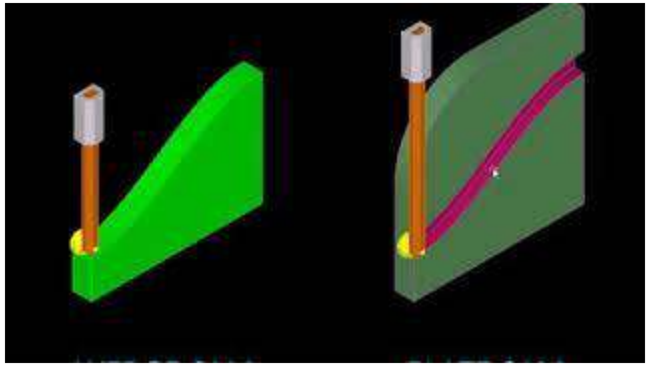
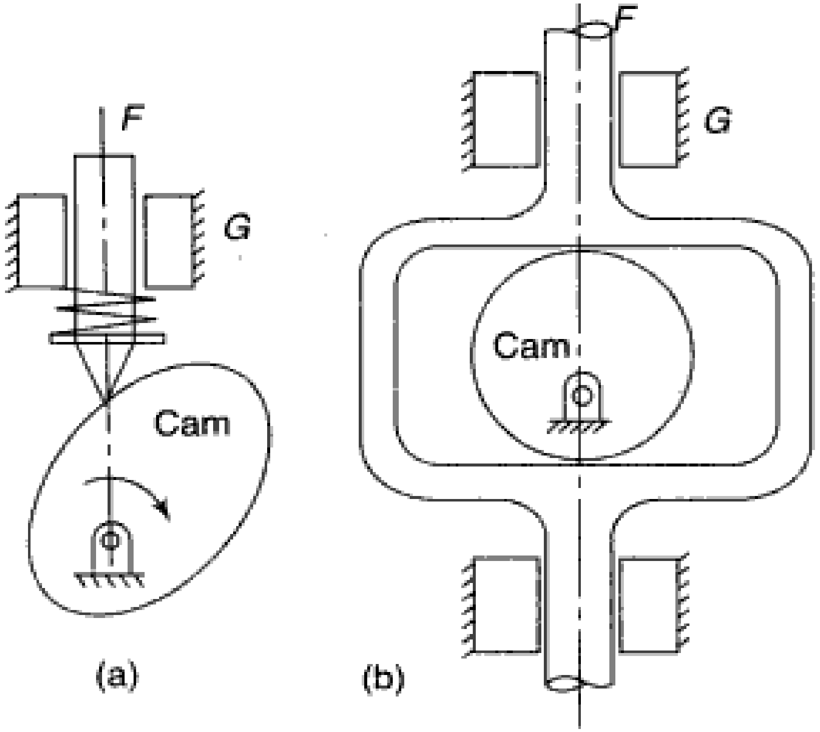


Types of cam and follower



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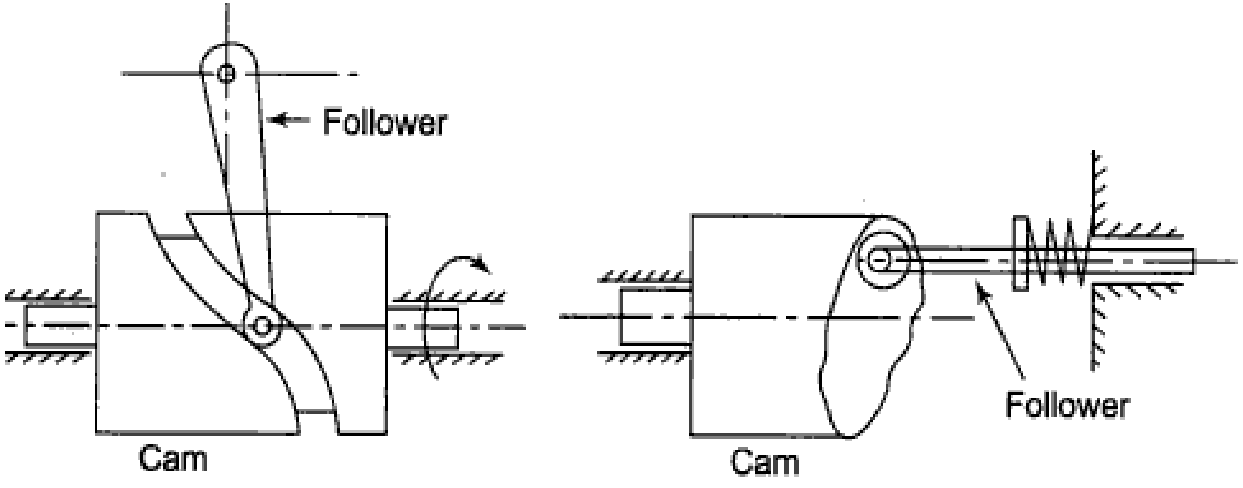
3.Cylindrical Cams

- In a cylindrical cam, a cylinder which has a circumferential contour cut in the surface, rotates about its axis.
- The follower motion can be of two types as follows: In the first type, a groove is cut on the surface of the cam and a roller follower has a constrained (or positive)oscillating motion [Fig.3.(a)].
- Another type is an end cam in which the end of the cylinder is the working surface(b).
- A spring-loaded follower translates along or parallel to the axis of the rotating cylinder.



(a) (b)

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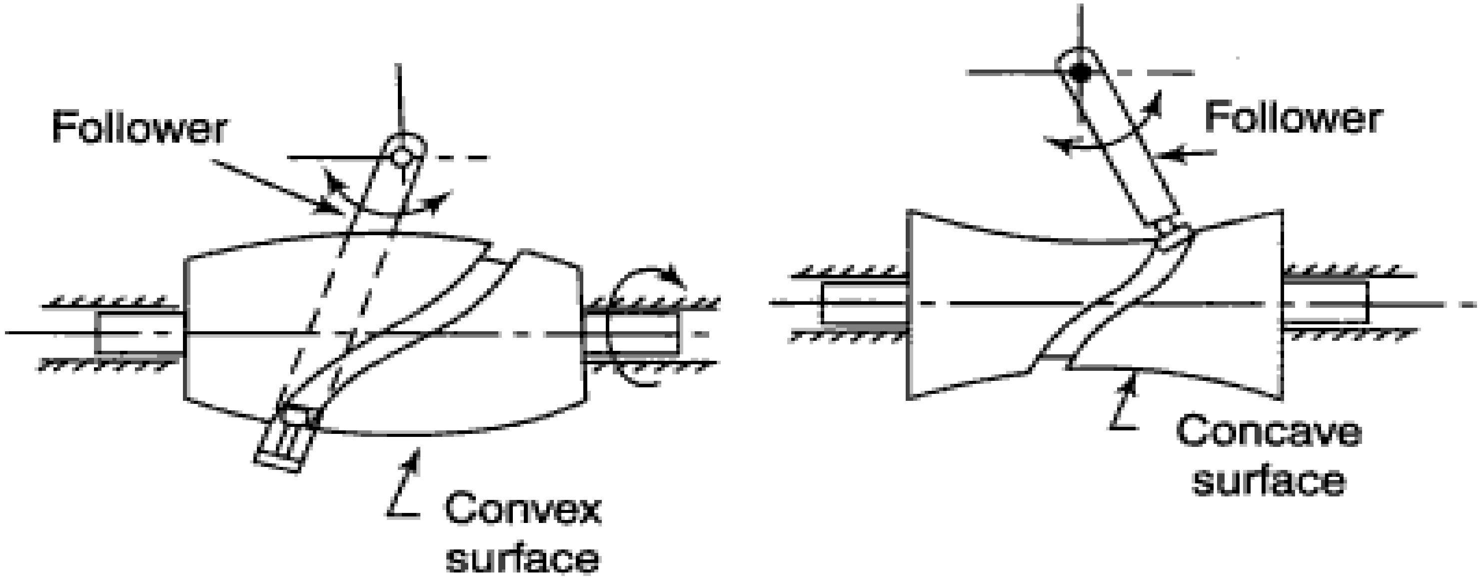
4. Globoidal Cams

- A globoidal cam can have two types of surfaces, convex or concave.
- A circumferential contour is cut on the surface of rotation of the cam to impart motion to the follower which has an oscillatory motion (Fig.4).
- The application of such cams is limited to moderate speeds and where the angle of oscillation of the follower is large.

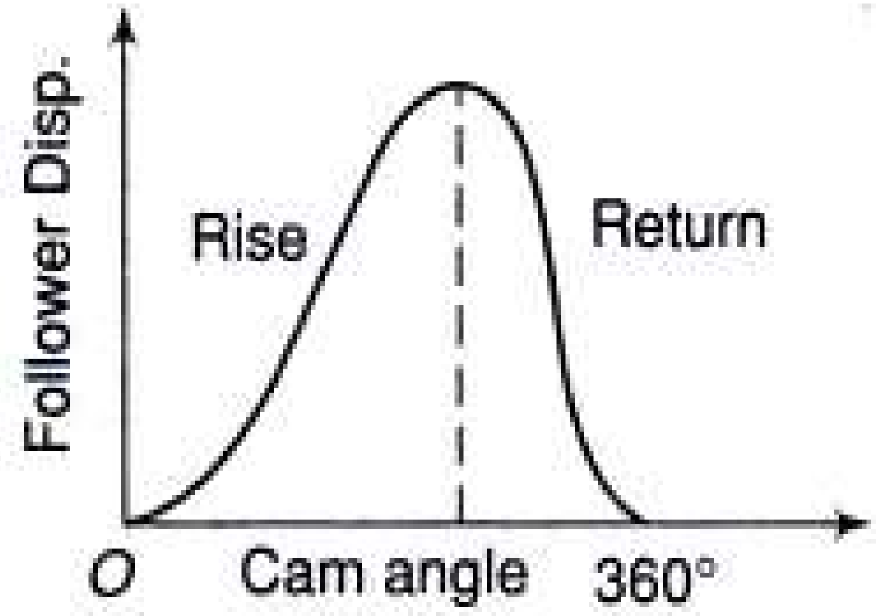


II. According to Follower Movement

- The motions of the followers are distinguished from each other by the dwells they have.
- A dwell is the zero displacement or the absence of motion of the follower during the motion of the cam.
- Cams are classified according to the motions of the followers in the following ways:

1. Rise-Return-Rise (R-R-R)

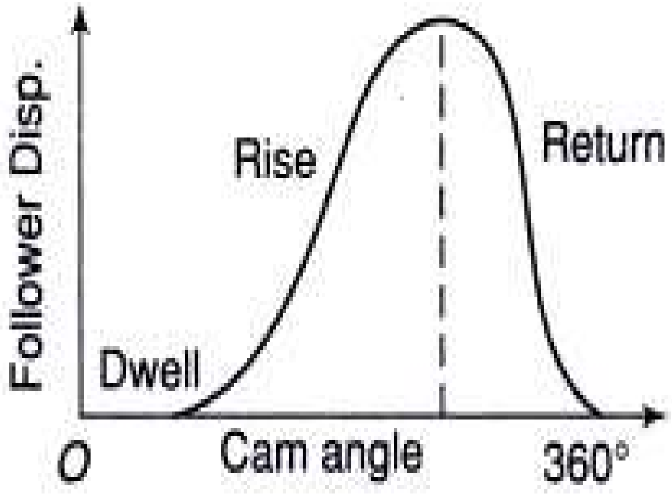
- In this, there is alternate rise and return of the follower with no periods of dwells (Fig. a).
- Its use is very limited in the industry.
- The follower has a linear or an angular displacement.



(a)

2. Dwell-Rise-Return-Dwell (D-R-R-D)

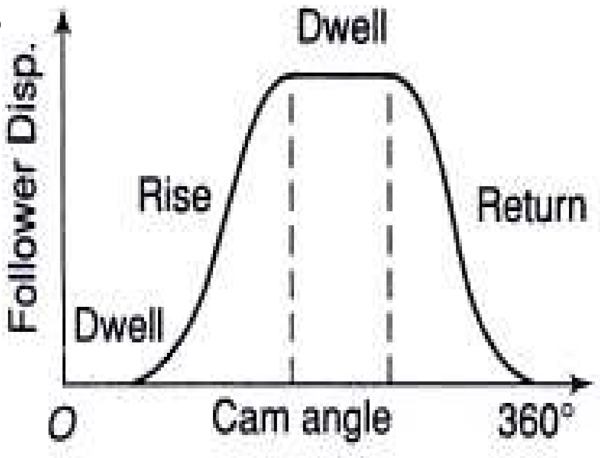
- In such a type of cam, there is rise and return of the follower after a dwell Fig.(b).
- This type is used more frequently than the R-R-R type of cam.



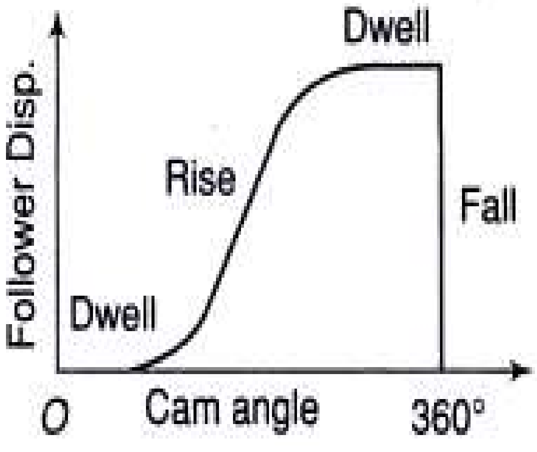
(b)

3. Dwell-Rise-Dwell-Return-Dwell (D-R-D-R-D)

- It is the most widely used type of cam.
- The dwelling of the cam is followed by rise and dwell and subsequently by return and dwell as shown in fig. (c).
- In case the return of the follower is by a fall [Fig.(d)], the motion may be known as Dwell-Rise-Dwell (D-R-D).



(c)



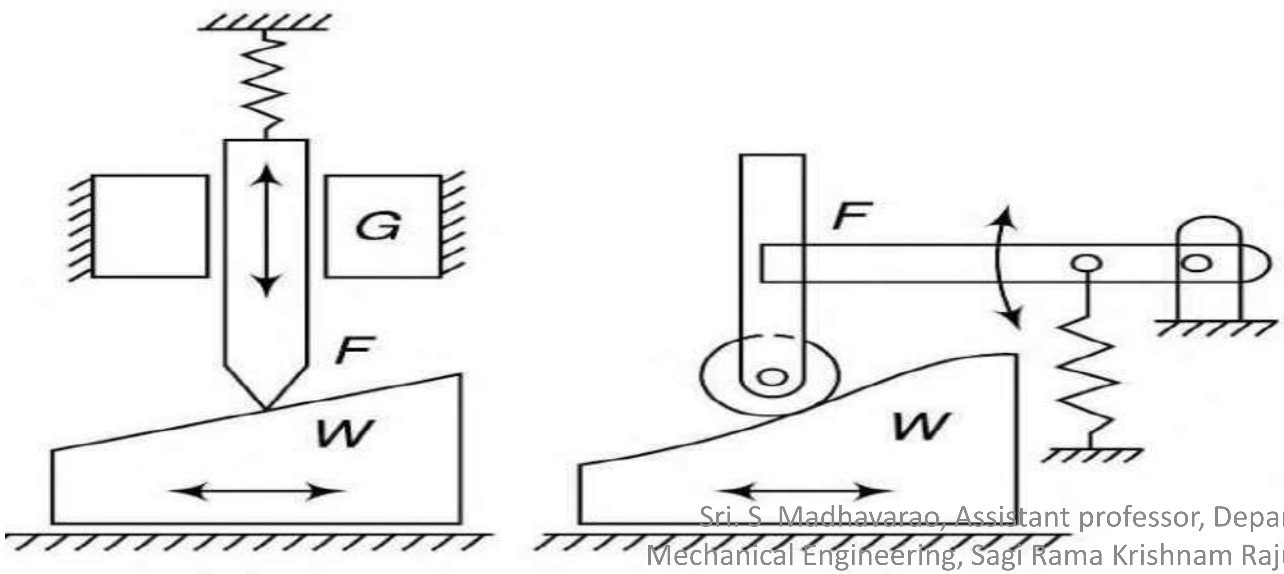
(d)

III. According to Manner of Constraint of the Follower

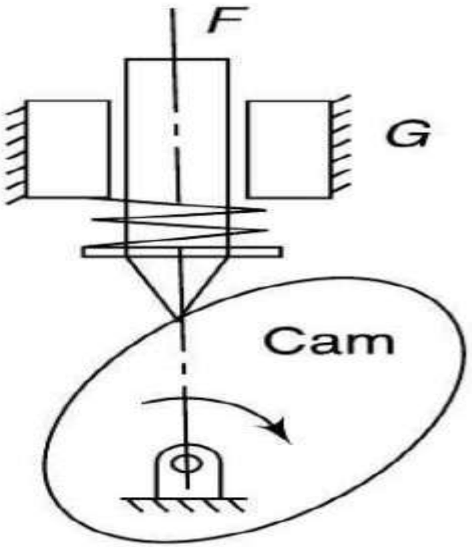
- To reproduce exactly the motion transmitted by the cam to the follower, it is necessary that the two remain in touch at all speeds and at all times.
- The cams can be classified according to the manner in which this is achieved.

1. Pre-loaded Spring Cam

- A pre-loaded compression spring is used for the purpose of keeping the contact between the cam and the follower. [Fig.1.(a)] and [Fig.(b)]., Fig.2. (a)]



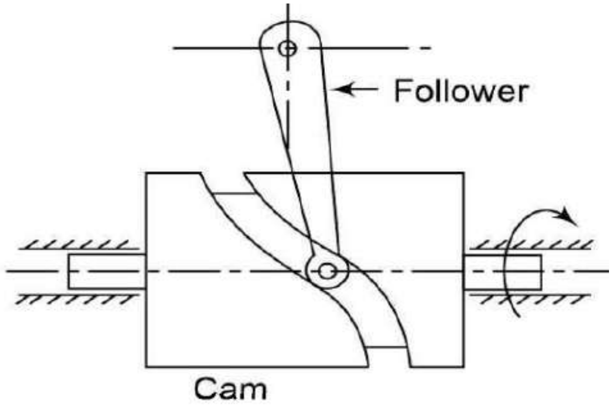
(a) (b)



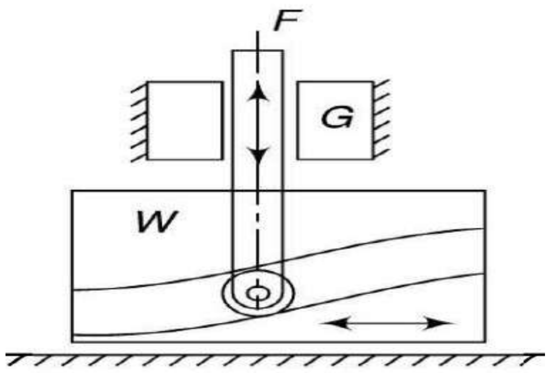
(a)

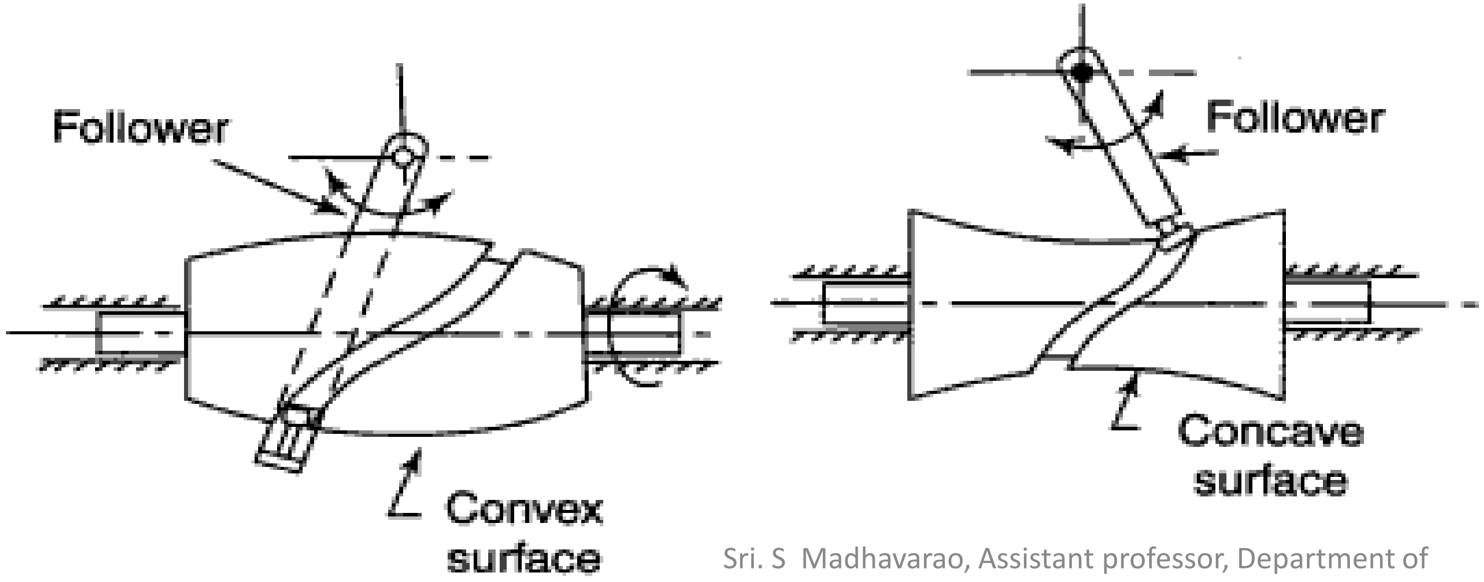
2. Positive-drive Cam

- In this type, constant touch between the cam and the follower is maintained by a roller follower operating in the groove of a cam. [Fig.3.(a), Fig.4.]. The follower cannot go out of this groove under the normal working operations.
- A constrained or positive drive is also obtained by the use of a conjugate cam



(a)





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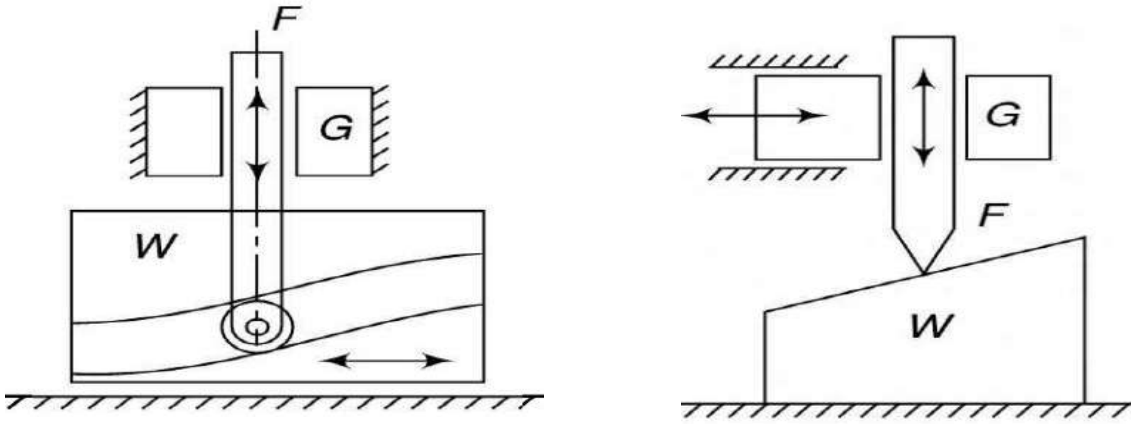
3. Gravity Cam

- If the rise of the cam is achieved by the rising surface of the cam and the return by the force of gravity or due to the weight of the cam, the cam is known as a gravity cam. Fig.1.(c), show such a cam . However, these cams are not preferred due to their uncertain behavior



(c)

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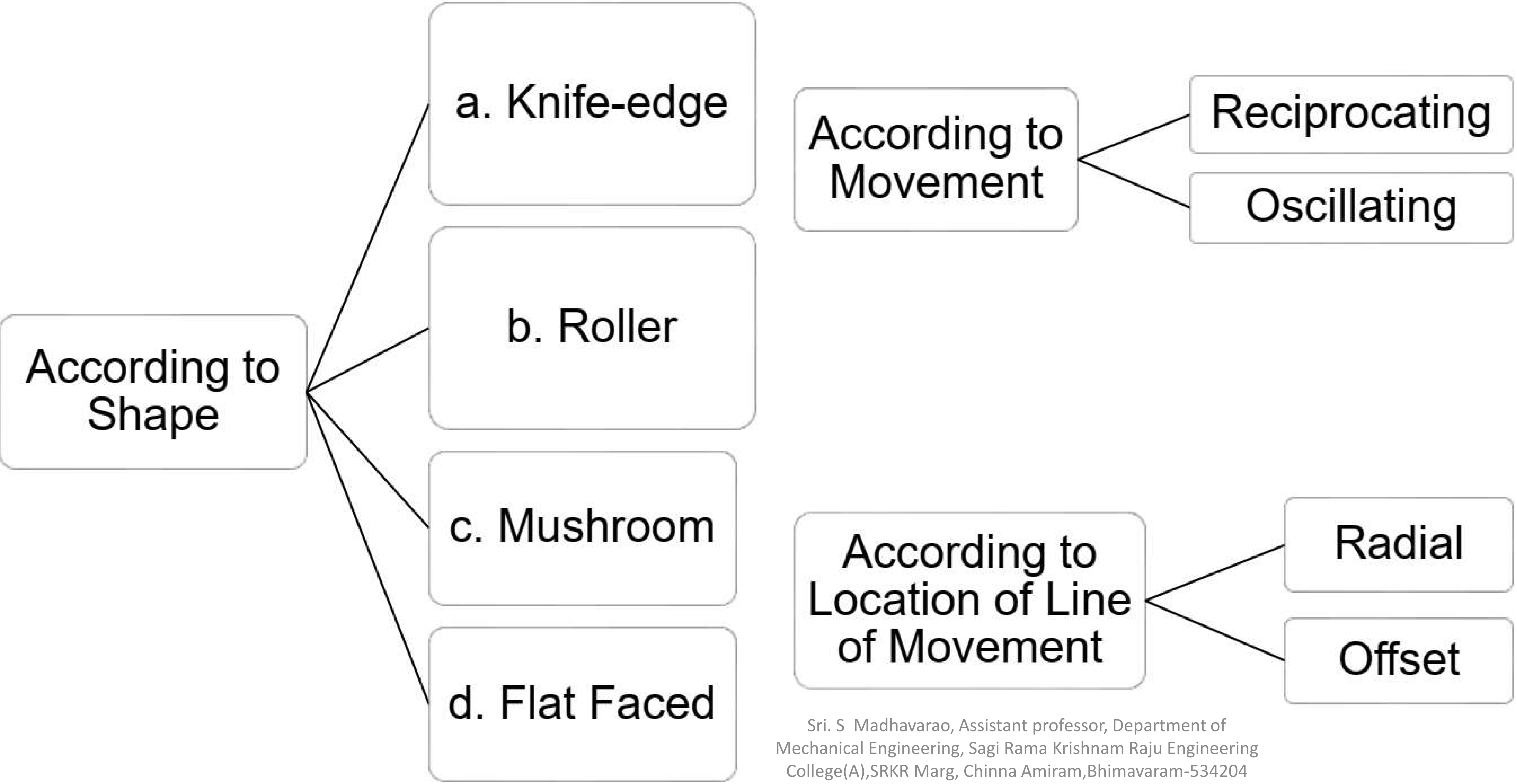
Classification of Followers



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Classification of Followers

1. According to the surface in contact.

a) Knife edge follower.

When the contacting end of the follower has a sharp knife edge, it is called a knife edge follower, as shown in Fig.(a). The sliding motion takes place between the contacting surfaces (i.e. the knife edge and the cam surface).

- It is seldom used in practice because the small area of contacting surface results in excessive wear. In knife edge followers, a considerable side thrust exists between the follower and the guide

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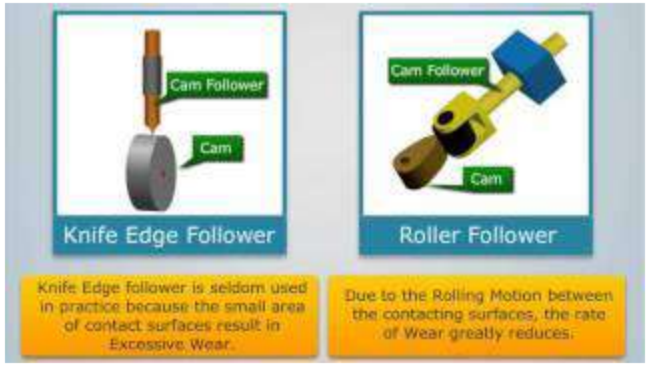
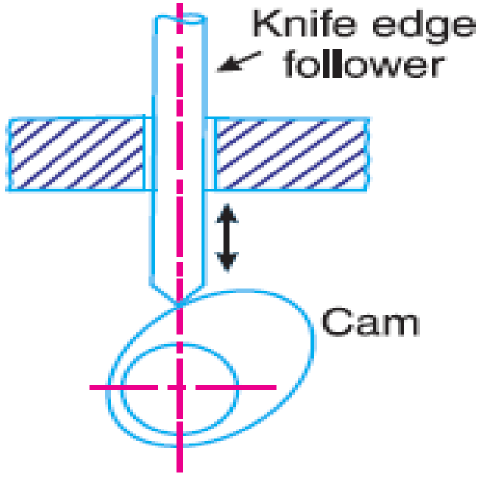


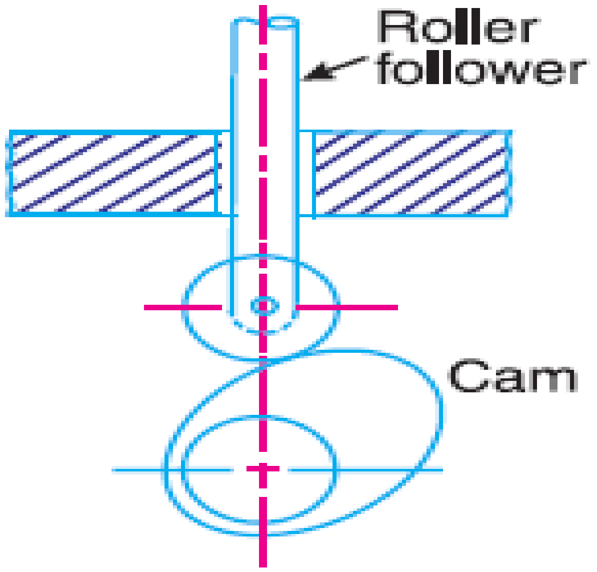
Cam and Follower Mechanism in Engine



(a) Cam with knife edge follower.

(b) Roller follower.

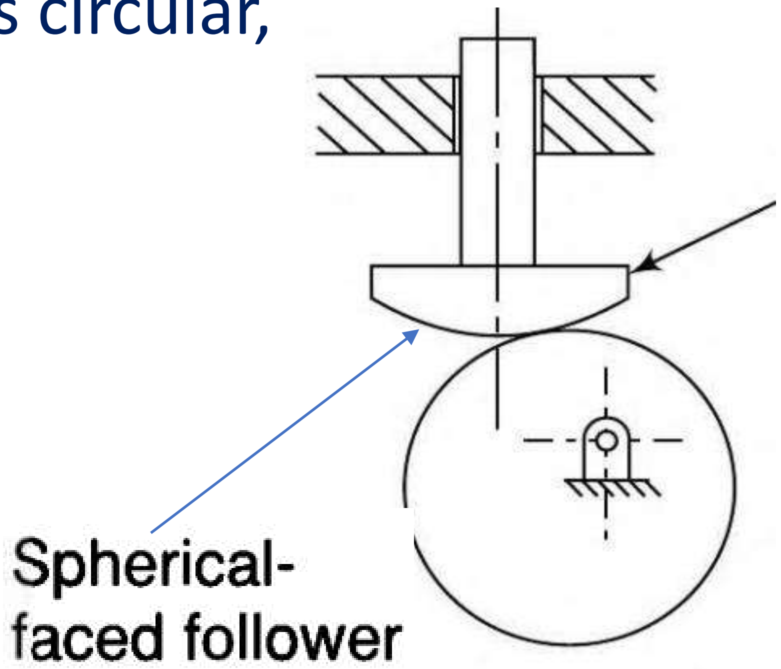
- **When the contacting end of the follower is a roller, it is called a roller follower, as** shown in Fig. (b). Since the rolling motion takes place between the contacting surfaces (i.e. the roller and the cam), therefore the rate of wear is greatly reduced.
- In roller followers also the side thrust exists between the follower and the guide.
- The roller followers are extensively used where more space is available such as in stationary gas and oil engines and aircraft engines.

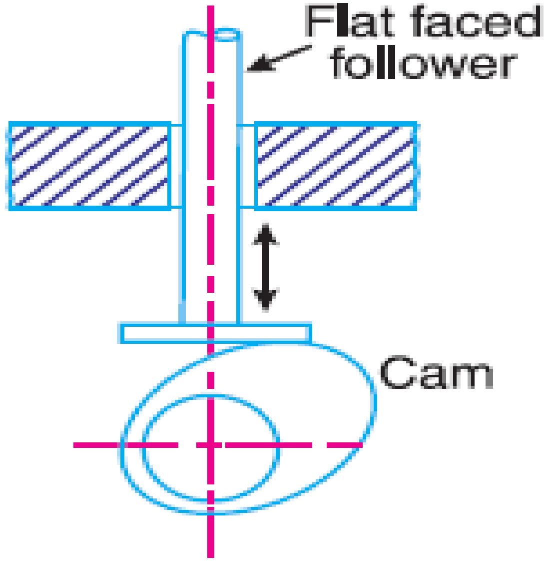


(b) Cam with roller follower.

(c) Flat faced or mushroom follower.

- When the contacting end of the follower is a perfectly flat face, it is called a flat faced follower, as shown in Fig. 20.1 (c). It may be noted that the side thrust between the follower and the guide is much reduced in case of flat faced followers.
- The only side thrust is due to friction between the contact surfaces of the follower and the cam. The flat faced followers are generally used where space is limited such as in cams which operate the valves of automobile engines.
- Note : When the flat faced follower is circular, it is then called a mushroom follower.





(c) Cam with flat faced follower.

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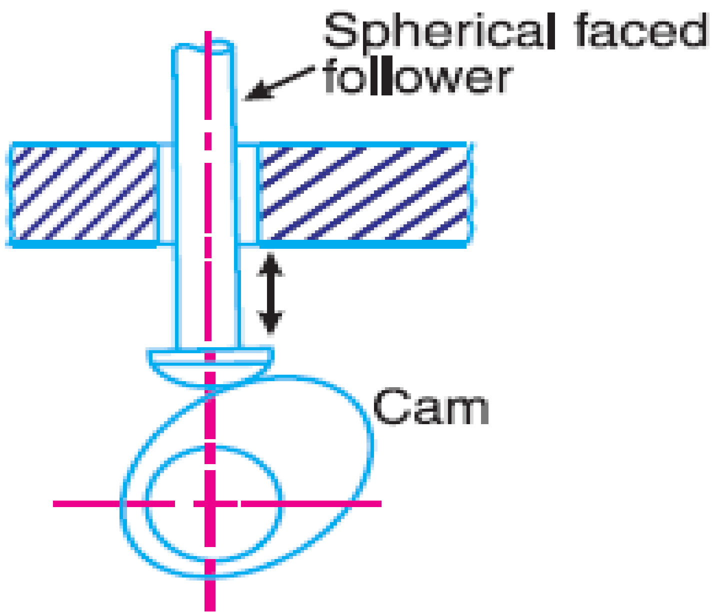
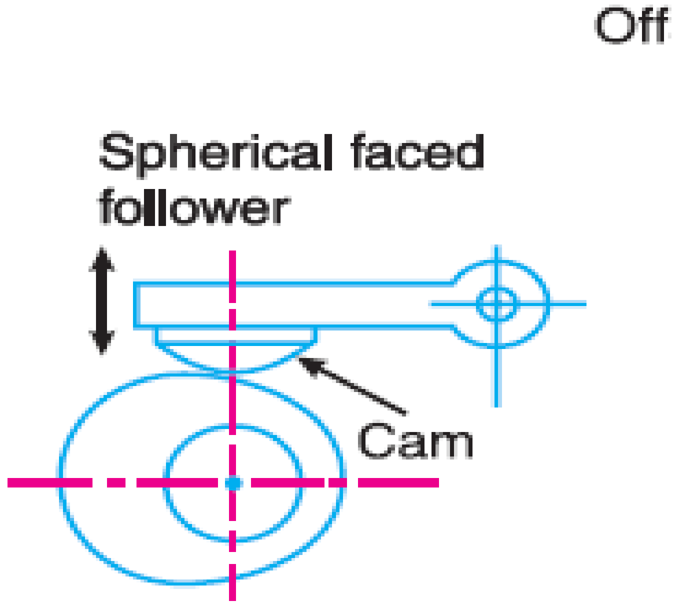
(d) Spherical faced follower.

- When the contacting end of the follower is of spherical shape, it is called a spherical faced follower, as shown in Fig. (d). It may be noted that when a flat-faced follower is used in automobile engines, high surface stresses are produced.
- In order to minimize these stresses, the flat end of the follower is machined to a spherical shape.



(*d*) Cam with spherical faced follower.

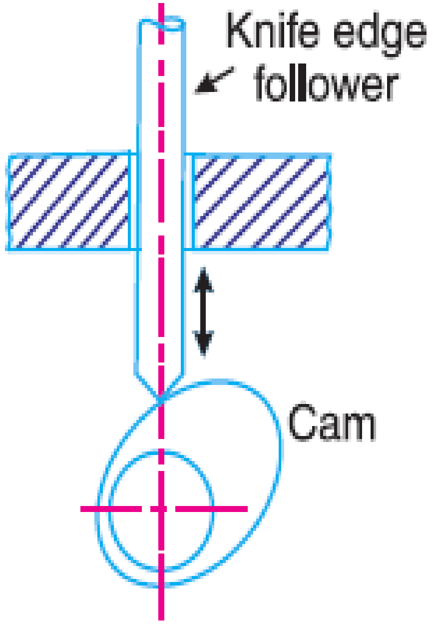


(*e*) Cam with spherical faced follower.

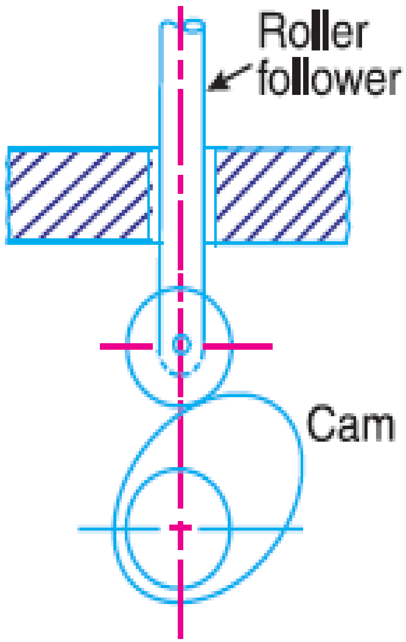
2. According to the motion of the follower

(a) Reciprocating or translating follower.

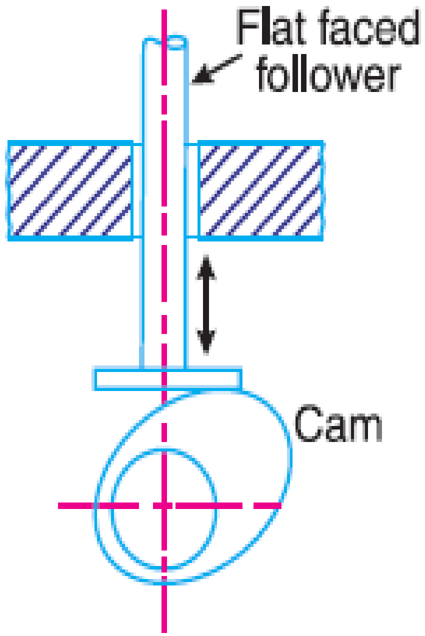
- When the follower reciprocates in guides as the cam rotates uniformly, it is known as reciprocating or translating follower.
- The followers as shown in Fig. (a) to (c) are all reciprocating or translating followers.



(*a*) Cam with knife edge follower.



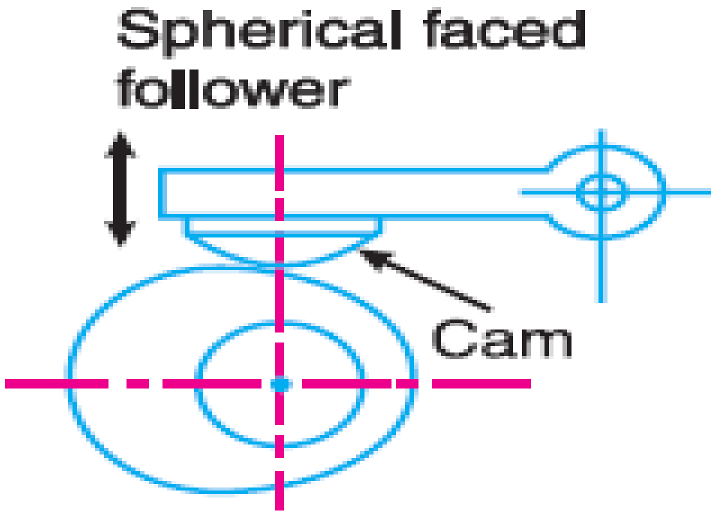
(*b*) Cam with roller follower.



(*c*) Cam with flat faced follower.

(b) Oscillating or rotating follower.

- When the uniform rotary motion of the cam is converted into predetermined oscillatory motion of the follower, it is called oscillating or rotating follower.
- The follower, as shown in (e), is an oscillating or rotating follower.

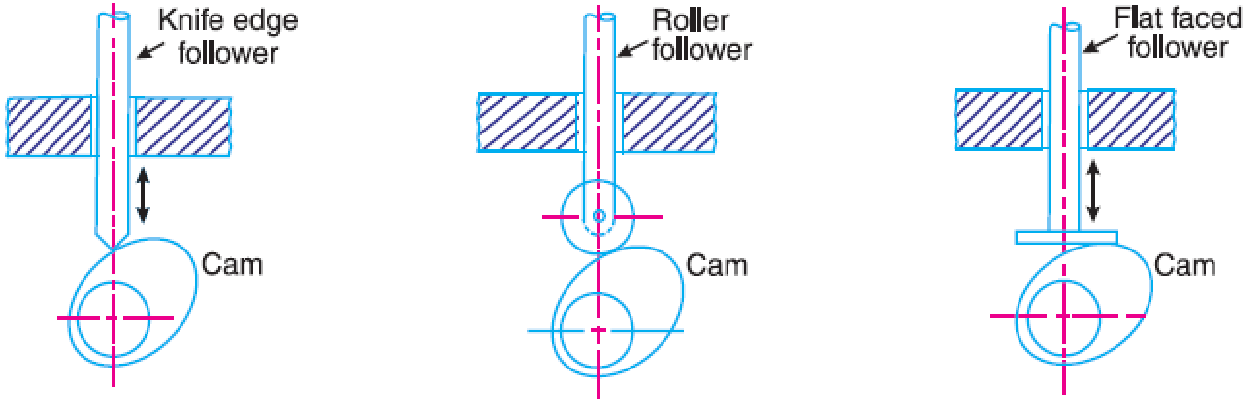


(e) Cam with spherical faced follower.

3. According to the path of motion of the follower.

(a) Radial follower: When the motion of the follower is along an axis passing through the centre of the cam, it is known as radial follower.

The followers, as shown in Fig. (a) to (c), are all radial followers.

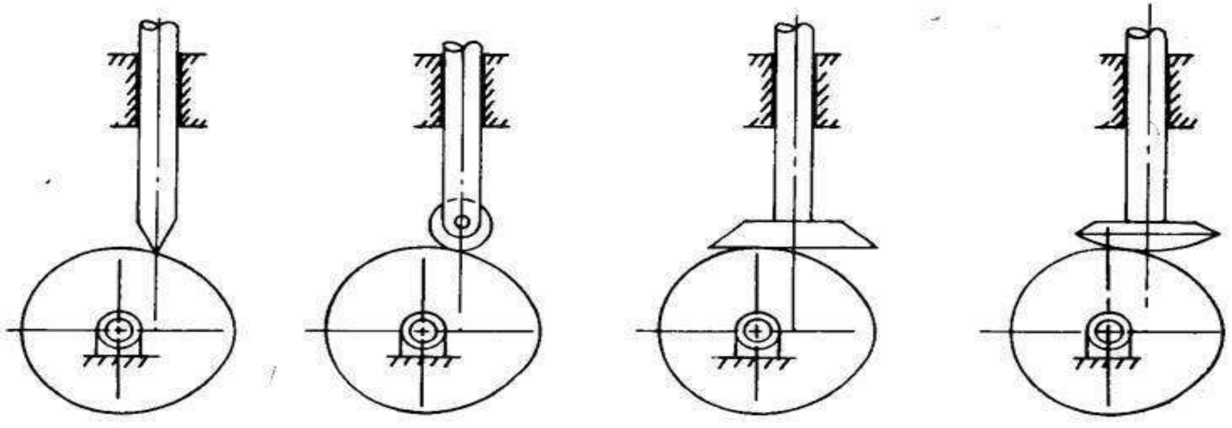


(*a*) Cam with knife edge follower. (*b*) Cam with roller follower. (*c*) Cam with flat faced follower.

(b) Off-set follower.

- When the motion of the follower is along an axis away from the axis of the cam centre, it is called off-set follower.

The follower, as shown in Fig. (a) to (d), is an off-set follower.



(a) (b) (c) (d)

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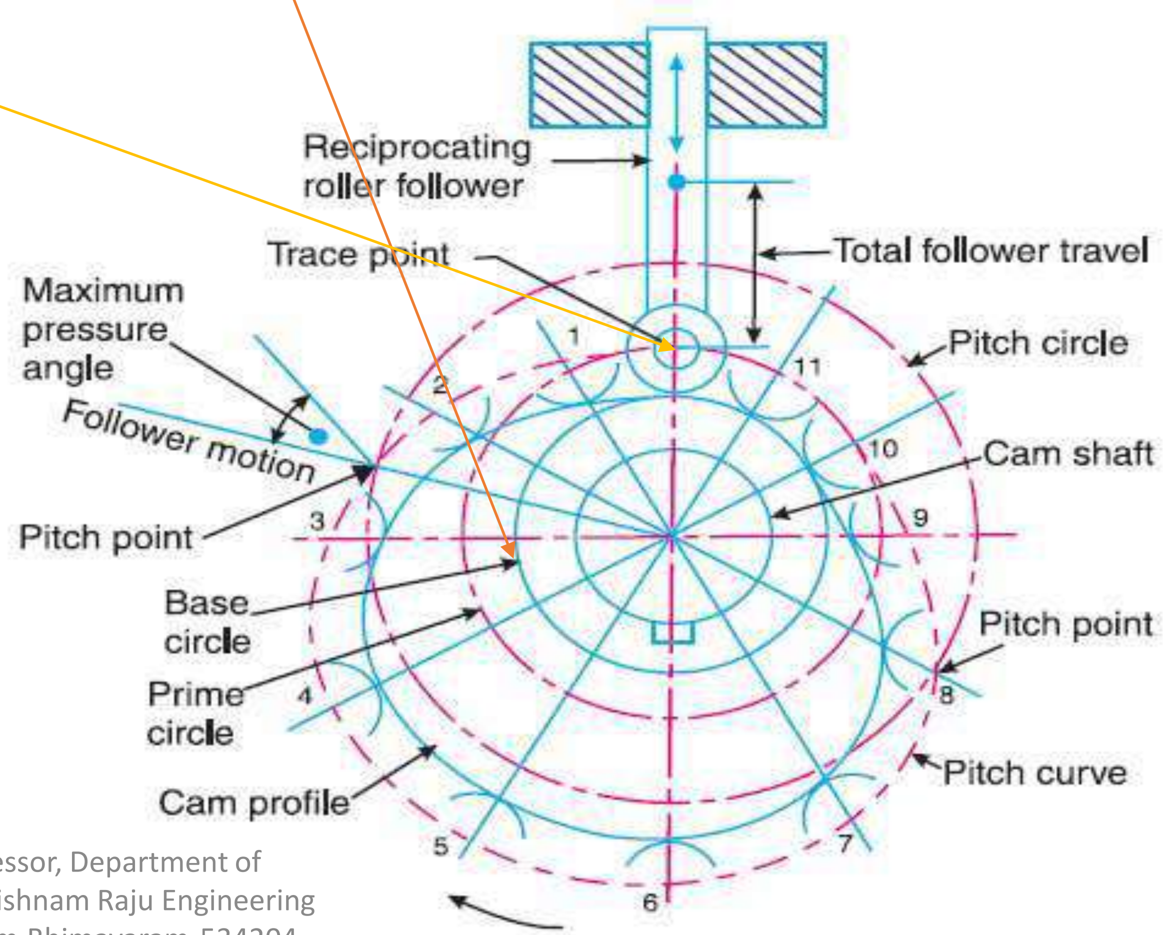
Terms Used in Radial Cams

- Fig. shows a radial cam with reciprocating roller follower. The following terms are important in order to draw the cam profile.

1. **Base circle.** It is the smallest circle that can be drawn to the cam profile.

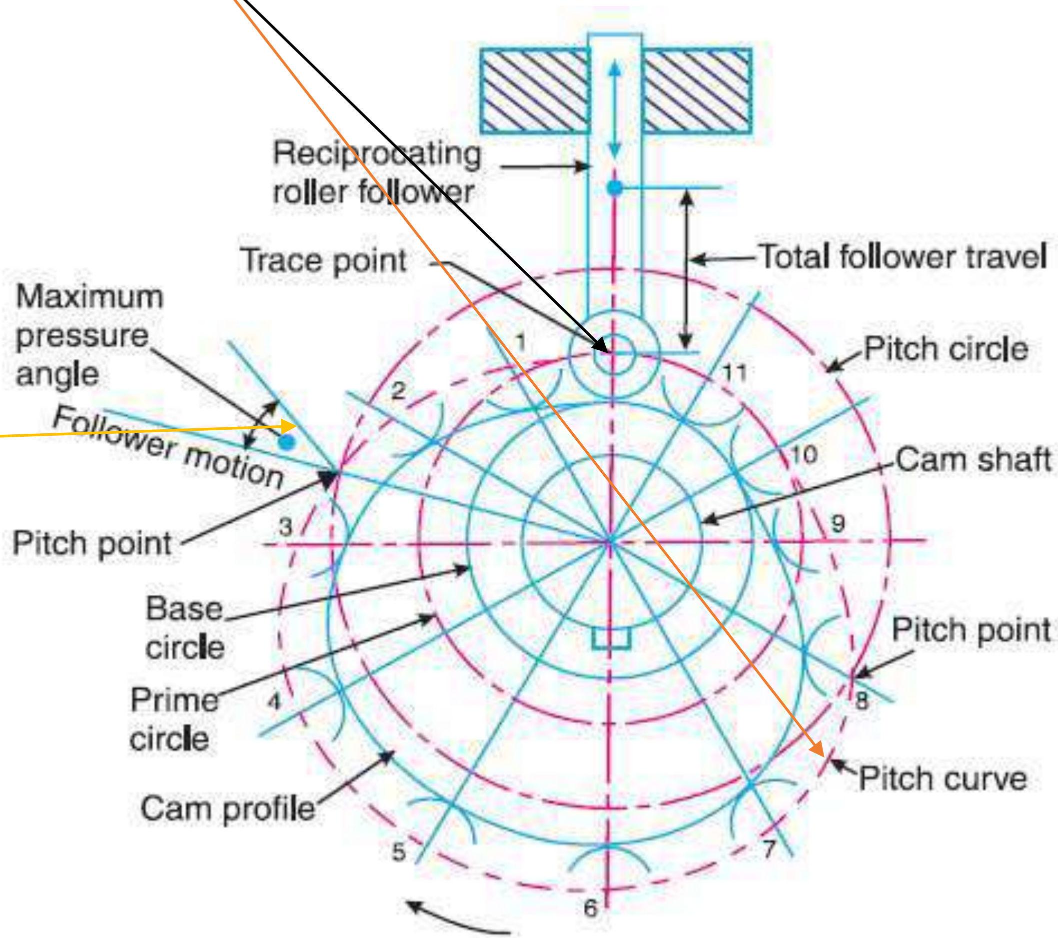
2. **Trace point.** It is a reference point on the follower and is used to generate the pitch curve. In case of knife edge follower, the knife edge represents the trace point and the pitch curve corresponds to the cam profile. In a roller follower, the centre of the roller represents the trace point.



Terms Used in Radial Cams

3. Pitch curve. It is the curve generated by the trace point as the follower moves relative to the cam. For a knife edge follower, the pitch curve and the cam profile are same whereas for a roller follower, they are separated by the radius of the roller

4.Pressure angle. It is the angle between the direction of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile. If the pressure angle is too large, a reciprocating follower will jam in its bearings.



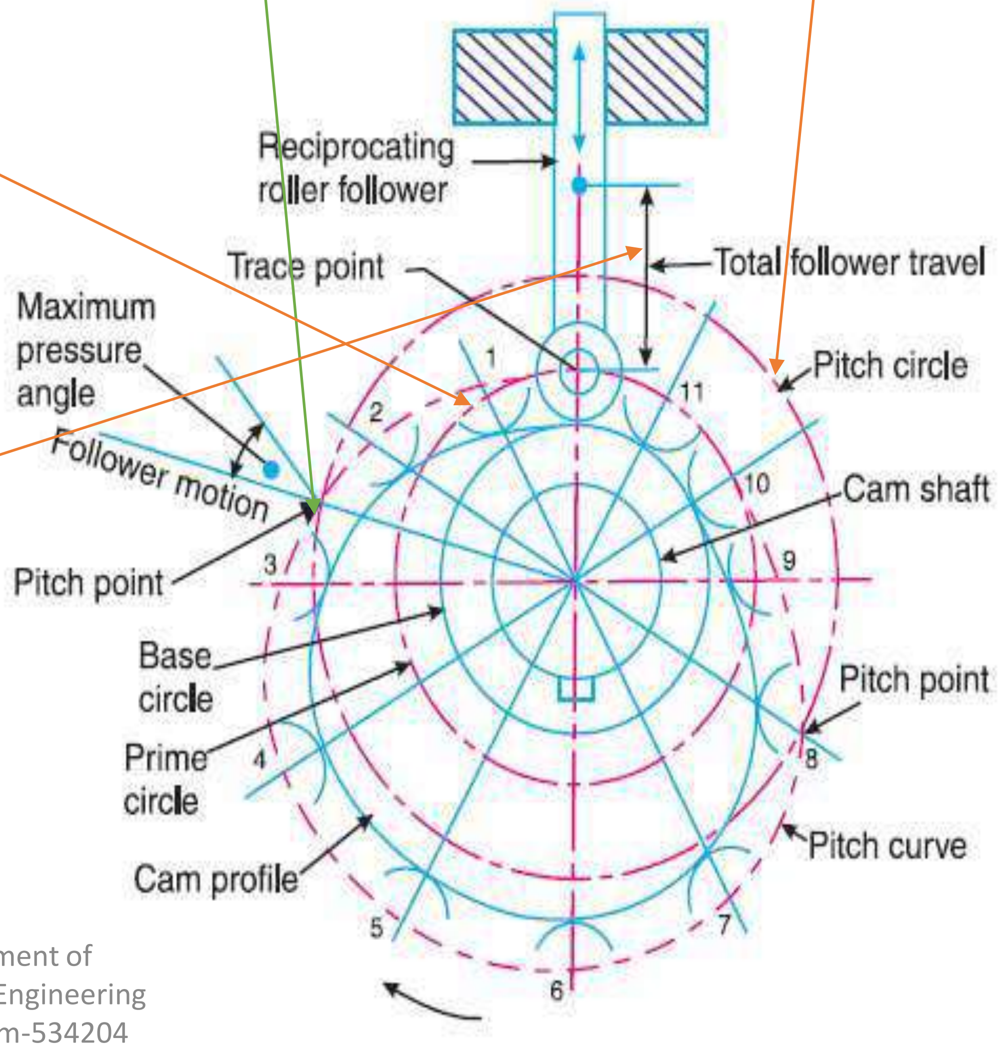
Terms Used in Radial Cams

5 . **Pitch point.** It is a point on the pitch curve having the maximum pressure angle.

6. **Pitch circle.** It is a circle drawn from the Centre of the cam through the pitch points.

7. **Prime circle.** It is the smallest circle that can be drawn from the centre of the cam and tangent to the pitch curve. For a knife edge and a flat face follower, the prime circle and the base circle are identical. For a roller follower, the prime circle is larger than the base circle by the radius of the roller.

8. **Lift or stroke.** It is the maximum travel of the follower from its lowest position to the top most position.



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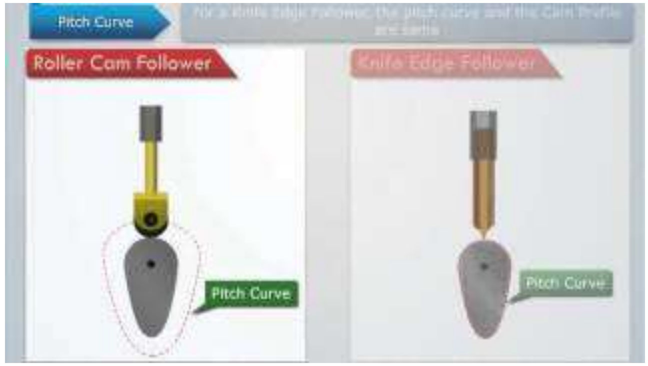

Terms Used in Cam and Follower

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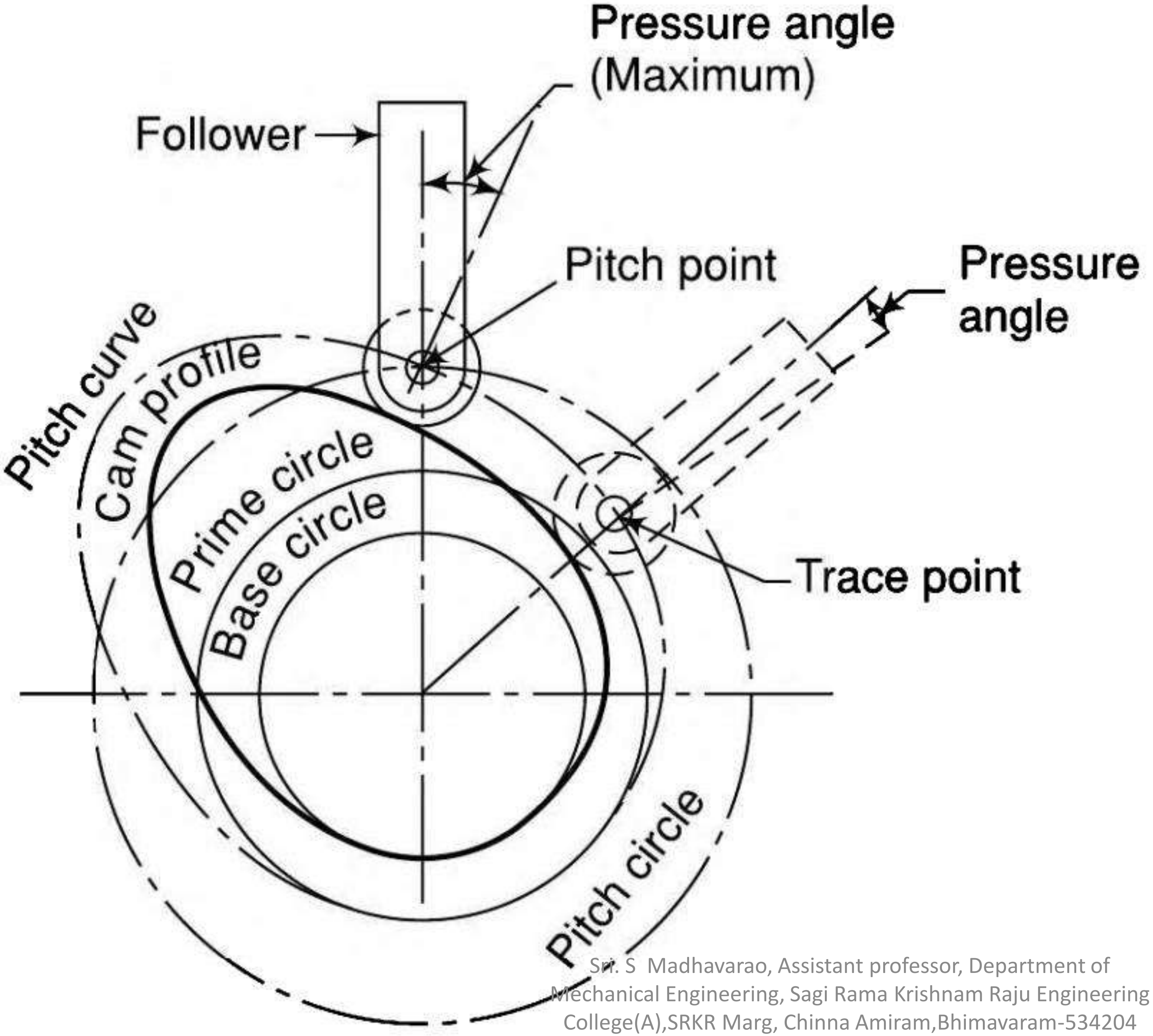
CAM TERMINOLOGIES



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Motion of the Follower

The follower, during its travel, may have one of the following motions.

– **Uniform velocity**

– **Simple harmonic motion**

– **Uniform acceleration and retardation**

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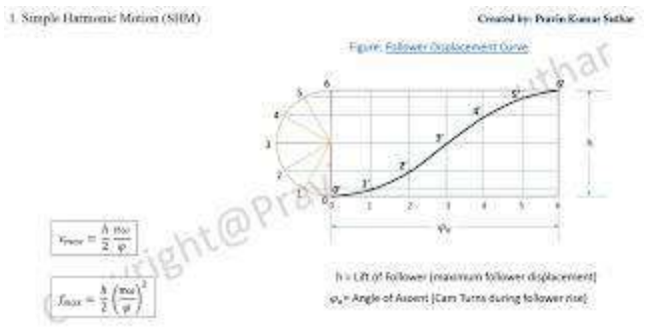

displacement diagram for cam, follower motion, Cam profile, Follower, cam

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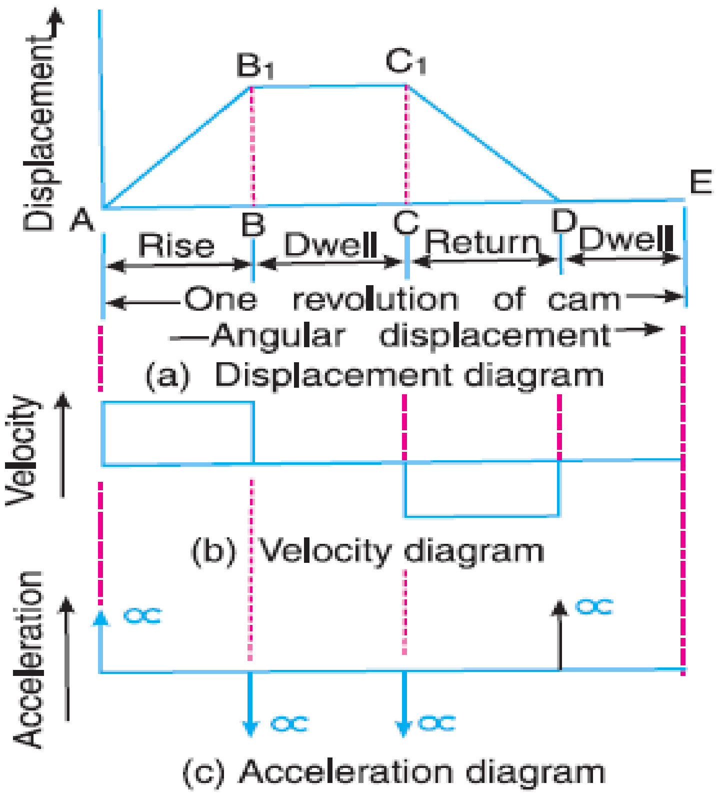
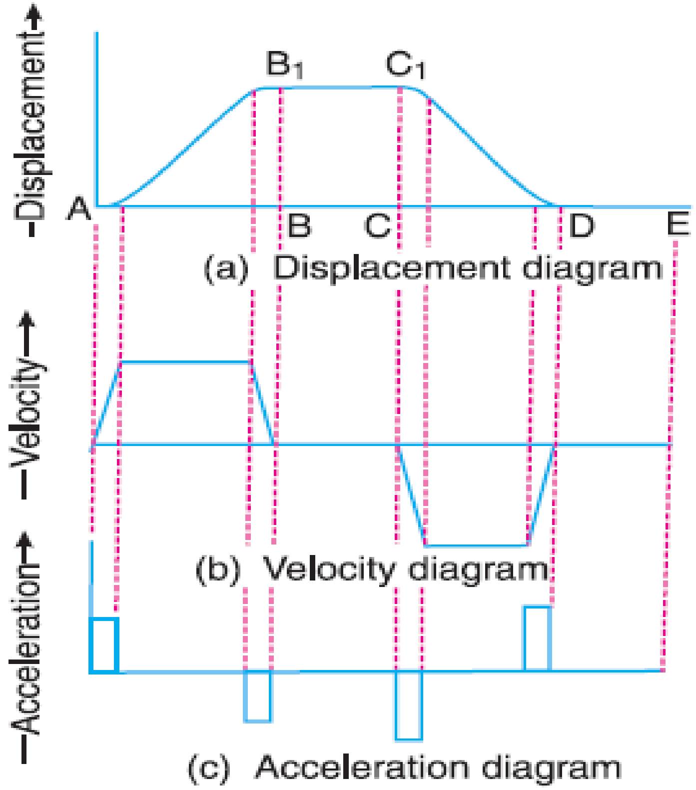
Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Velocity

- The displacement, velocity and acceleration diagrams when a knife-edged follower moves with uniform velocity are shown in Fig. (a), (b) and (c) respectively.
- The abscissa x-axis (base) represents the time (i.e. the number of seconds required for the cam to complete one revolution) or it may represent the angular displacement of the cam in degrees. The ordinate(y axis) represents the displacement, or velocity or acceleration of the follower.

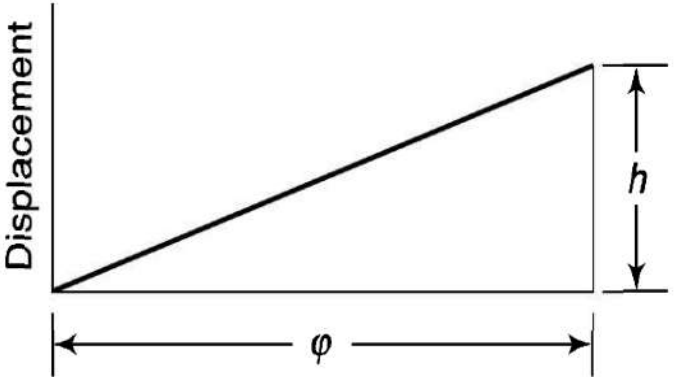




Displacement diagram: constant velocity of the follower implies that the displacement of the follower is proportional to the cam displacement and slope of the displacement curve is constant[fig.(a)]

Displacement of the follower for the angular displacement Θ of the cam is given by

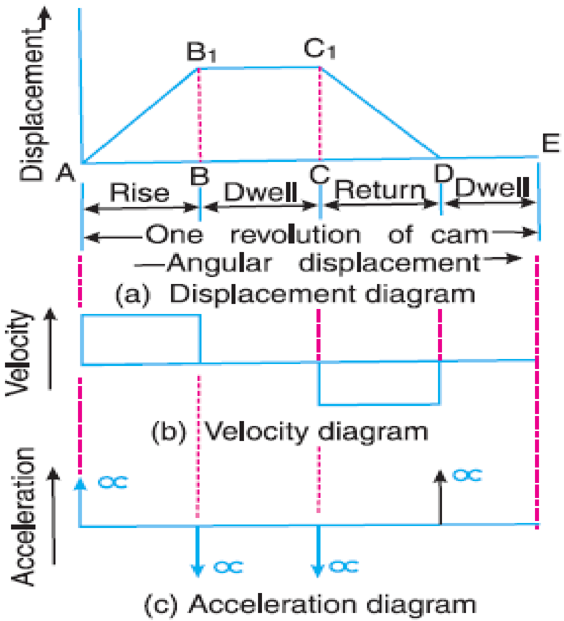
$$x = \frac{s}{\Theta 0} * \Theta \qquad s = h\frac{\theta}{\varphi} = h\frac{\omega t}{\varphi}$$

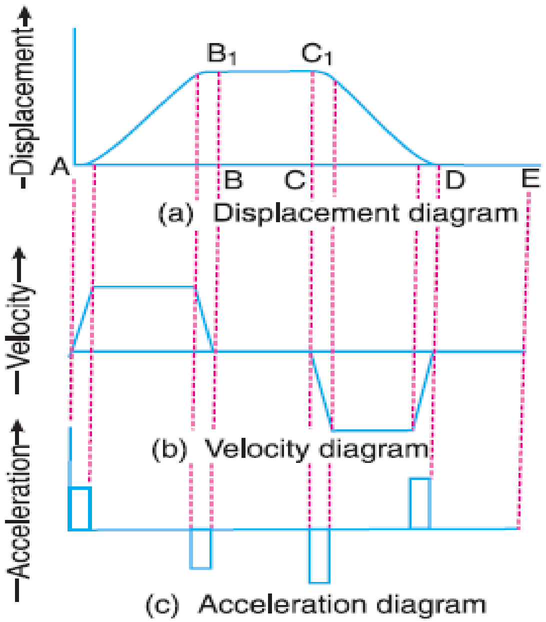


Velocity diagram: when the follower moves with uniform velocity. The velocity is Constant during rise and return stroke. Hence the velocity will be represented by horizontal lines during rise and return stroke

$$v = \frac{s}{\Theta 0} * \omega$$

$$v = \frac{ds}{dt} = \frac{h\omega}{\varphi} \text{ constant}$$

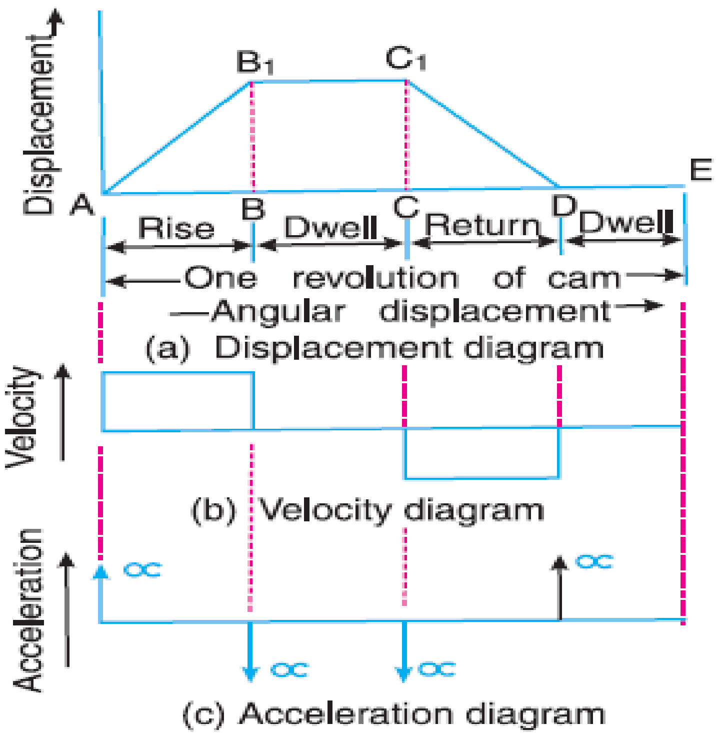




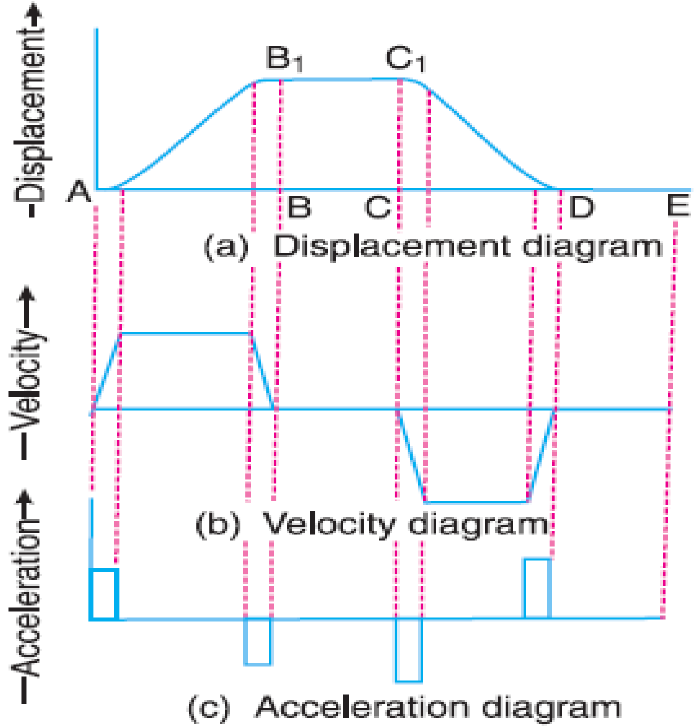
- Acceleration diagram: when the follower moves with uniform velocity .At the beginning and at the end of rise stroke, the velocity change suddenly(change in velocity takes place in zero time).This means the acceleration is in infinite in magnitude at beginning of the rise stroke ,which is physically impossible
- Modified acceleration: Hence the conditions which govern the follower motion ,must be modified so that the acceleration and retardation are having finite values. This is done by rounding off the sharp corner s of the displacement diagram at the beginning and at the end of each stroke

a=0

$$f = \frac{dv}{dt} = 0$$

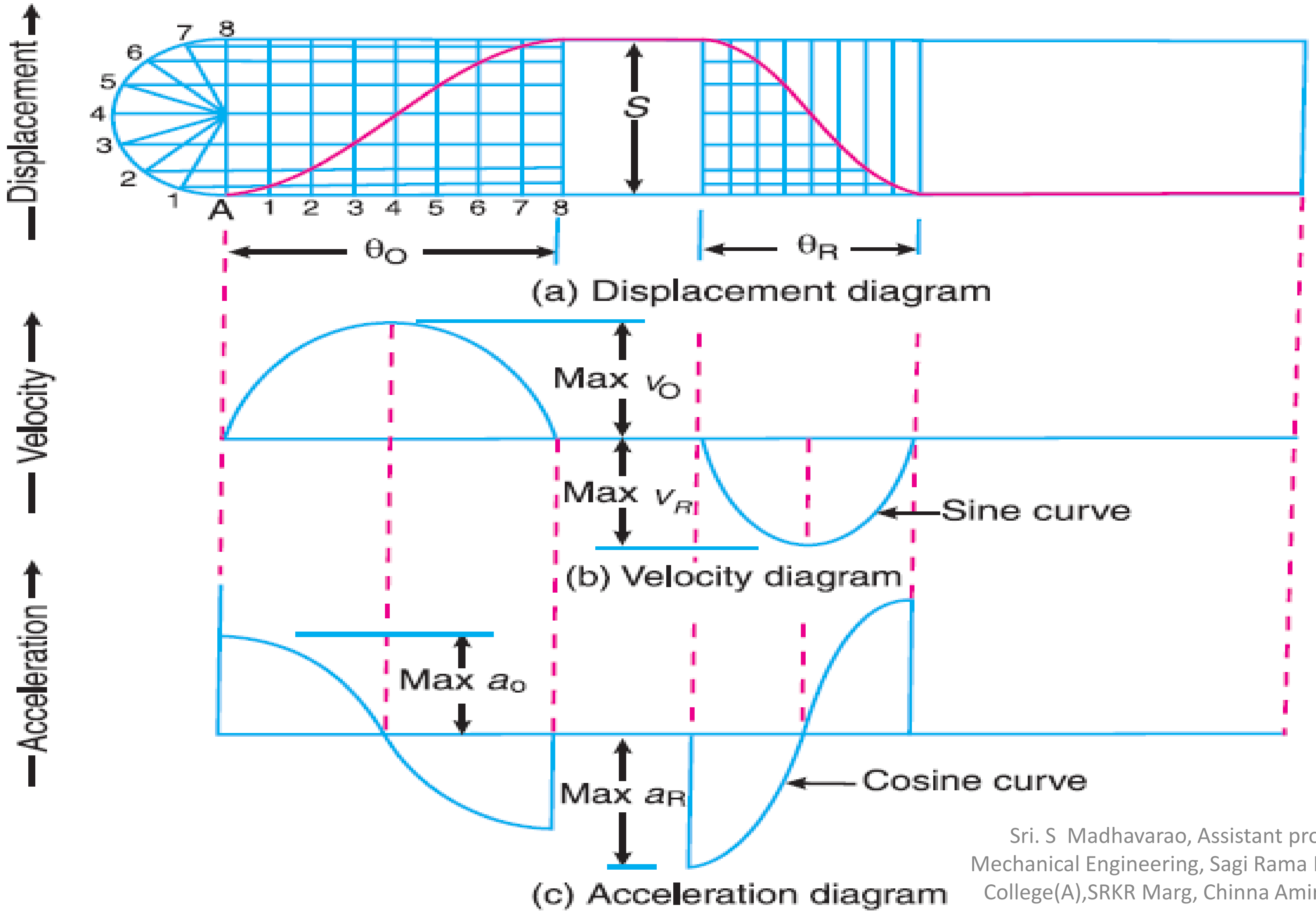


(a) Displacement diagram
(b) Velocity diagram
(c) Acceleration diagram



(a) Displacement diagram
(b) Velocity diagram
(c) Acceleration diagram

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Simple Harmonic Motion



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- **Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Simple Harmonic Motion**

Let $h = S$ = Stroke of the follower,

θ_O and θ_R = Angular displacement of the cam during out stroke and return stroke of the follower respectively, in radians, and

ω = Angular velocity of the cam in rad/s.

∴ Time required for the out stroke of the follower in seconds,

$$t_O = \theta_O / \omega$$

Consider a point P moving at a uniform speed ω_P radians per sec round the circumference of a circle with the stroke S as diameter, as shown in Fig. 20.7. The point P' (which is the projection of a point P on the diameter) executes a simple harmonic motion as the point P rotates. The motion of the follower is similar to that of point P'.

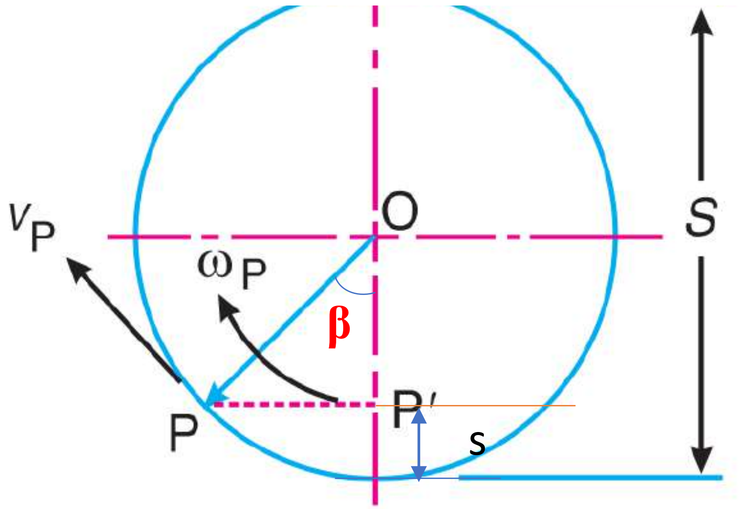

Fig. 20.7. Motion of a point.

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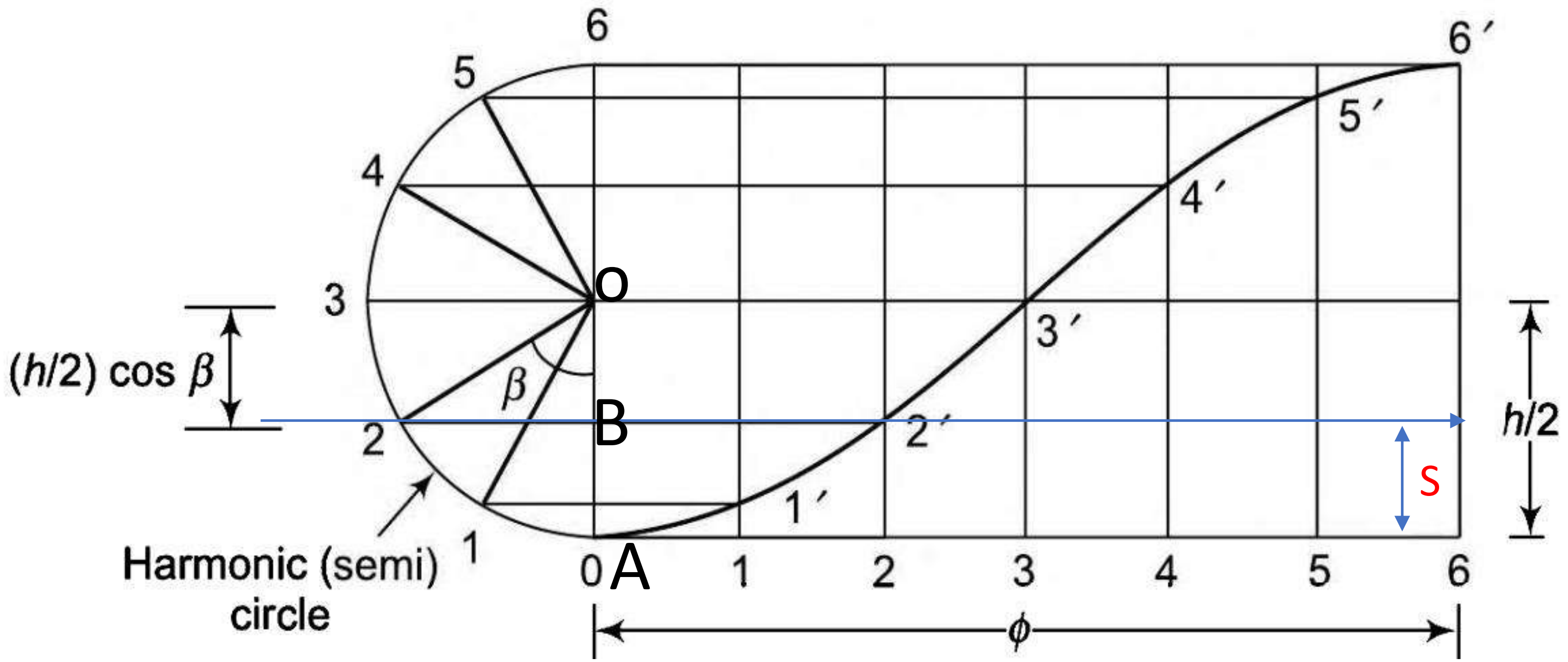

$$S = OA - OB$$

$$s = \frac{h}{2} - \frac{h}{2}\cos\beta$$

$$= \frac{h}{2}(1-\cos\beta)$$

For the rise (or fall) h of the follower displacement, the cam is rotated through an angle φ whereas a point on the harmonic semicircle traverses an angle π. Thus, the cam rotation is proportional to the angle turned by the point on the harmonic semicircle, i.e.

$$\beta = \pi\frac{\theta}{\varphi}$$

Thus β can be replaced by θ and φ in Eq. (i) above,

$$s = \frac{h}{2}\left(1-\cos\frac{\pi\theta}{\varphi}\right)$$

The expression is also valid for β more than 90°. In that case, cos β or cos $\pi\theta/\varphi$ becomes negative so that s is again positive and more than $h/2$.

Let ω = Angular velocity of the cam

$$\therefore \qquad \theta = \omega t$$

$$s = \frac{h}{2}\left(1 - \cos\frac{\pi\omega t}{\varphi}\right)$$

$$v = \frac{ds}{dt} = \frac{h}{2}\frac{\pi\omega}{\varphi}\sin\frac{\pi\omega t}{\varphi}$$

$$= \frac{h}{2}\frac{\pi\omega}{\varphi}\sin\frac{\pi\theta}{\varphi}$$

$$v_{max} = \frac{h}{2}\frac{\pi\omega}{\varphi} \text{ at } \theta = \frac{\varphi}{2}$$

$$f = \frac{dv}{dt} = \frac{h}{2}\left(\frac{\pi\omega}{\varphi}\right)^2\cos\frac{\pi\omega t}{\varphi}$$

$$= \frac{h}{2}\left(\frac{\pi\omega}{\varphi}\right)^2\cos\frac{\pi\theta}{\varphi}$$

$$f_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\varphi}\right)^2 \text{ at } \theta = 0$$

∴ Peripheral speed of the point P',

$$v_P = \frac{\pi S}{2} \times \frac{1}{t_O} = \frac{\pi S}{2} \times \frac{\omega}{\theta_O}$$

and maximum velocity of the follower on the outstroke,

$$v_O = v_P = \frac{\pi S}{2} \times \frac{\omega}{\theta_O} = \frac{\pi \omega S}{2\theta_O}$$

- Maximum acceleration of the follower on the outstroke,

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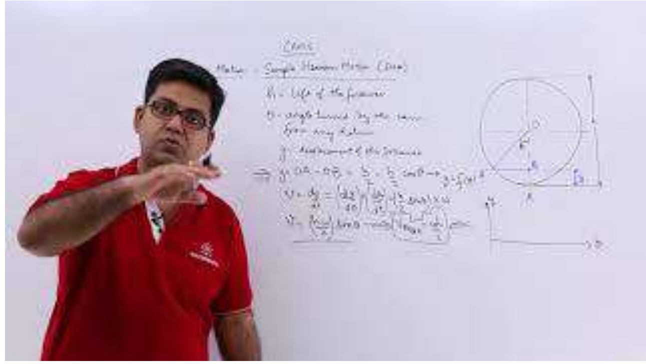

$$a_O = a_P = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2}$$

Similarly, maximum velocity of the follower on the return stroke,

$$v_R = \frac{\pi \omega . S}{2\theta_R}$$

and maximum acceleration of the follower on the return stroke,

$$a_R = \frac{\pi^2 \omega^2 . S}{2(\theta_R)^2}$$

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- **Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Acceleration and Retardation**

Mean velocity of the follower during outstroke

$V = S/t_O$

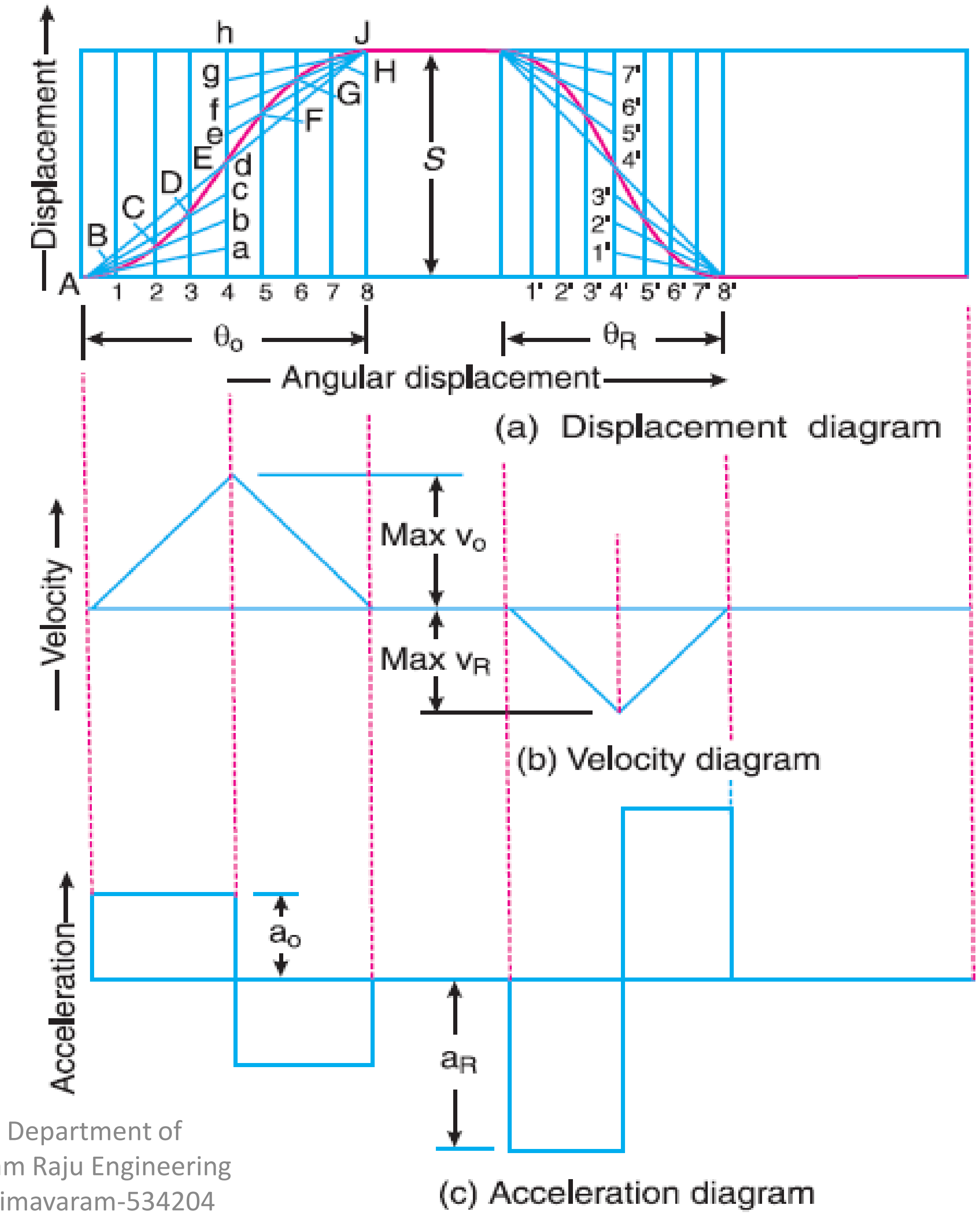
and mean velocity of the follower during return stroke

$= S/t_R$

Since the maximum velocity of the follower is equal to twice

The mean velocity, therefore maximum velocity of the follower during outstroke

$$v_O = \frac{2S}{t_O} = \frac{2\omega.S}{\theta_O}$$



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We see from the acceleration diagram, as shown in Fig. 20.8 (*c*), that during first half of the outstroke there is uniform acceleration and during the second half of the out stroke there is uniform retardation. Thus, the maximum velocity of the follower is reached after the time $t_O/2$ (during out stroke) and $t_R/2$ (during return stroke).

velocity of the follower during outstroke,

$$v_O = \frac{2\omega S}{\theta_O}$$

maximum velocity of the follower during return

$$v_R = \frac{2\omega S}{\theta_R}$$

Maximum acceleration of the follower during outstroke,

$$a_O = \frac{4\omega^2 . S}{(\theta_O)^2}$$

Similarly, maximum acceleration of the follower during return

$$a_R = \frac{4\omega^2 . S}{(\theta_R)^2}$$

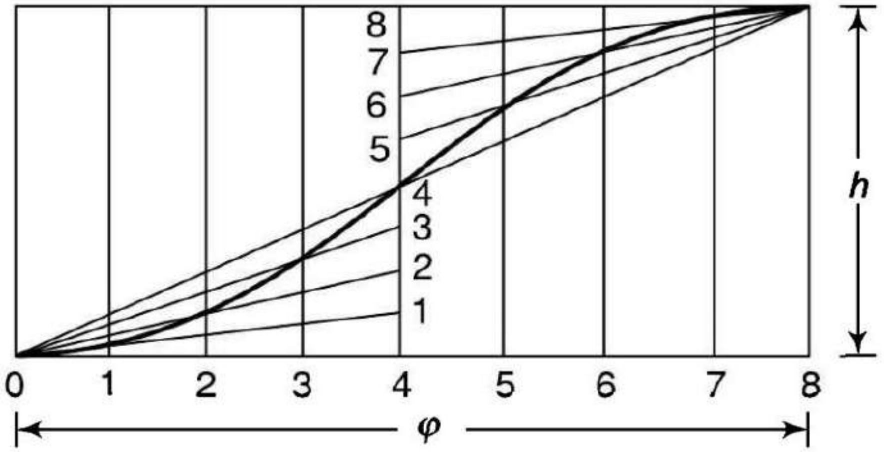
$s = ut + 1/2ft^2$ $s = 1/2ft^2$ Initial velocity is u is zero at rise and fall

or $$f = \frac{2s}{t^2} = \text{constant}$$

As f is constant during the accelerating period, considering the follower at the midway,

$$s = \frac{h}{2} \quad \text{and} \quad t = \frac{\varphi/2}{\omega}$$

$$\therefore \quad f = \frac{2h/2}{\varphi^2/4\omega^2} = \frac{4h\omega^2}{\varphi^2}$$



($\theta = \omega t$)

The velocity is linear during the period and is given by

$$v = \frac{ds}{dt} = \frac{1}{2} \times 2ft = ft$$

$$= \frac{4h\omega^2}{\varphi^2} \frac{\theta}{\omega}$$

$$= \frac{4h\omega}{\varphi^2} \theta$$

(

The velocity is maximum when θ is maximum or the follower is at the midway, i.e., when $\theta = \varphi/2$.

$$v_{max} = \frac{4h\omega}{\varphi^2} \frac{\varphi}{2} = \frac{2h\omega}{\varphi}$$

During the second half of the follower motion, the follower is decelerated at constant rate so that the velocity reduces to zero at the end.

Construction of Cam Profile for a Radial Cam

In order to draw the cam profile for a radial cam,

(1) first of all the displacement diagram for the given motion of the follower is drawn.

(2) Then by constructing the follower in its proper position at each angular position, the profile of the working surface of the cam is drawn.

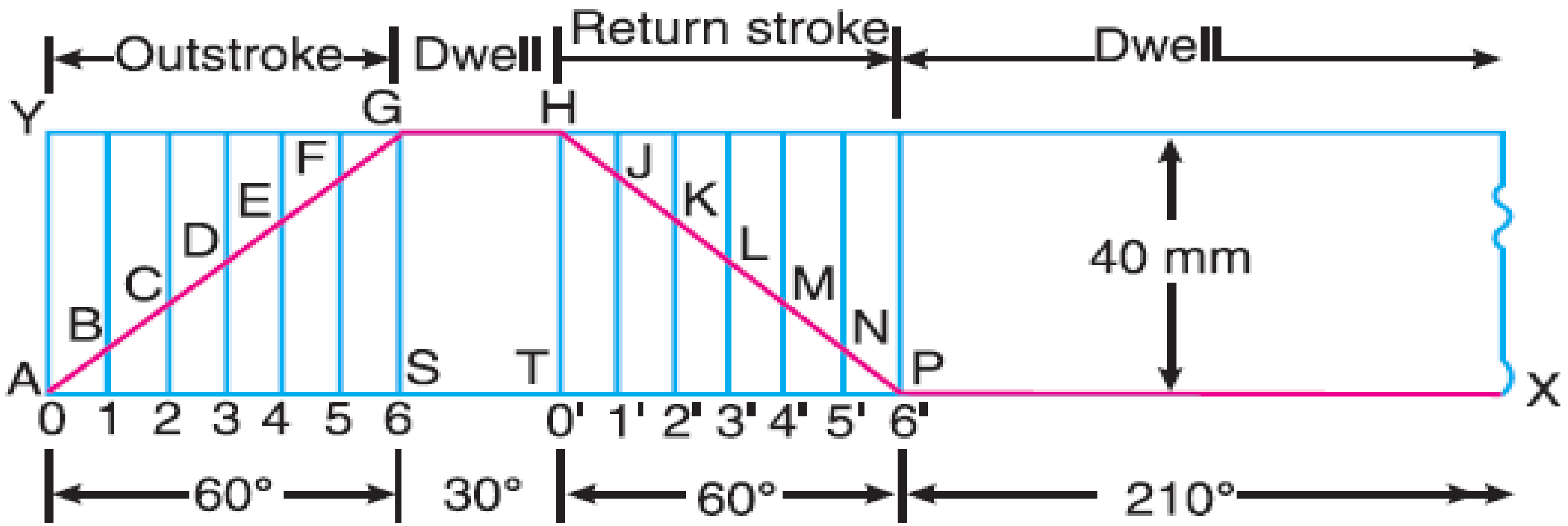
In constructing the cam profile, the principle of kinematic inversion is used, i.e. the cam is imagined to be stationary and **the follower is allowed to rotate in the opposite direction to the cam rotation.**

The construction of cam profiles for different types of follower with different types of motions

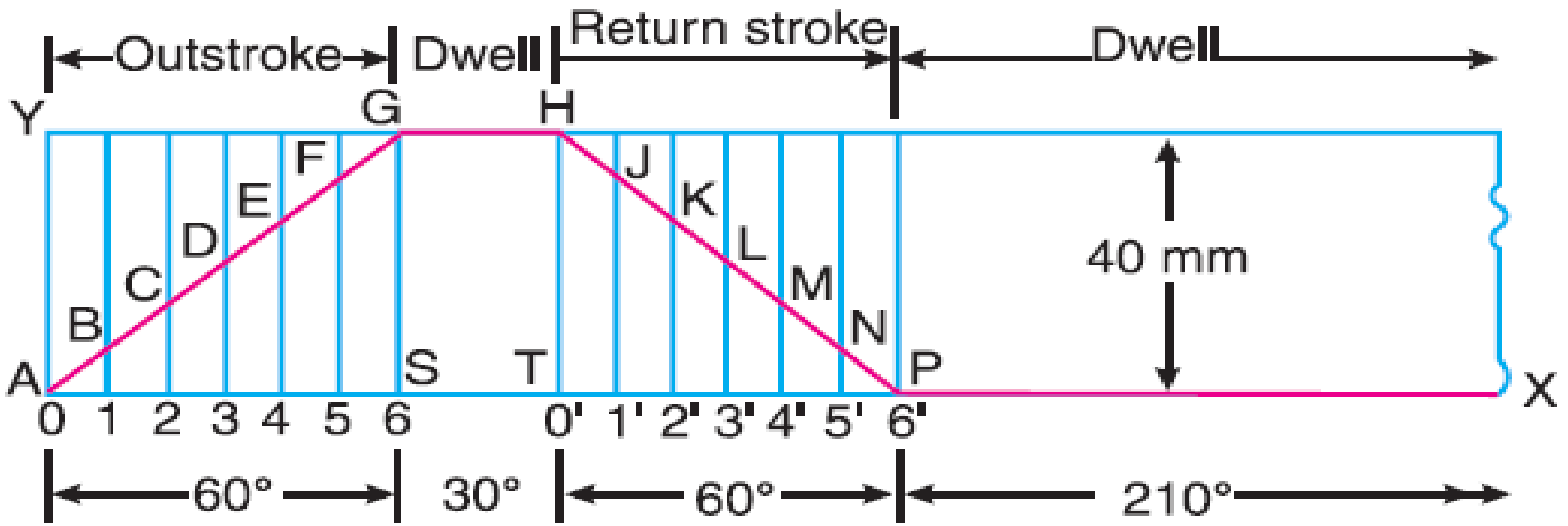
Q1. A cam is to give the following motion to a knife-edged follower: 1. Outstroke during 60° of cam rotation ; 2. Dwell for the next 30° of cam rotation ; 3. Return stroke during next 60° of cam rotation, and 4. Dwell for the remaining 210° of cam rotation. The stroke of the follower is 40 mm and the minimum radius of the cam is 50 mm. The follower moves with uniform velocity during both the outstroke and return strokes. Draw the profile of the cam when (a) the axis of the follower passes through the axis of the cam shaft, and (b) the axis of the follower is offset by 20 mm from the axis of the cam shaft

Sol: Scale x-Axis 1cm=10^0 Y-Axis 1cm=1cm

Construction



Construction



1. Draw a horizontal line AX = 360° to some suitable scale. On this line, mark AS = 60° to represent outstroke of the follower, ST = 30° to represent dwell, TP = 60° to represent return stroke and PX = 210° to represent dwell.

2. Draw vertical line AY equal to the stroke of the follower (i.e. 40 mm) and complete the rectangle as shown in Fig.

3. Divide the angular displacement during outstroke and return stroke into any equal number of even parts (say six) and draw vertical lines through each point.

4. Since the follower moves with uniform velocity during outstroke and return stroke, therefore the displacement diagram consists of straight lines. Join AG and HP.

5. The complete displacement diagram is shown by AGHPX in Fig.

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(a) Profile of the cam when the axis of follower passes through the axis of cam shaft

- The profile of the cam when the axis of the follower passes through the axis of the cam shaft.
- **Now set off 1B, 2C, 3D ... etc. and 0 H,1 J ... etc. from the displacement diagram.**
- Join the points A, B, C,... M, N, P with a smooth curve. The curve AGHPA is the complete profile of the cam

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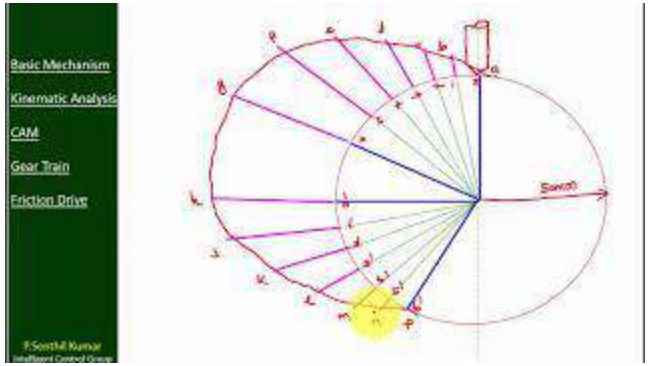
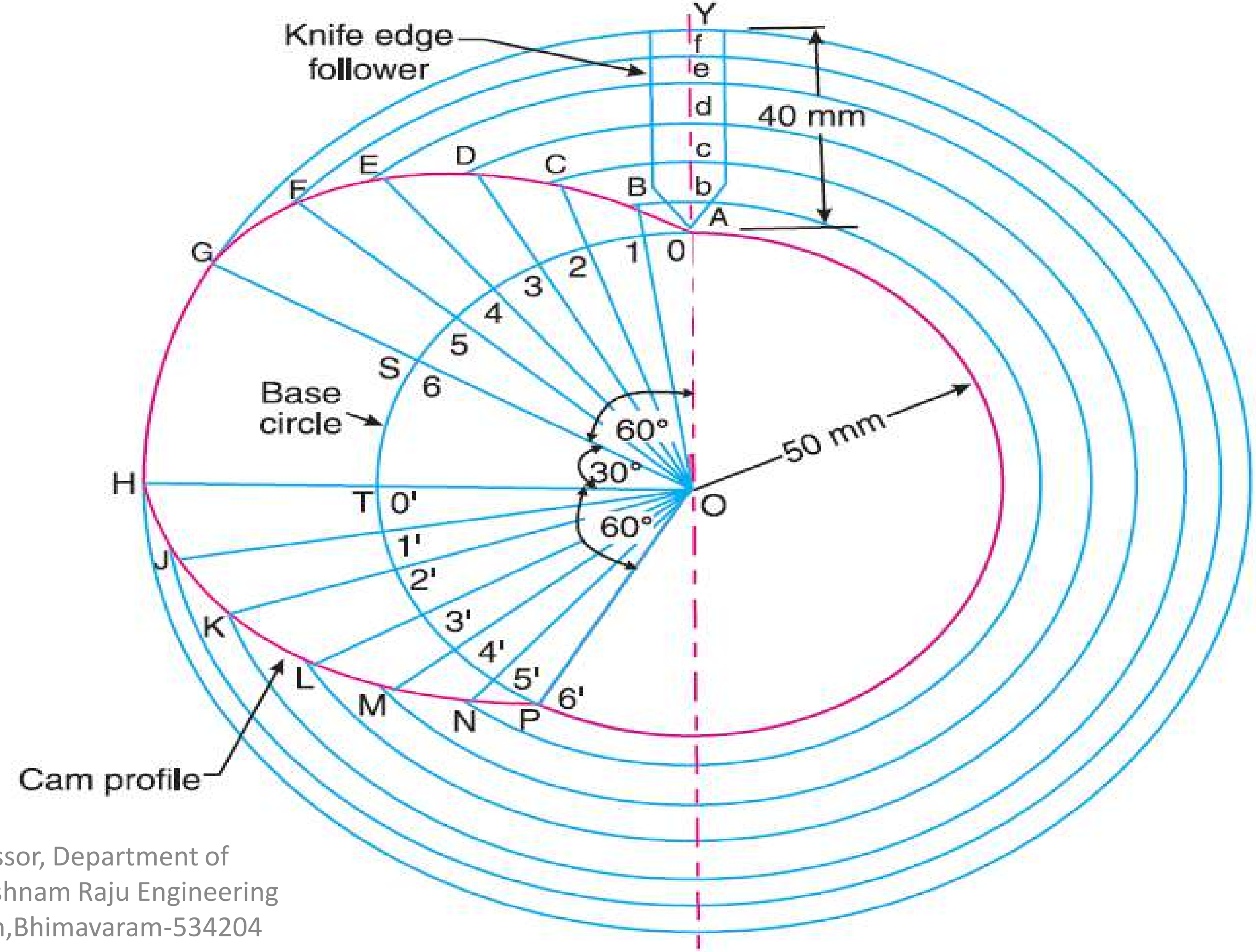


Cam profile for knife edge follower with uniform velocity motion



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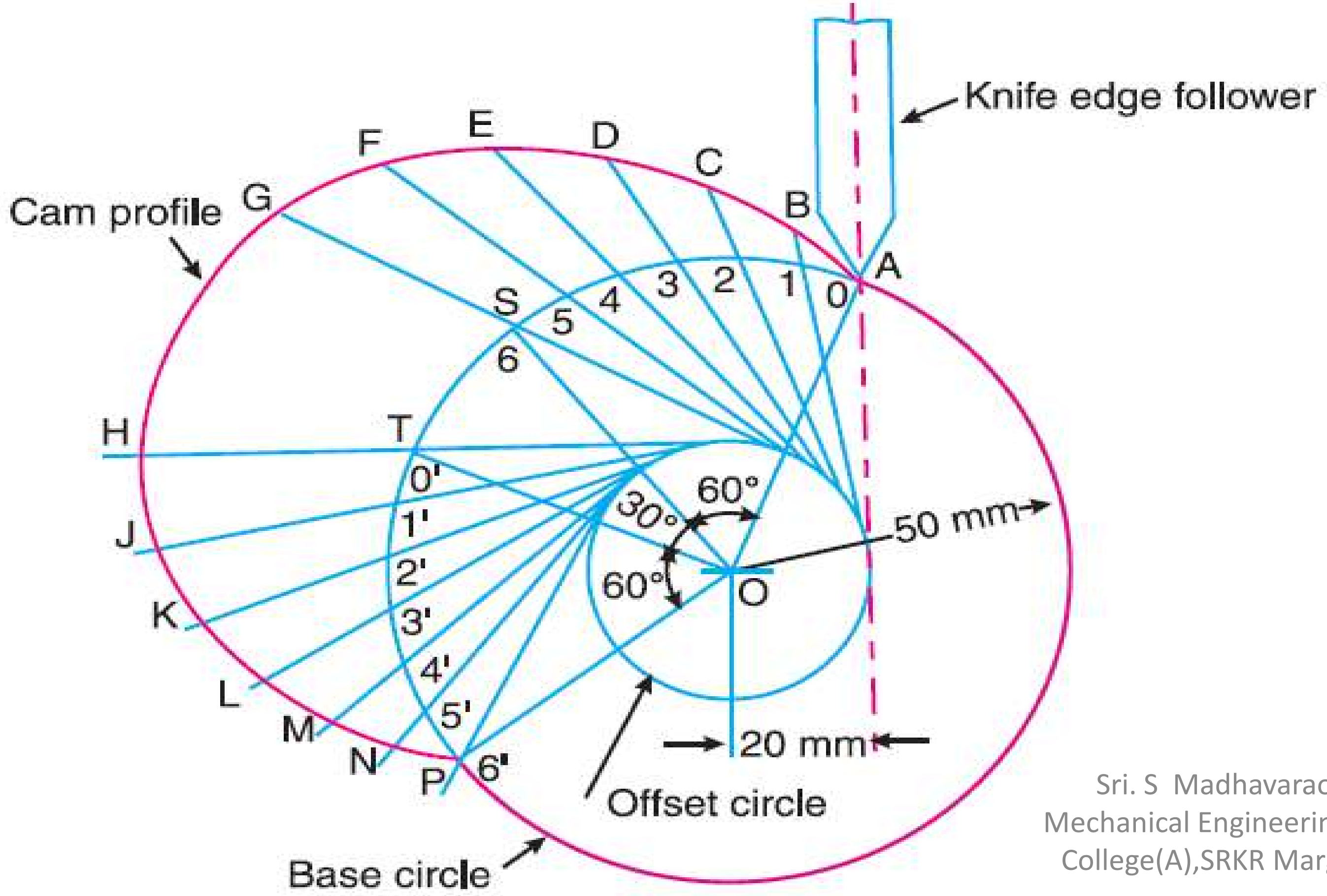
- **(b) Profile of the cam when the axis of the follower is offset by 20 mm from the axis of the cam shaft**
- The profile of the cam when the axis of the follower is offset from the axis of the cam shaft
- **1**.Now from the points 1, 2, 3 ... etc. and 0,1, 2,3 ... etc. on the base circle, draw tangents to the offset circle and produce these tangents beyond the base circle as shown in Fig
- **2**.Now set off 1B, 2C, 3D ... etc. and 0 H,1 J ... etc. from the displacement diagram



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Q2. A cam is to be designed for a knife edge follower with the following data

1. Cam lift = 40 mm during 90° of cam rotation with simple harmonic motion.
2. Dwell for the next 30°. 3. During the next 60° of cam rotation, the follower returns to its original position with simple harmonic motion. 4. Dwell during the remaining 180°. Draw the profile of the cam when

(a) the line of stroke of the follower passes through the axis of the cam shaft, and

(b) the line of stroke is offset 20 mm from the axis of the cam shaft.

The radius of the base circle of the cam is 40 mm. Determine the maximum velocity and acceleration of the follower during its ascent and descent, if the cam rotates at 240 r.p.m.

SOL: Scale x-Axis $1cm=10^0$ Y-Axis 1cm=1cm

Solution. Given : $S = 40$ mm $= 0.04$ m; $\theta_O = 90° = \pi/2$ rad $= 1.571$ rad ; $\theta_R = 60° = \pi/3$ rad $= 1.047$ rad ; $N = 240$ r.p.m.

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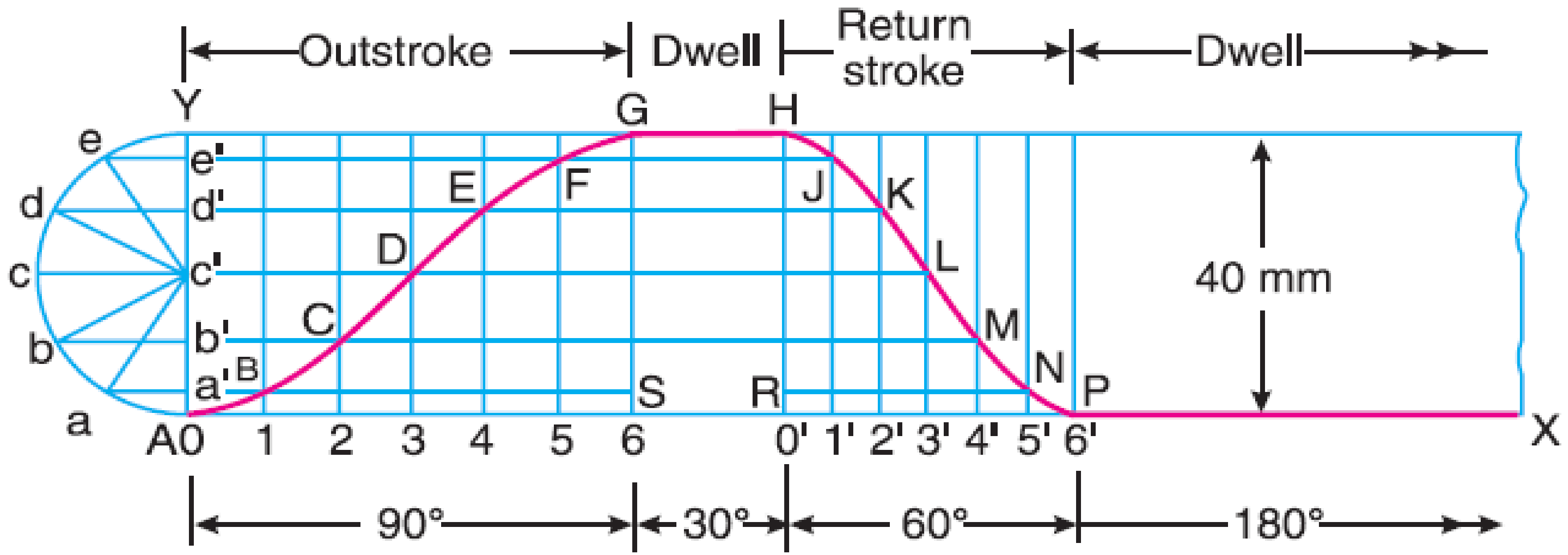

1. Draw horizontal line AX = 360° to some suitable scale. On this line, mark AS = 90° to represent out stroke ; SR = 30° to represent dwell ; RP = 60° to represent return stroke and PX = 180° to represent dwell.

2. Draw vertical line AY = 40 mm to represent the cam lift or stroke of the follower and complete the rectangle as shown in Fig.

3. Divide the angular displacement during out stroke and return stroke into any equal number of even parts (say six) and draw vertical lines through each point.

4. Since the follower moves with simple harmonic motion, therefore draw a semicircle with AY as diameter and divide into six equal parts.

5. From points a, b, c ... etc. draw horizontal lines intersecting the vertical lines drawn through1, 2, 3 ... etc. and 0 ,1 , 2 ...etc. at B, C, D ... M, N, P.

6. Join the points A, B, C ... etc. with a smooth curve as shown in Fig. . This is the required displacement diagram

(a) Profile of the cam when the line of stroke of the follower passes through the axis of the cam shaft

- The profile of the cam when the line of stroke of the follower passes through the axis of the cam shaft,

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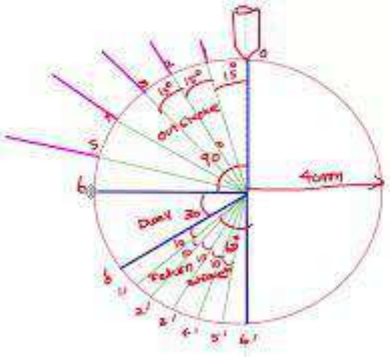

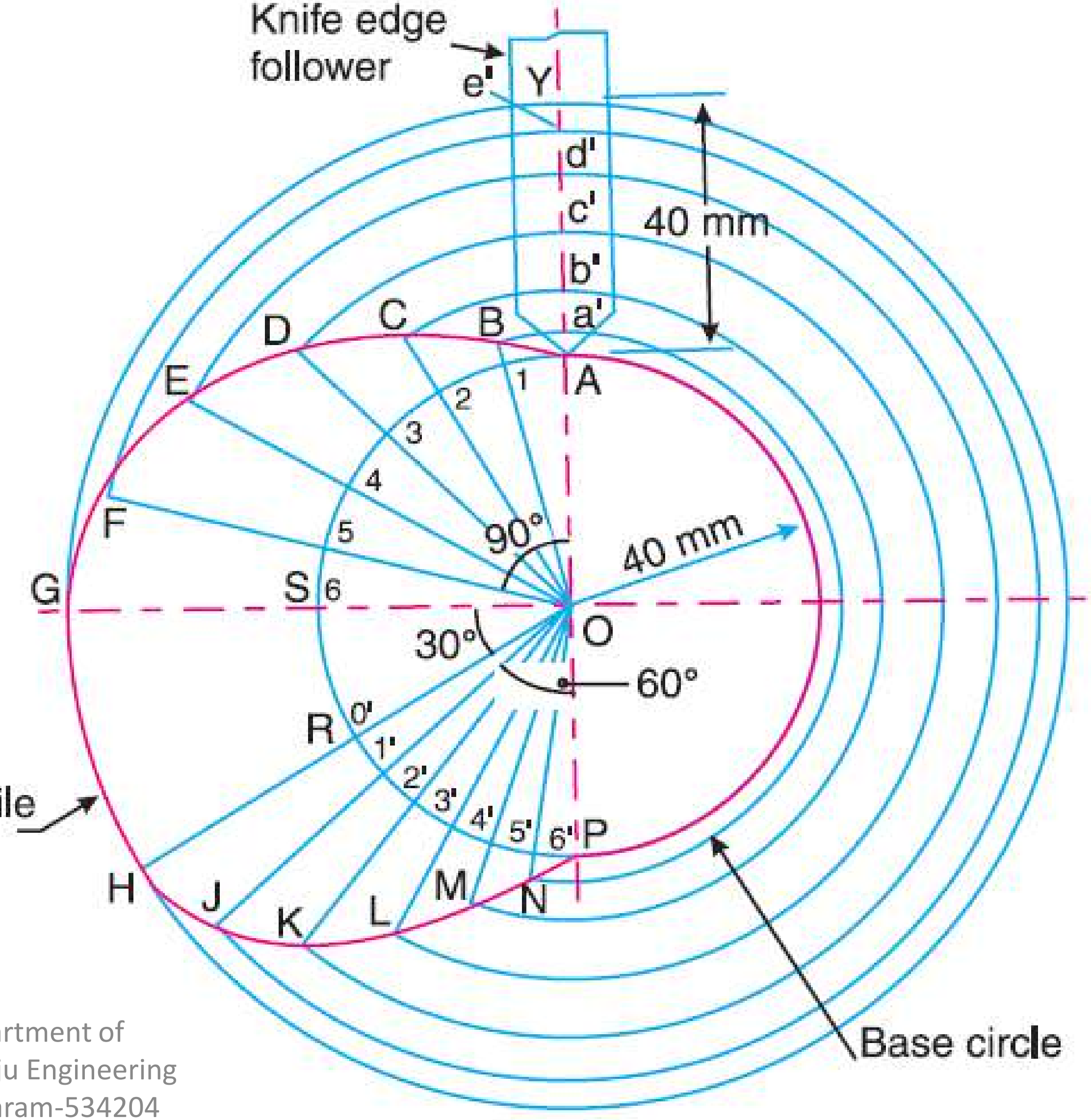

3.2 cam profile for knife edge follower with simple harmonic motion

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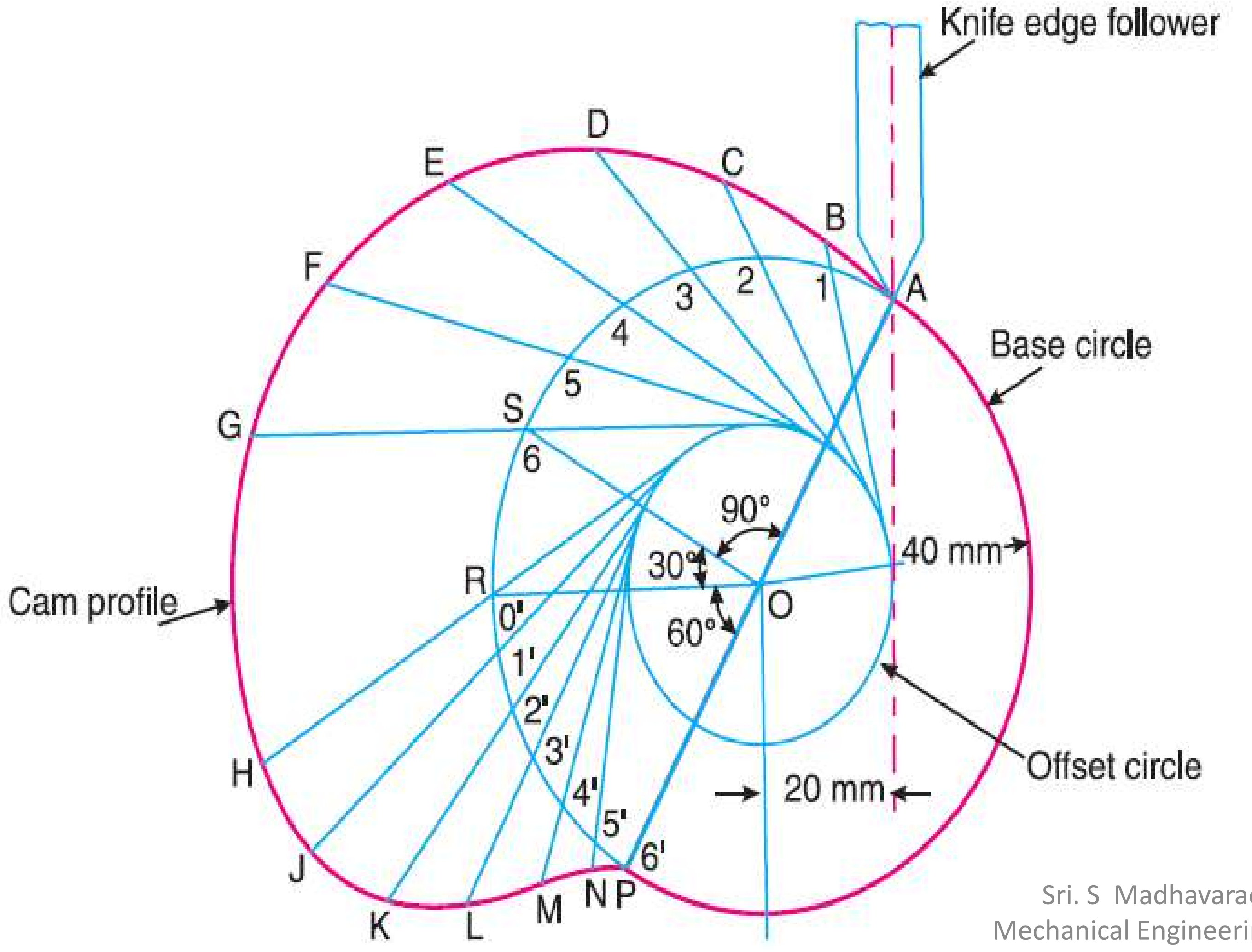
(b) Profile of the cam when the line of stroke of the follower is offset 20 mm from the axis of the cam shaft

- The profile of the cam when the line of stroke of the follower is offset 20 mm from the axis of the cam shaft



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Maximum velocity of the follower during its ascent and descent

We know that angular velocity of the cam,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 25.14 \text{ rad/s}$$

We also know that the maximum velocity of the follower during its ascent,

$$v_O = \frac{\pi \omega . S}{2\theta_O} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.571} = 1 \text{ m/s} \quad \textbf{Ans.}$$

and maximum velocity of the follower during its descent,

$$v_R = \frac{\pi \omega . S}{2\theta_R} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.047} = 1.51 \text{ m/s} \quad \textbf{Ans.}$$

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Maximum acceleration of the follower during its ascent and descent

We know that the maximum acceleration of the follower during its ascent,

$$a_O = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2} = \frac{\pi^2 (25.14)^2 0.04}{2(1.571)^2} = 50.6 \text{ m/s}^2 \textbf{ Ans.}$$

and maximum acceleration of the follower during its descent,

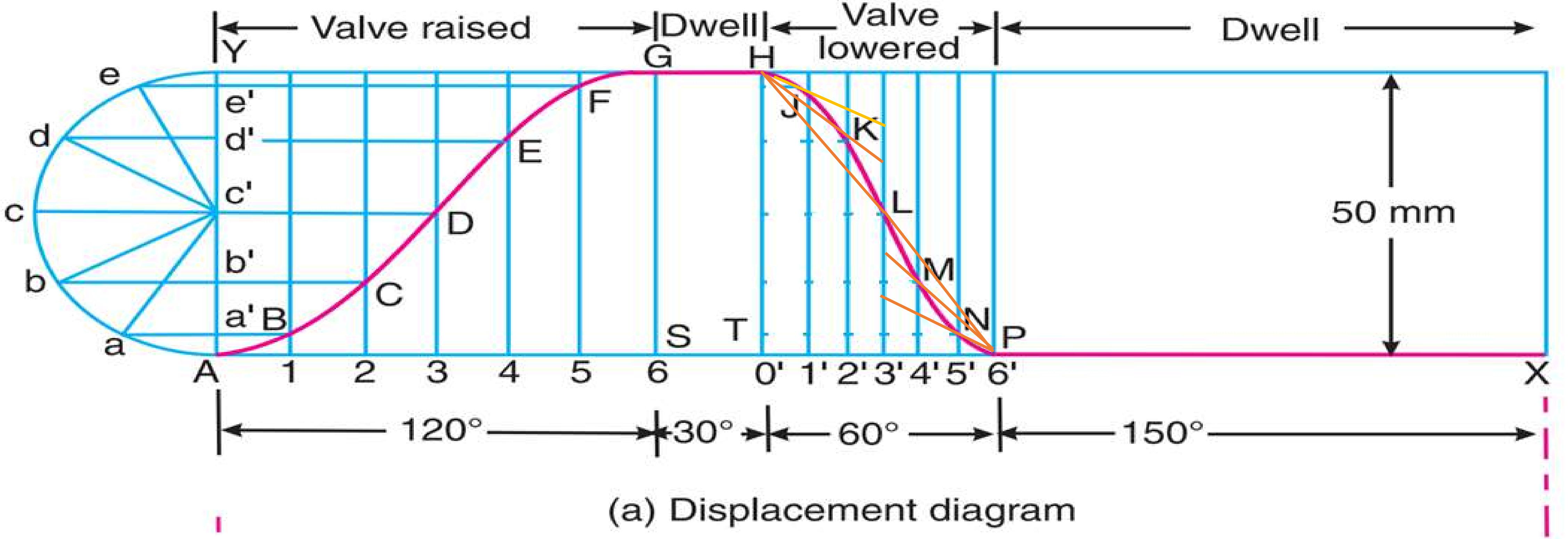
$$a_R = \frac{\pi^2 \omega^2 . S}{2(\theta_R)^2} = \frac{\pi^2 (25.14)^2 0.04}{2(1.047)^2} = 113.8 \text{ m/s}^2 \textbf{ Ans.}$$

Q3. A cam, with a minimum radius of 25 mm, rotating clockwise at a uniform speed is to be designed to give a roller follower, at the end of a valve rod, motion described below :

1. To raise the valve through 50 mm during 120° rotation of the cam ;
2. To keep the valve fully raised through next 30°;
3. To lower the valve during next 60°; and
4. To keep the valve closed during rest of the revolution i.e. 150° ;

The diameter of the roller is 20 mm and the diameter of the cam shaft is 25 mm. Draw the profile of the cam when (a) the line of stroke of the valve rod passes through the axis of the cam shaft, and (b) the line of the stroke is offset 15 mm from the axis of the cam shaft. The displacement of the valve, while being raised and lowered, is to take place with simple harmonic motion during outstroke and uniform retardation during return stroke.

Solution. Given : $S = 50$ mm $= 0.05$ m ; $\theta_O = 120° = 2\pi/3$ rad $= 2.1$ rad ; $\theta_R = 60° = \pi/3$ rad $= 1.047$ rad ; $N = 100$ r.p.m.



(a) Displacement diagram

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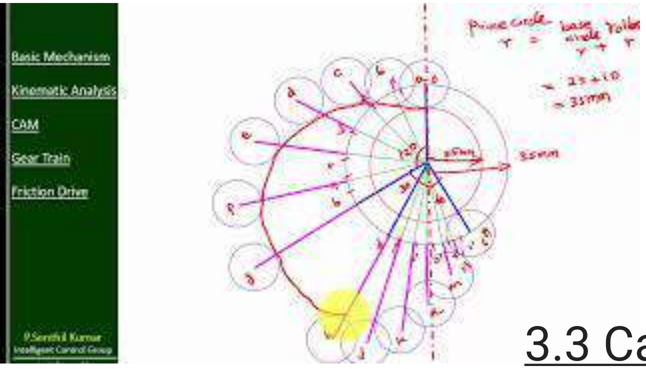


3.3 Cam profile for roller follower with simple harmonic and uniform retardation motion

(a) Profile of the cam when the line of stroke of the valve rod passes through the axis of the cam shaft

1.Draw a base circle with centre O and radius equal to the minimum radius of the cam (i.e. 25 mm).

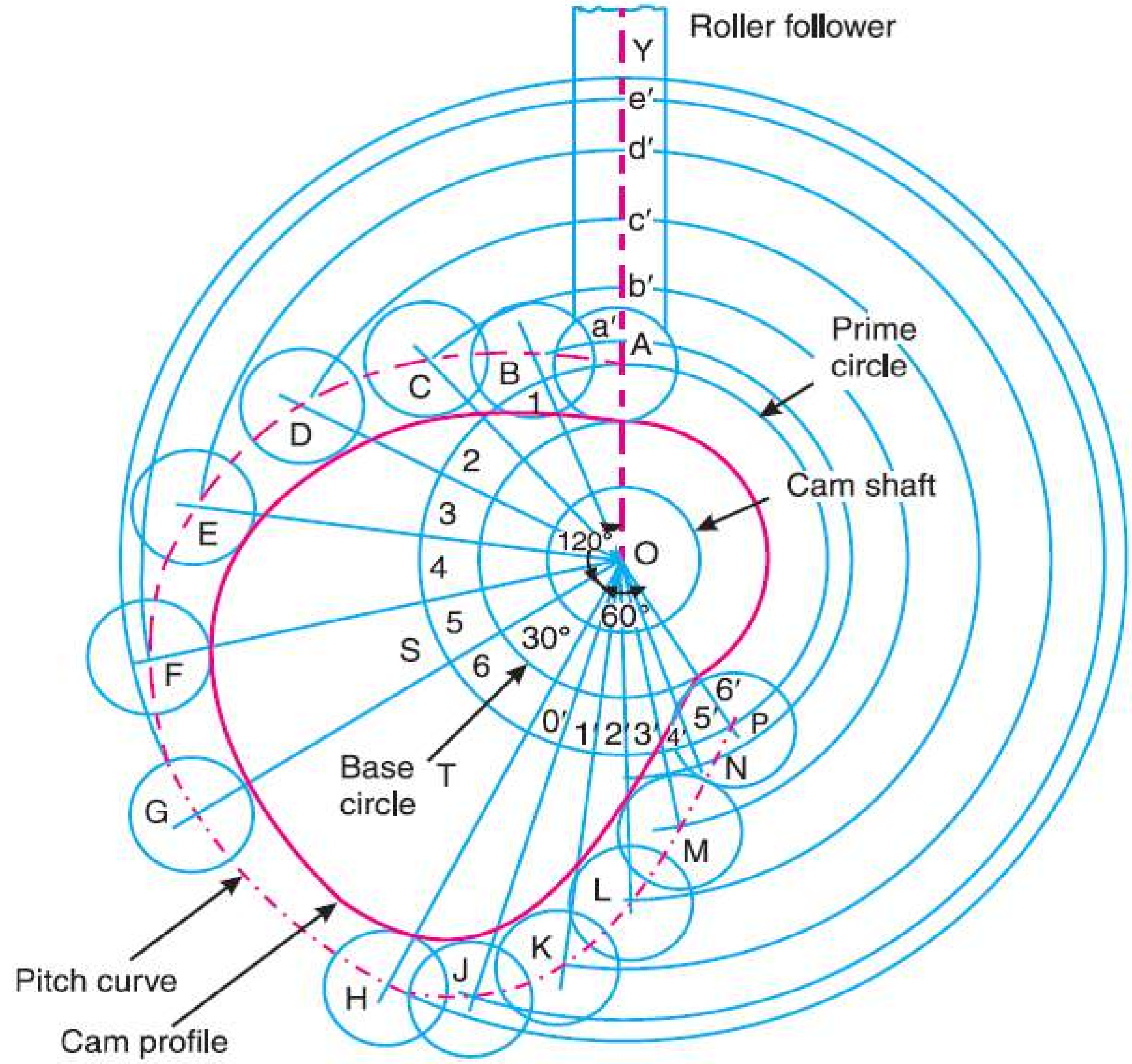
2. Join the points 1, 2, 3, etc. with the centre O and produce the lines beyond prime circle as shown in Fig.

3. Set off 1B, 2C, 3D etc. equal to the displacements from displacement diagram

4.Join the points A, B, C ... N, P, A. The curve drawn through these points is known as **pitch curve**

5.From the points A, B, C ... N, P, draw circles of radius equal to the radius of the roller

6.Join the bottoms of the circles with a smooth curve as shown in Fig. This is the required profile of the cam.

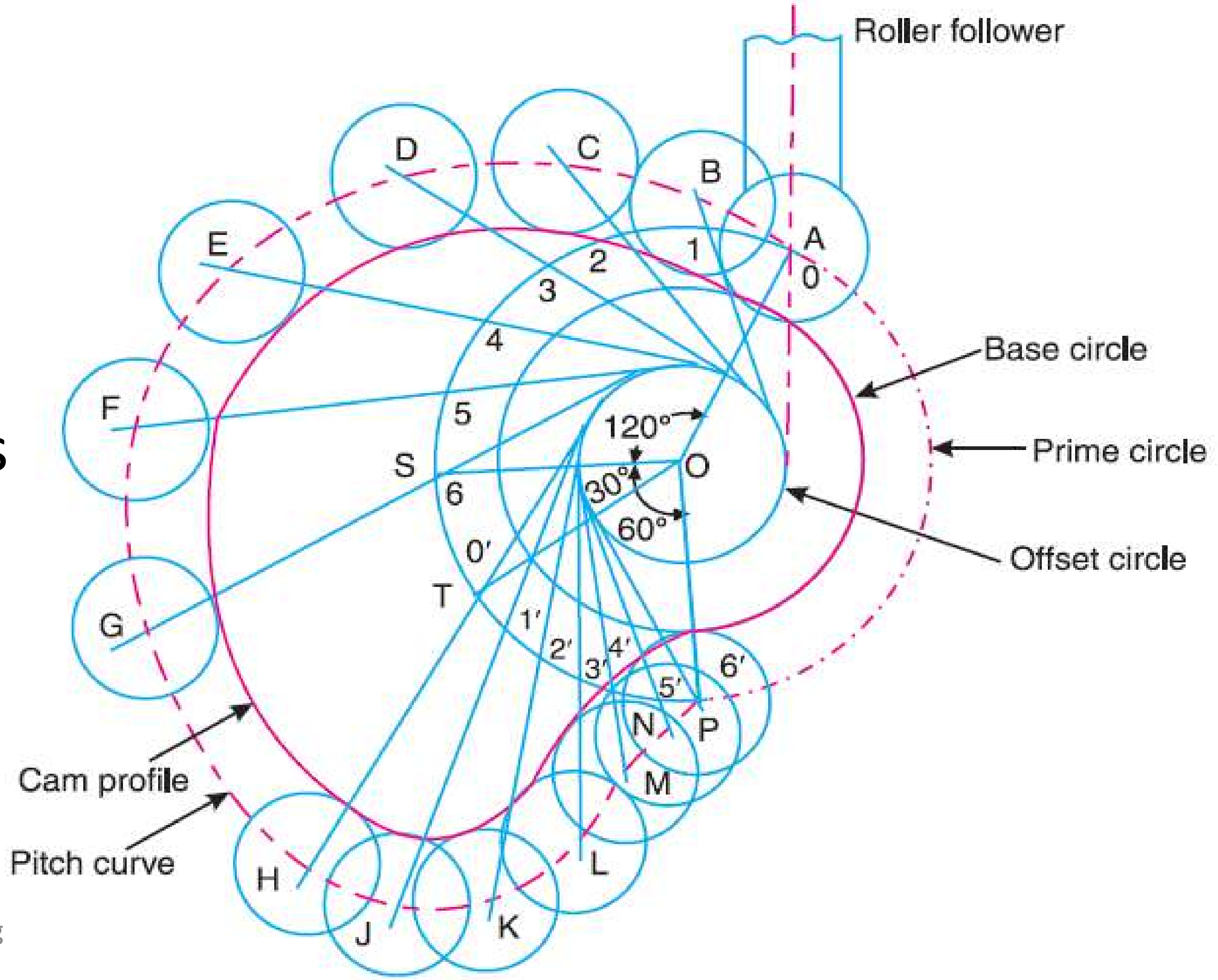


(b) Profile of the cam when the line of stroke is offset 15 mm from the axis of the cam shaft

1.From points 1, 2, 3 .... etc. and 0 ,1 , 3 , ...etc. on the prime circle, draw tangents to the offset circle

2.Set off 1B, 2C, 3D... etc. equal to displacements as measured from displacement diagram

3.Now A, B, C...etc. as centre, draw circles with radius equal to the radius of roller



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Q4.A cam, with a minimum radius of 50 mm, rotating clockwise at a uniform speed, is required to give a knife edge follower the motion as described below :

1. To move outwards through 40 mm during 100° rotation of the cam

2. To dwell for next80° ;

3. To return to its starting position during next 90°, and 4. To dwell for the rest period of a revolution i.e. 90°. Draw the profile of the cam

(i) when the line of stroke of the follower passes through the centre of the cam shaft, and (ii) when the line of stroke of the follower is off-set by 15 mm.

The displacement of the follower is to take place with uniform acceleration and uniform retardation. Determine the maximum velocity and acceleration of the follower when the cam shaft rotates at 900 r.p.m. Draw the displacement, velocity and acceleration diagrams for one complete revolution of the cam.

Solution. Given : $S = 40$ mm $= 0.04$ m; $\theta_O = 100° = 100 \times \pi/180 = 1.745$ rad ; $\theta_R = 90° = \pi/2 = 1.571$ rad ; $N = 900$ r.p.m.

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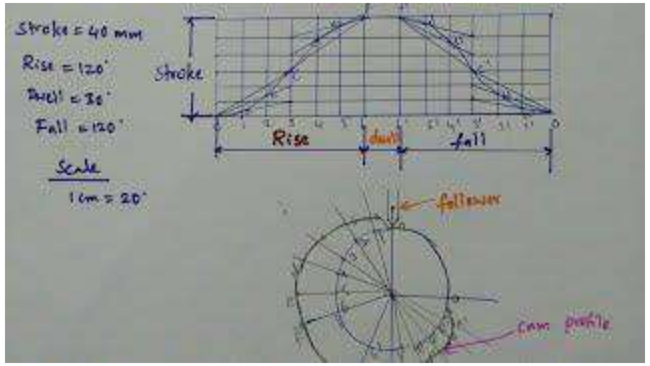
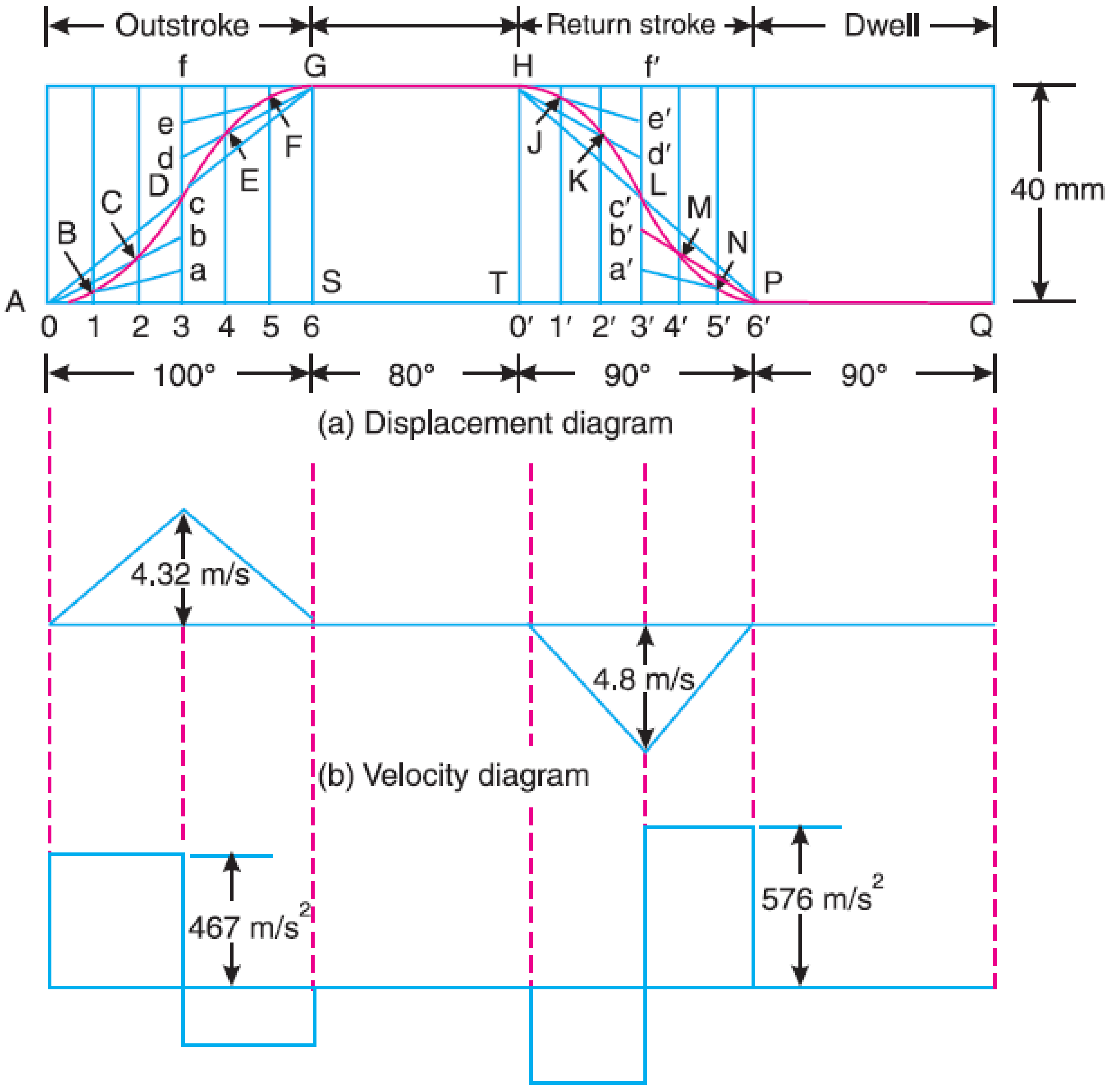


Cam profile for uniform acceleration and retardation



(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram

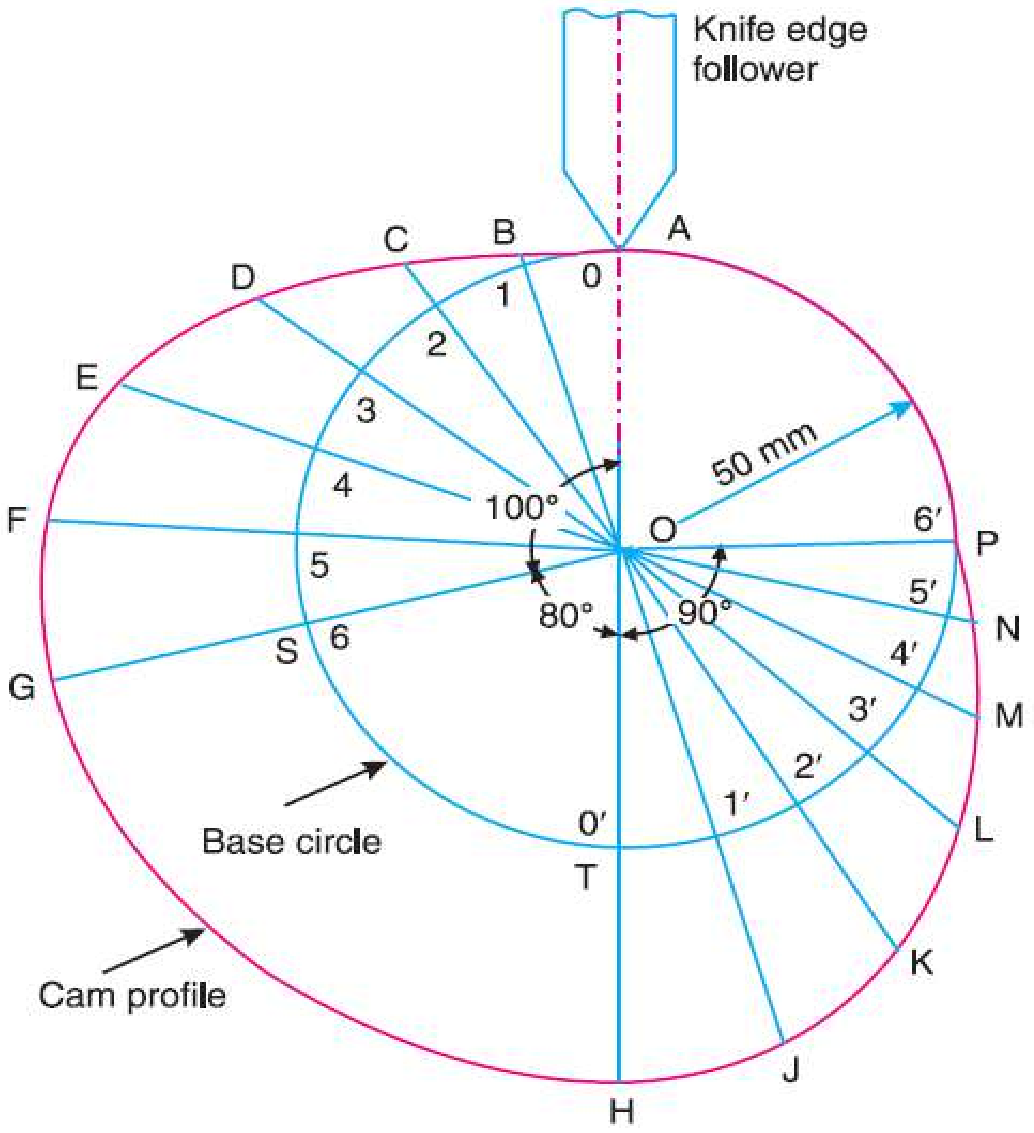
The curve A B C . . . N P Q is the required displacement diagram

(i) Profile of the cam when the line of stroke of the follower passes through th1.e centre of the cam shaft

1.Join the points 1, 2, 3 . . . and 1 , 2 , 3 , . . . with centre O and produce these lines beyond the base circle

2.From points 1, 2, 3 . . . and 1 , 2 , 3 , . . . mark the displacements 1B, 2C, 3D . . . etc. as measured from the displacement diagram.

3.Join the points A, B, C . . . M, N, P with a smooth curve as shown in Fig. This is the required profile of the cam



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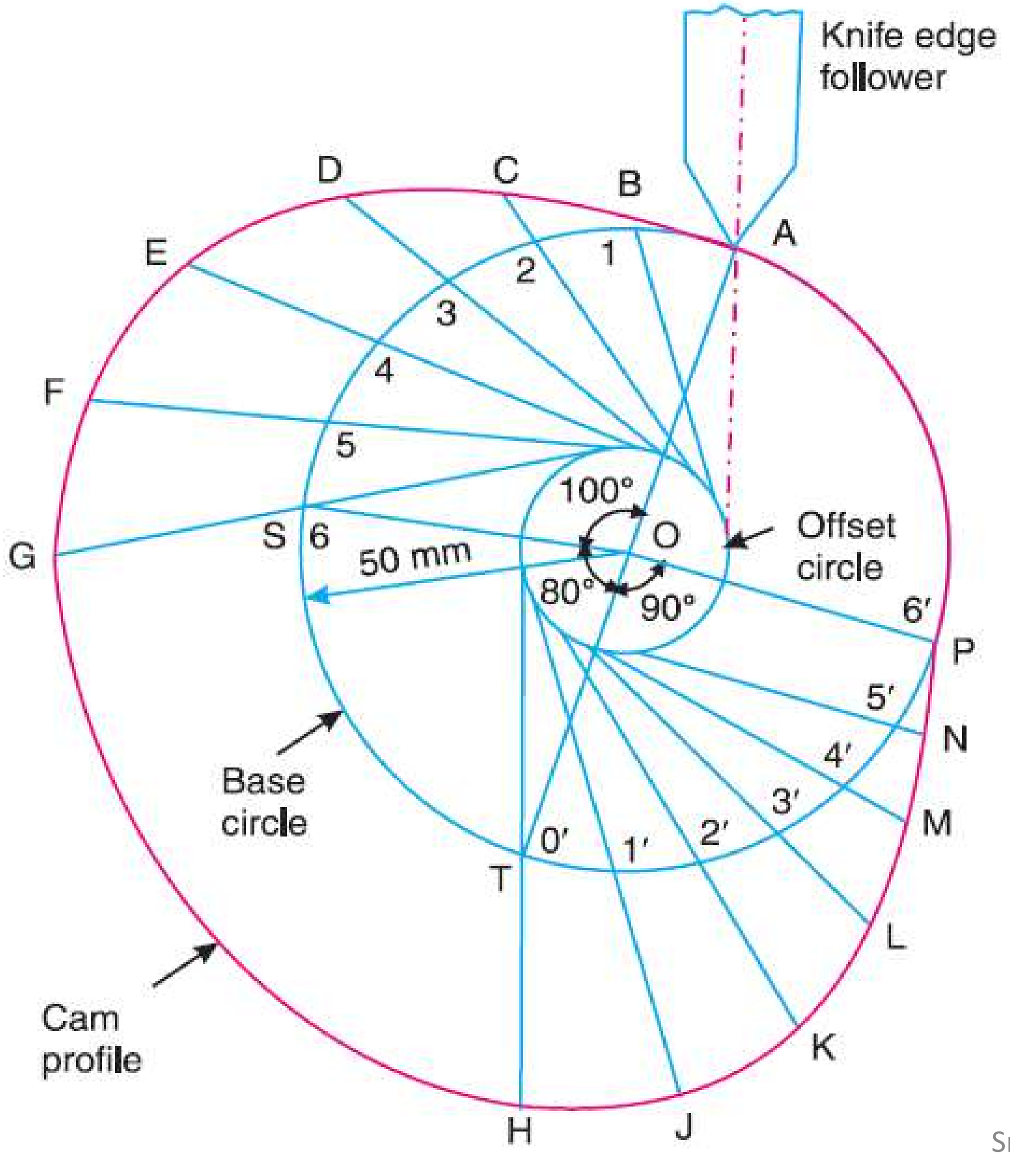
(ii) Profile of the cam when the line of stroke of the follower is offset by 15 mm



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Maximum velocity of the follower during out stroke and return stroke

We know that angular velocity of the cam shaft,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 900}{60} = 94.26 \text{ rad/s}$$

We also know that the maximum velocity of the follower during out stroke,

$$v_O = \frac{2\omega S}{\theta_O} = \frac{2 \times 94.26 \times 0.04}{1.745} = 4.32 \text{ m/s} \quad \textbf{Ans.}$$

and maximum velocity of the follower during return stroke,

$$v_R = \frac{2\omega S}{\theta_R} = \frac{2 \times 94.26 \times 0.04}{1.571} = 4.8 \text{ m/s} \quad \textbf{Ans.}$$

Maximum acceleration of the follower during out stroke and return stroke

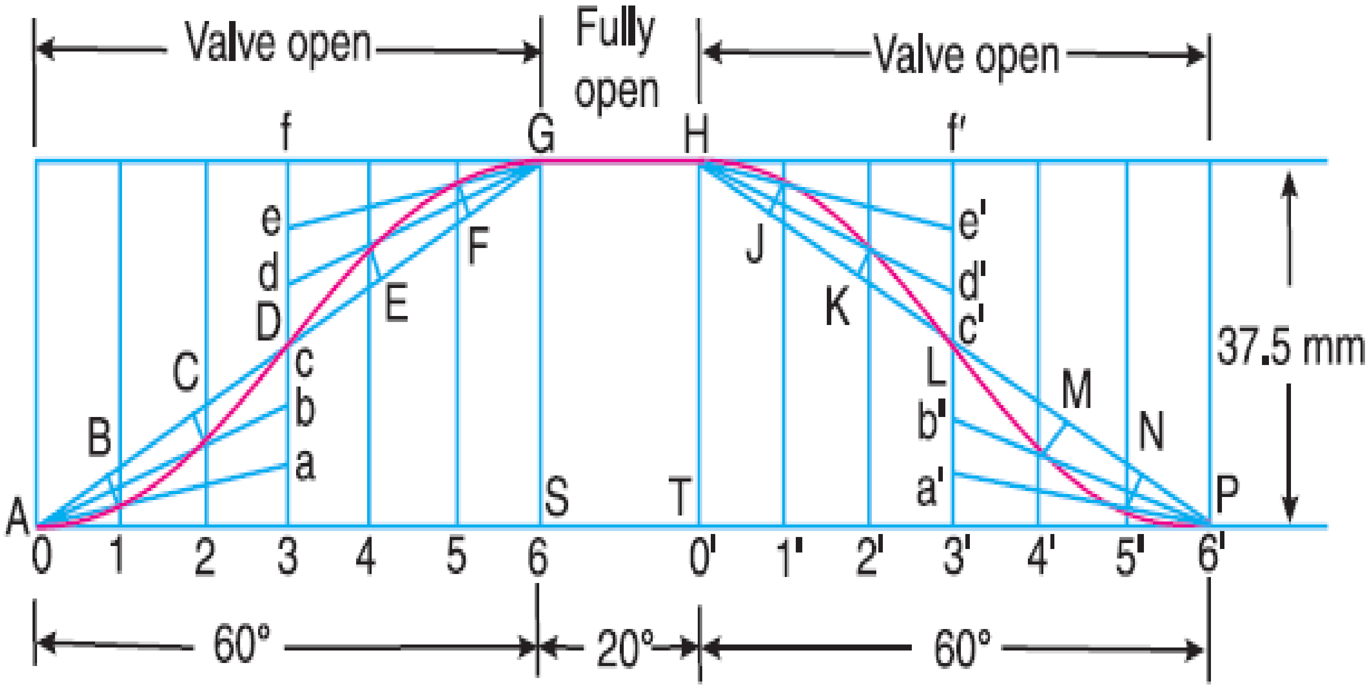
We know that the maximum acceleration of the follower during out stroke,

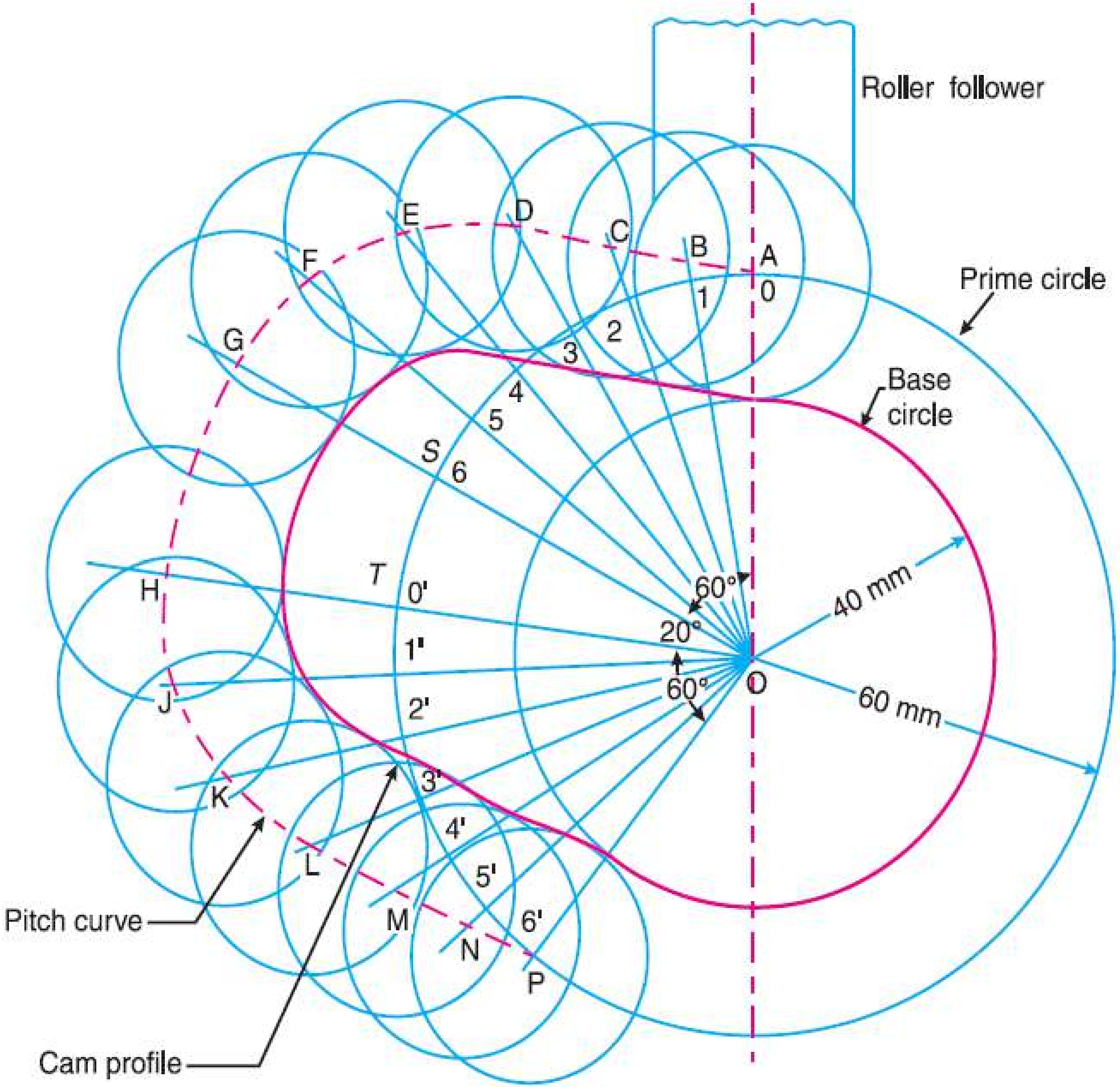
$$a_O = \frac{4\omega^2 . S}{(\theta_O)^2} = \frac{4(94.26)^2 0.04}{(1.745)^2} = 467 \text{ m/s}^2 \text{ **Ans.**}$$

and maximum acceleration of the follower during return stroke,

$$a_R = \frac{4\omega^2 . S}{(\theta_R)^2} = \frac{4(94.26)^2 0.04}{(1.571)^2} = 576 \text{ m/s}^2 \text{ **Ans.**}$$

Q5.Design a cam for operating the exhaust valve of an oil engine. It is required to give equal uniform acceleration and retardation during opening and closing of the valve each of which corresponds to 60° of cam rotation. The valve must remain in the fully open position for20° of cam rotation. The lift of the valve is 37.5 mm and the least radius of the cam is 40 mm. The follower is provided with a roller of radius 20 mm and its line of stroke passes through the axis of the cam.





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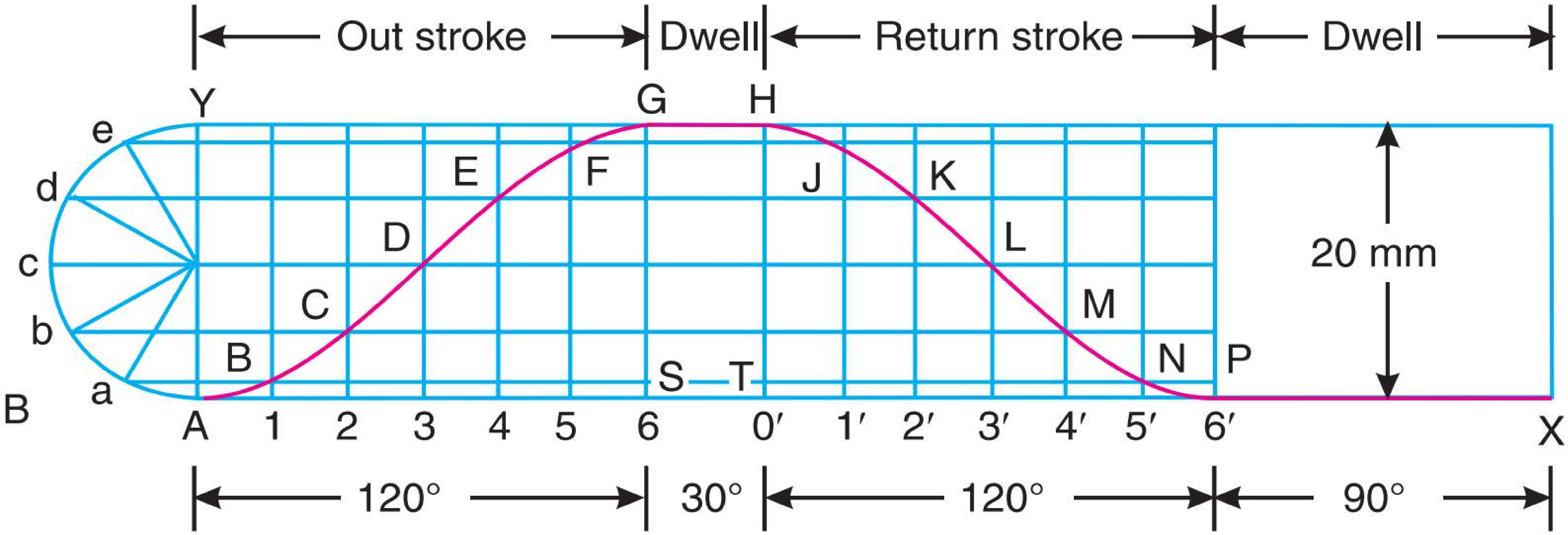
- A cam drives a flat reciprocating follower in the following manner :During first 120° rotation of the cam, follower moves outwards through a distance of 20 mm with simple harmonic motion. The follower dwells during next 30° of cam rotation. During next 120° of cam rotation, the follower moves inwards with simple harmonic motion. The follower dwells for the next 90° of cam rotation. The minimum radius of the cam is 25 mm. Draw the profile of the cam



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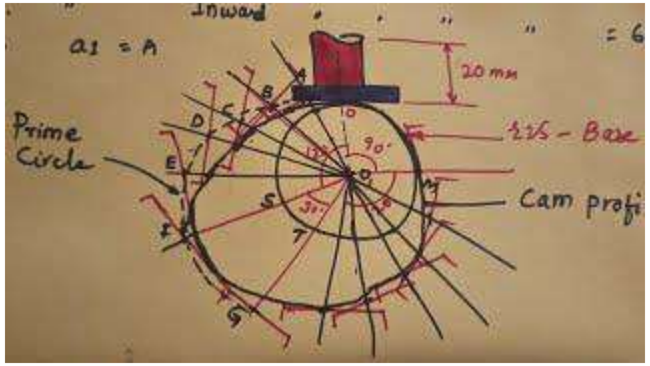
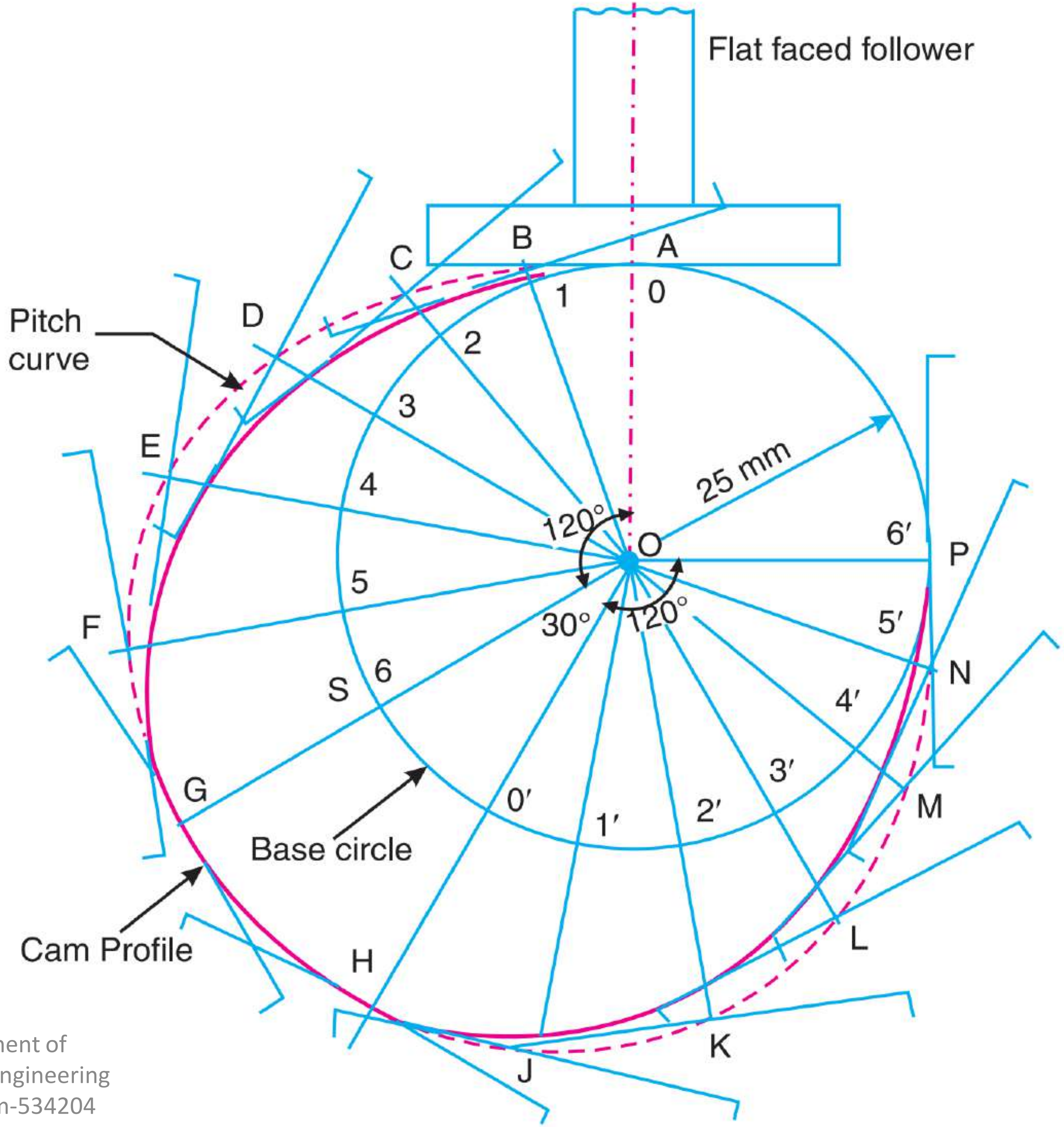


Flat Follower Cam Profile



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Q6. A cam rotating clockwise at a uniform speed of 1000 r.p.m. is required to give a roller follower the motion defined below :

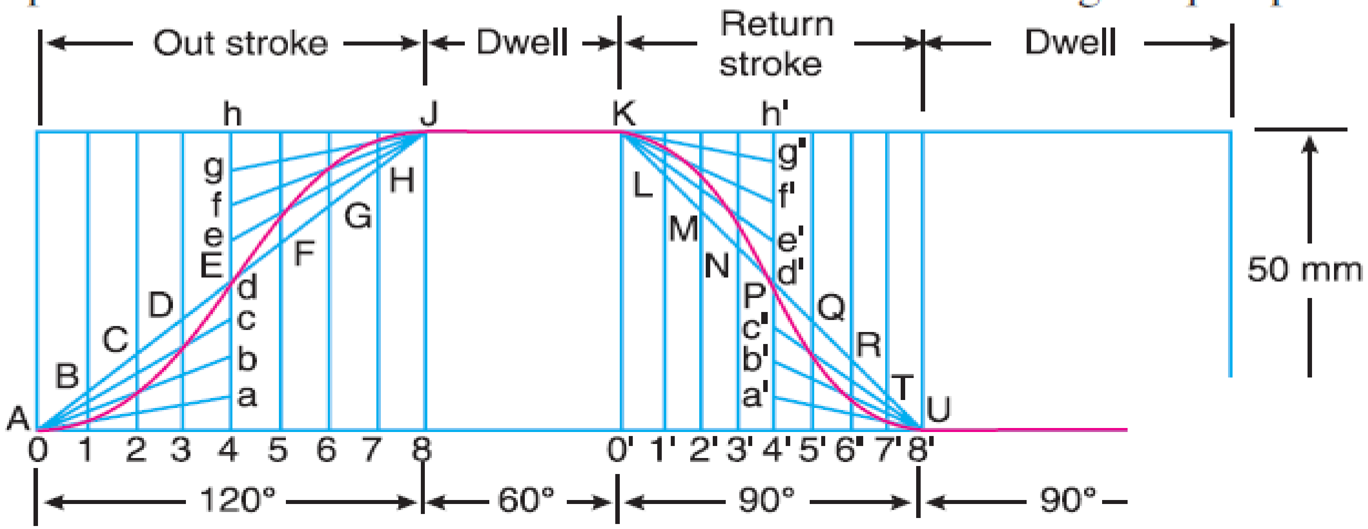
1. Follower to move outwards through 50 mm during 120° of cam rotation,

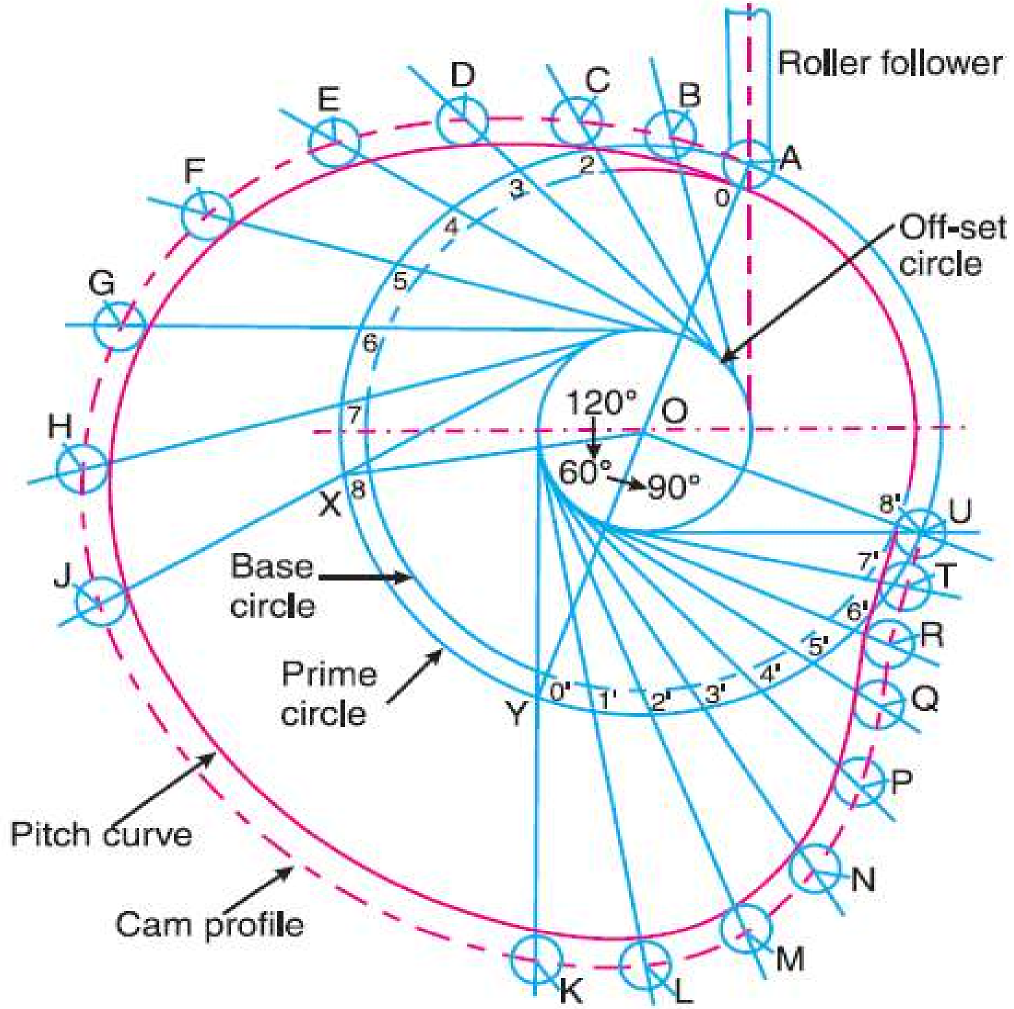
2. Follower to dwell for next 60° of cam rotation,

3. Follower to return to its starting position during next 90° of cam rotation,

4. Follower to dwell for the rest of the cam rotation.

The minimum radius of the cam is 50 mm and the diameter of roller is 10 mm. The line of stroke of the follower is off-set by 20 mm from the axis of the cam shaft. If the displacement follower takes place with uniform and equal acceleration and retardation on both the outward and return strokes, draw profile of the cam and find the maximum velocity and acceleration during outstroke and return stroke





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Maximum velocity of the follower during out stroke and return stroke

We know that angular velocity of the cam,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1000}{60} = 104.7 \text{ rad/s.}$$

We also know that the maximum velocity of the follower during outstroke,

$$v_O = \frac{2\omega S}{\theta_O} = \frac{2 \times 104.7 \times 0.05}{2.1} = 5 \text{ m/s} \quad \textbf{Ans.}$$

and maximum velocity of the follower during return stroke,

$$v_R = \frac{2\omega S}{\theta_R} = \frac{2 \times 104.7 \times 0.05}{1.571} = 6.66 \text{ m/s} \quad \textbf{Ans.}$$

Maximum acceleration of the follower during out stroke and return stroke

We know that the maximum acceleration of the follower during out stroke,

$$a_O = \frac{4\omega^2 . S}{(\theta_O)^2} = \frac{4(104.7)^2 0.05}{(2.1)^2} = 497.2 \ \text{m/s}^2 \ \textbf{Ans.}$$

and maximum acceleration of the follower during return stroke,

$$a_R = \frac{4\omega^2 . S}{(\theta_R)^2} = \frac{4(104.7)^2 0.05}{(1.571)^2} = 888 \ \text{m/s}^2 \ \textbf{Ans.}$$

Example 7.4 *Draw the profile of a cam operating a roller reciprocating follower and with the following data:*

Minimum radius of cam = 25 mm

Lift = 30 mm

Roller diameter = 15 mm

The cam lifts the follower for 120° with SHM followed by a dwell period of 30°. Then the follower lowers down during 150° of the cam rotation with uniform acceleration and deceleration followed by a dwell period. If the cam rotates at a uniform speed of 150 rpm, calculate the maximum velocity and acceleration of the follower during the descent period.

Solution:

$h = 30$ mm $\qquad \varphi_a = 120°$

$N = 150$ mm $\qquad \delta_1 = 30°$

$r_c = 25$ mm $\qquad \varphi_d = 150°$

$r_r = 7.5$ mm $\qquad \delta_2 = 60°$

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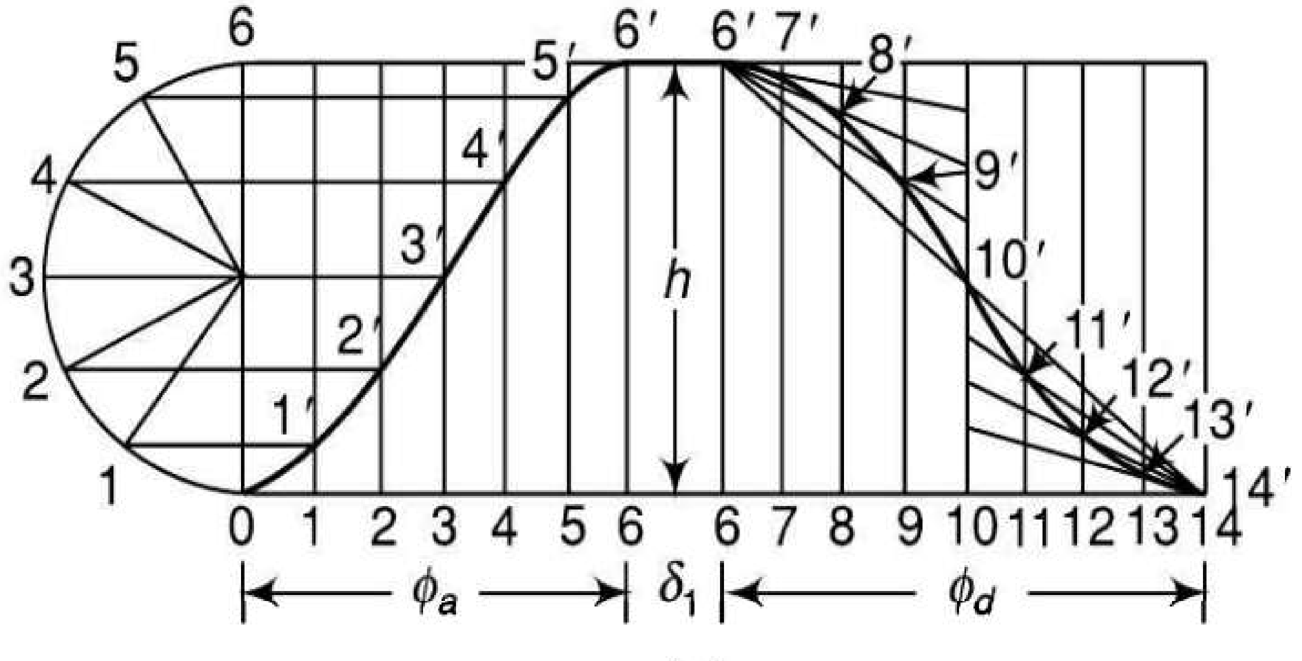
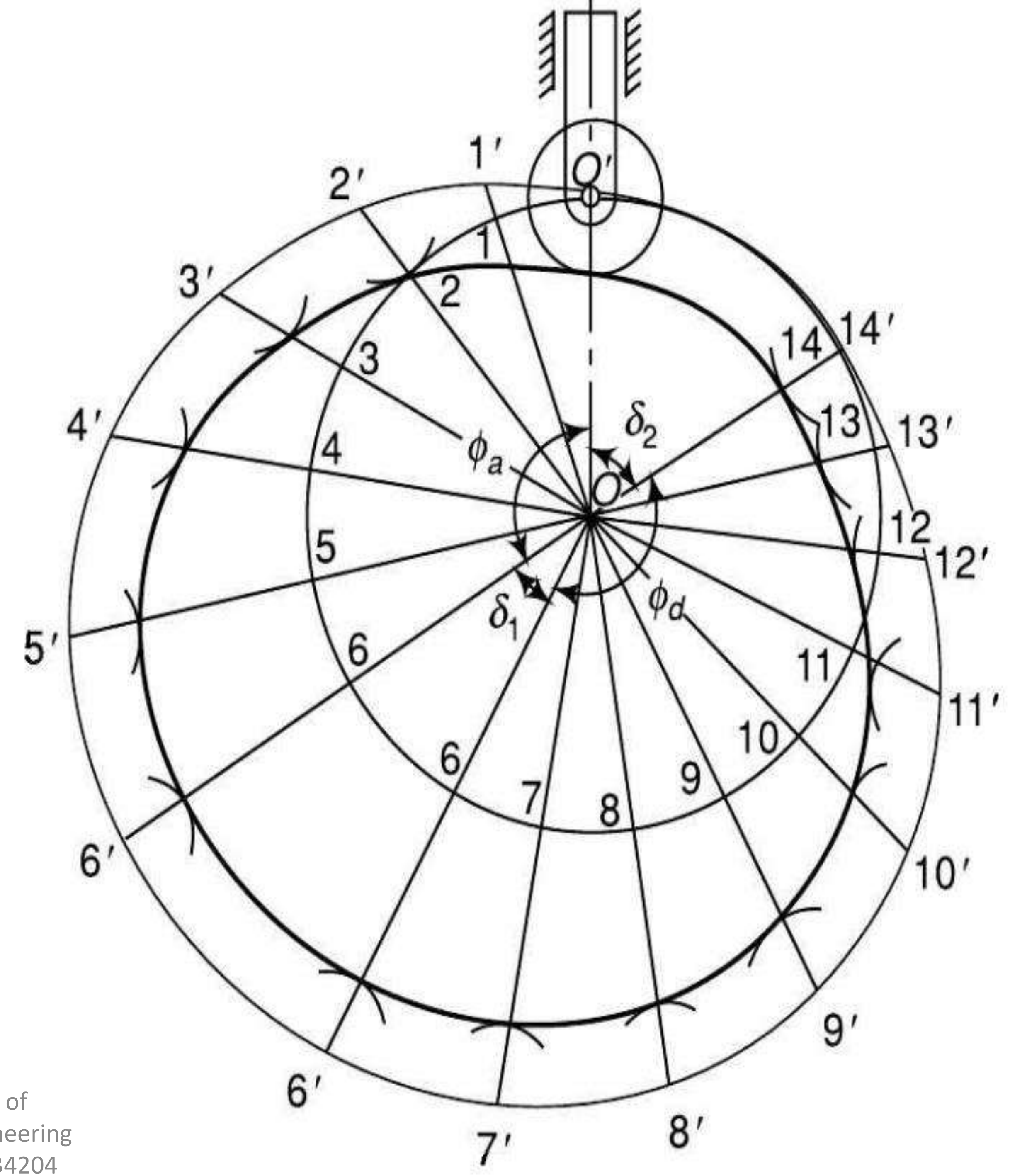


(a)



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$$v_{\max} = 2h\frac{\omega}{\varphi_d} \qquad \text{Eq. (7.8)}$$

or

$$v_{\max} = 2 \times 30 \times \frac{\dfrac{2\pi \times 150}{60}}{150 \times \dfrac{\pi}{180}} = \underline{360\ \text{m/s}}$$

$$f_{\max} = f_{\text{uniform}} = \frac{4h\omega^2}{\varphi_d^2} \qquad \text{Eq. (7.6)}$$

$$f_{\max} = \frac{4 \times 30 \times \left(\dfrac{2\pi \times 150}{60}\right)^2}{\left(150 \times \dfrac{\pi}{180}\right)^2} = 4320\ \text{mm/s}^2$$

or $\underline{4.32\ \text{m/s}^2}$

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UNIT IV BIT BANK

1. The size of a cam depends upon (a)

(a) base circle (b) pitch circle (c) prime circle (d) pitch curve

2. The angle between the direction of the follower motion and a normal to the pitch curve is called (d)

(a) pitch angle (b) prime angle (c) base angle (d) pressure angle

3. A circle drawn with centre as the cam centre and radius equal to the distance between the cam centre and the point on the pitch curve at which the pressure angle is maximum, is called (b)

(a) base circle (b) pitch circle (c) prime circle (d) none of these

4. The cam follower generally used in automobile engines is (c)

(a) knife edge follower (b) flat faced follower

(c) spherical faced follower (d) roller follower

5. The cam follower extensively used in air-craft engines is (d)

(a) knife edge follower (b) flat faced follower

(c) spherical faced follower (d) roller follower

6. In a radial cam, the follower moves (a)

(a) in a direction perpendicular to the cam axis (b) in a direction parallel to the cam axis

(c) in any direction irrespective of the cam axis (d) along the cam axis

7. A radial follower is one (a)

(a) that reciprocates in the guides (b) that oscillates

(c) in which the follower translates along an axis passing through the cam centre of rotation. (d) none of the above

8. Offset is provided to a cam follower mechanism to (a)

(a) minimise the side thrust (b) accelerate (c) avoid jerk (d) none of these

9. For low and moderate speed engines, the cam follower should move with (b)

(a) uniform velocity (b) simple harmonic motion

(c) uniform acceleration and retardation (d) cycloidal motion

10. For high speed engines, the cam follower should move with (d)

(a) uniform velocity (b) simple harmonic motion

(c) uniform acceleration and retardation (d) cycloidal motion

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11.For a given lift of the follower of a cam follower mechanism, a smaller base circle diameter is desired. (d)

(a) because it will give a steeper cam and higher pressure angle.

(b) because it will give a profile with lower pressure angle

(c) because it will avoid jumping (d) none of the above.

12. The locus of the tracing point is known as the (d)

(a) base circle (b) pitch circle (c) prime circle (d) pitch curve

13. It is the smallest circle that can be drawn from the centre of the cam and tangent to the pitch curve (c)

(a) base circle (b) pitch circle (c) prime circle (d) pitch curve

14. Which of the following statements is false for SHM follower motion? (b)
(a) SHM can be used only for moderate speed purpose
(b) The acceleration is zero at the beginning and the end of each stroke
(c) The jerk is maximum at the mid of each stroke
(d) Velocity of follower is maximum at the mid of each stroke

15.Which of the following statements is/are true for cam profile? (b)

a. Pitch point on the pitch curve has minimum pressure angle

b. In case of roller follower, trace point represents centre of the roller

c. Pitch circle is drawn through trace point from the center of cam

d. All of the above

16.when the follower moves with uniform velocity. The velocity is ------during rise and return stroke (b)

(a) Zero (b) Constant (c) Fluctuating (d) none of the above

17 .In constructing the cam profile, the principle of kinematic inversion is used, i.e. the cam is imagined to be------ --and the follower is allowed to----------- in the opposite direction to the cam rotation. (a)

(a)Stationary& rotate (b) Rotate& Stationary

(c) Stationary &reciprocate (d) reciprocate & Stationary

18. When the motion of the follower is along an axis away from the axis of the cam centre, it is called----- (a)

(a)off-set follower. (b) radial follower (c) roller follower (d) none of the above

Question 1. In a cam design, the rise motion is given by a simple harmonic (SHM) $s = \frac{h}{2}\left(1 - \cos\frac{\pi\theta}{\beta}\right)$ where h is the total rise, θ is cam shaft angle, β is the total angle of the rise interval. The jerk given by

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(A) $\frac{h}{2}\left(1 - cos\frac{\pi\theta}{\beta}\right)$

(B) $\frac{\pi}{\beta}\frac{h}{2}sin\left(\frac{\pi\theta}{\beta}\right)$

(C) $\frac{\pi^2}{\beta^2}\frac{h}{2}cos\left(\frac{\pi\theta}{\beta}\right)$

(D) $-\frac{\pi^2}{\beta^2}\frac{h}{2}sin\left(\frac{\pi\theta}{\beta}\right)$

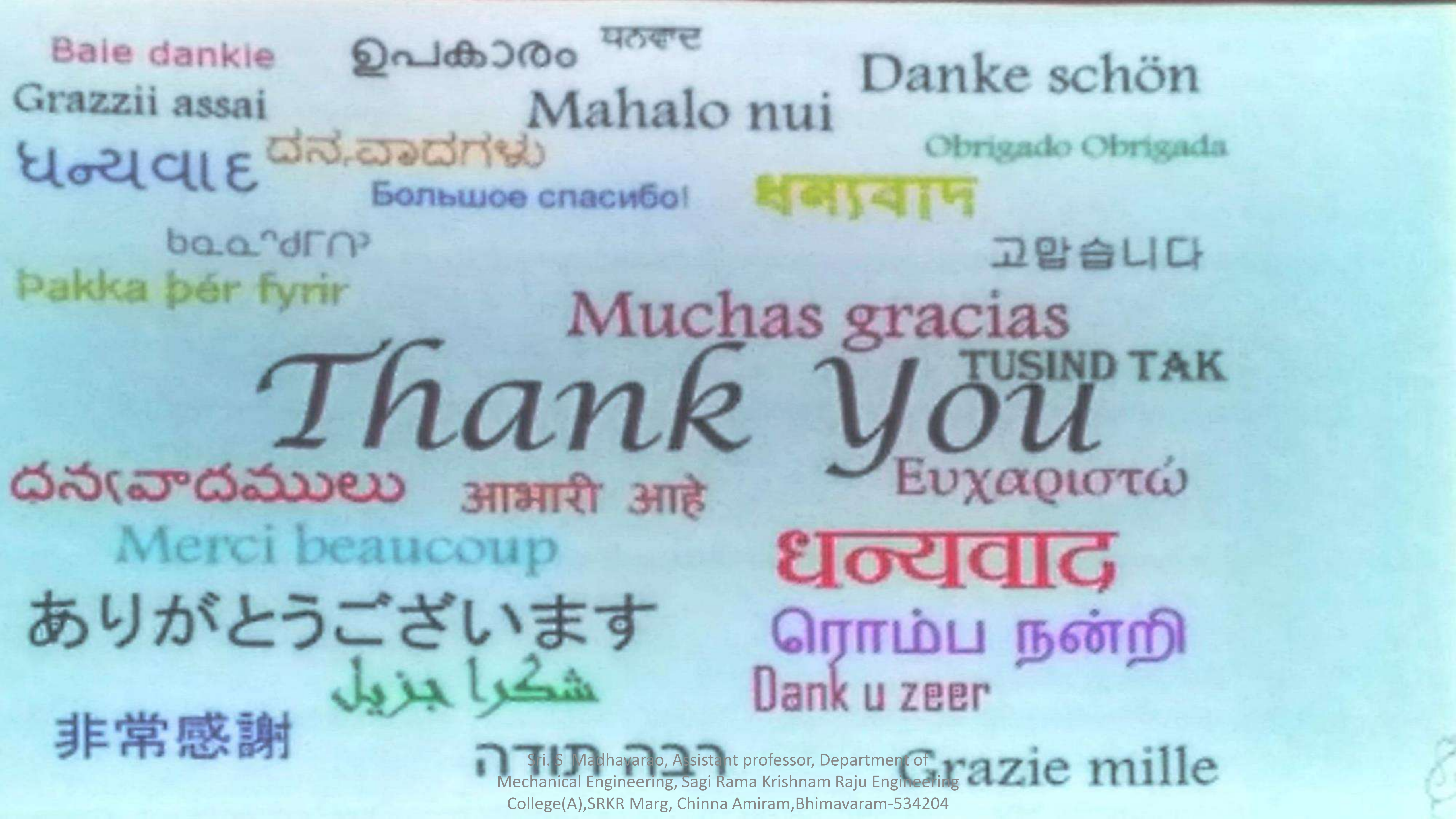
$$S = \frac{h}{2}\left[1 - \cos\frac{\pi\theta}{\beta}\right]$$

Jerk is given by

$$\frac{d^3S}{d\theta^3} = -\frac{\pi^3}{\beta^3}\frac{h}{2}\sin\frac{\pi\theta}{\beta} \quad where\ \theta = \omega t$$

References:

1. Theory of Machines, Rattan, Tata McGraw-Hill Education, 2009.
2. Theory of Machines, R S Kurmi, Eurasia Publishing House, 2005



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UNIT V
HIGHER PAIRS
(Toothed wheels)

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- **Higher pair:** Friction wheels and toothed gears – types – law of gearing, condition for constant velocity ratio for transmission of motion – involutes profiles – phenomena of interferences – Methods of interference. Condition for minimum number of teeth to avoid interference – expressions for arc of contact and path of contact of Pinion & Gear

WHAT IS GEAR ?

Gears are toothed members which transmit power/motion between two shafts by meshing without any slip. Hence, gear drives are also called positive drives. In any pair of gears,

the smaller one is called pinion and the larger one is called gear immaterial of which is driving the other

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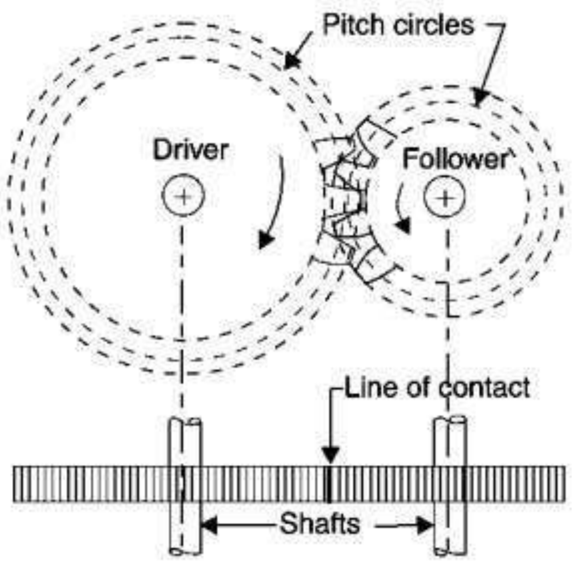
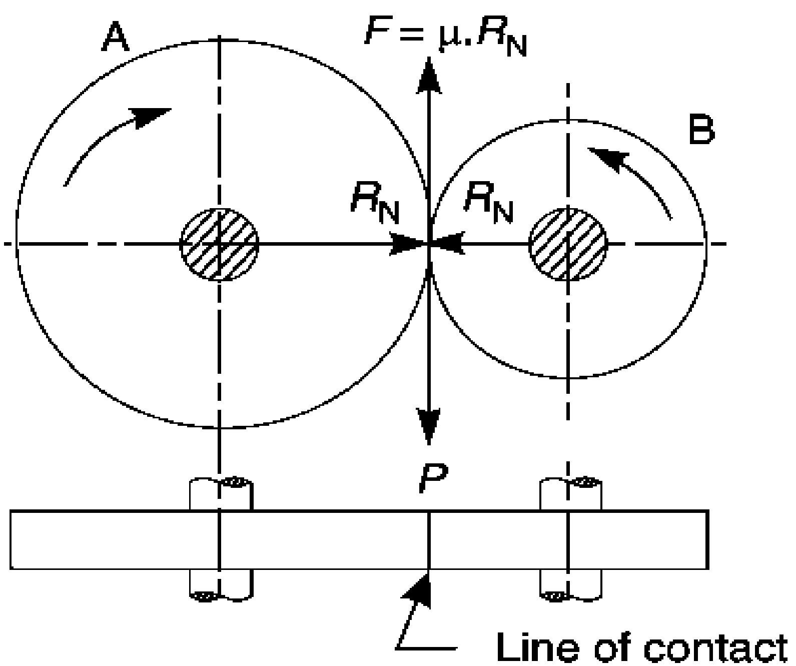
Friction wheels: The motion and power transmitted by gears is kinematically equivalent to that transmitted by frictional wheels or discs.

- Friction wheels can be used for small power transmission .These are mounted on the 2 shafts having sufficient rough surfaces and pressing against each other



(b) Toothed wheels.



(a)Friction wheel

- A friction wheel with the teeth cut on it is know as toothed wheel or gear

Applications of Gears

Toys and Small Mechanisms – small, low load, low cost

Appliance gears – long life, low noise & cost, low to moderate load

Power transmission – long life, high load and speed

Aerospace gears – light weight, moderate to high load

Control gears – long life, low noise, precision gears

Advantages and Disadvantages of Gear Drive

Advantages

- It transmits exact velocity ratio.
- It may be used to transmit large power.
- It may be used for small centre distances of shafts.
- It has high efficiency.
- It has reliable service.
- It has compact layout

Disadvantages

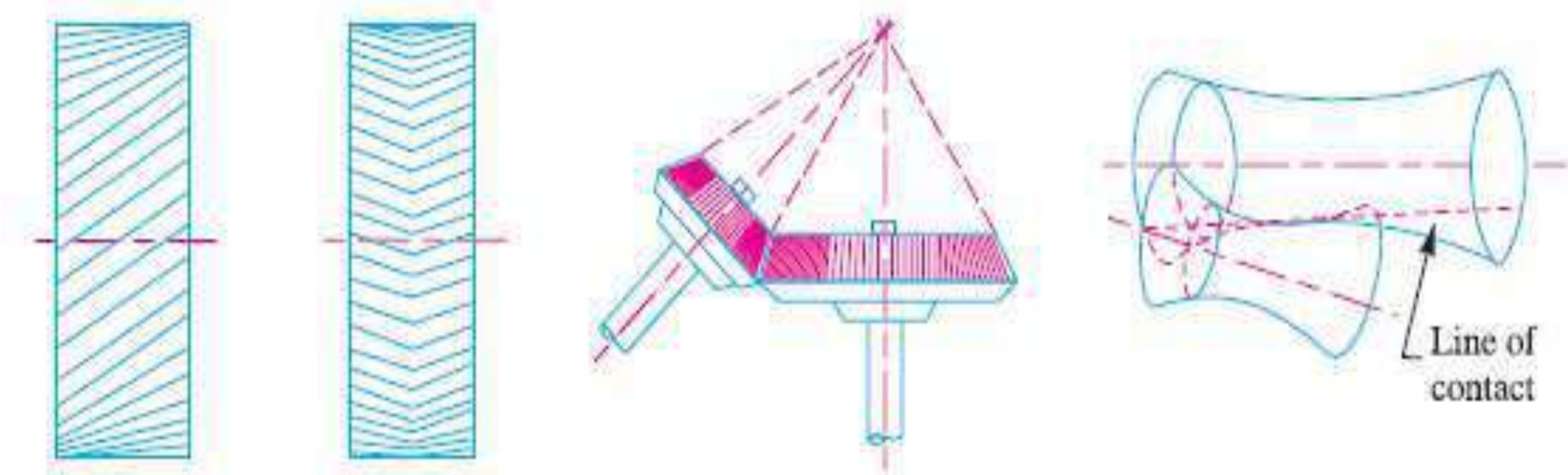
- Since the manufacture of gears require special tools and equipment, therefore it is costlier than other drives.
- The error in cutting teeth may cause vibrations and noise during operation.
- It requires suitable lubricant and reliable method of applying it, for the proper operation of gear drives.

TYPES OF GEARS (Classification of Gears)

1.According to the position of axes of the shafts

The axes of two shafts between which the motion is to be transmitted (a)Parallel- Spur/helical gear,

(b) Non-parallel or Intersecting- bevel/helical bevel gear, and (c) Non-intersecting and non-parallel- spiral/skew bevel/hyperboloid gear



(a) Single helical gear. (b) Double helical gear. (c) Bevel gear. (d) Spiral gear.

TYPES OF GEARS

SPUR GEAR

HELICAL GEAR

BEVEL GEAR

WORM GEAR

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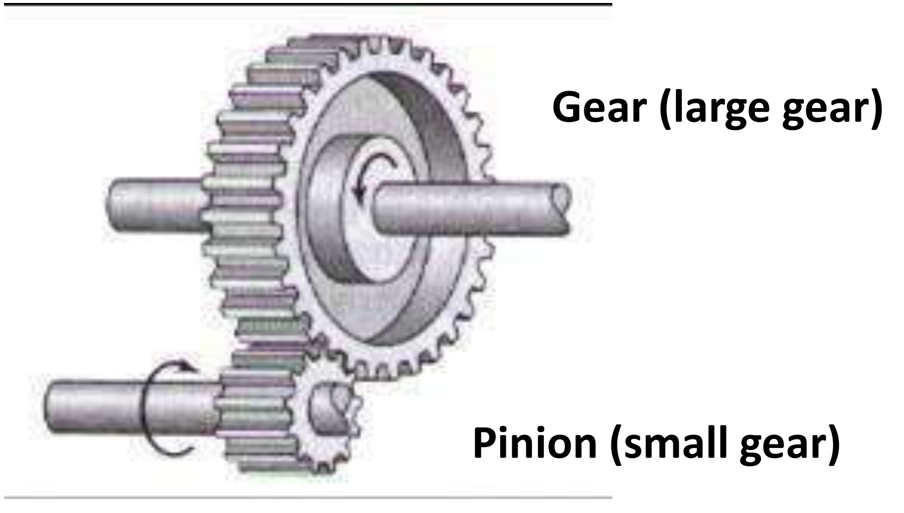
- **Spur gears *:*** Teeth are straight & parallel to shaft axis. Transmit power and motion between rotating parallel shafts.

Applications : Marine Engines

- Mechanical Clock and Watches
- Fuel Pumps
- Washing Machine



Helical Gear : Teeth are inclined to the axis of rotation, the angle provides more gradual engagement of the teeth during meshing, transmits motion between parallel shafts.

Applications :

- Fertilizer industries ,Printing industries
- Earth moving industries , Rolling Mill
- Power and Port Industries .
- Textile Industries
- Plastic Industries
- Automobile Gearboxes

Helical Gear

Herringbone Gears(double helical gears): Two helical gears operating together and so placed that the angle of teeth form a “V” shape; cancel out end- thrust forces. No thrust bearing is needed

Double Helical Gear

Bevel Gear: The bevel gears have a conical shape and are used to transmit forces between two Non-parallel or intersecting axes at a point (intersection axis).

- The bevel gear has a cone on its step surface and its saw teeth are cut along the cone.
- Bevel gears typically operate on shafts that are 90 degrees to each other .

Applications :

- Printing Machine
- Agriculture
- Bottling
- Material Handling
- Steering
- Differential Drives
- Hand Drill

Worm Gear :The screw shape that is cut on the shaft is a worm, the mesh gear is a helical gear, and the axes that do not intersect are called worm gears.

- Worm gears are gears which are widely used for transmitting power at high velocity ratios between non intersecting shafts that are generally, but not necessarily at right angles

- **Applications :**

Worm gears are used widely in material handling and transportation machinery, machine tools, automobiles etc

RACK GEAR : The rack is a bar containing teeth on one face for meshing with a gear.

- Racks with machined ends can be joined together to make any desired length. It is used to Change a rotary motion into a rectilinear motion

Applications :

A transfer system for machine tools, printing press, robots, etc.

- Steering system in vehicles

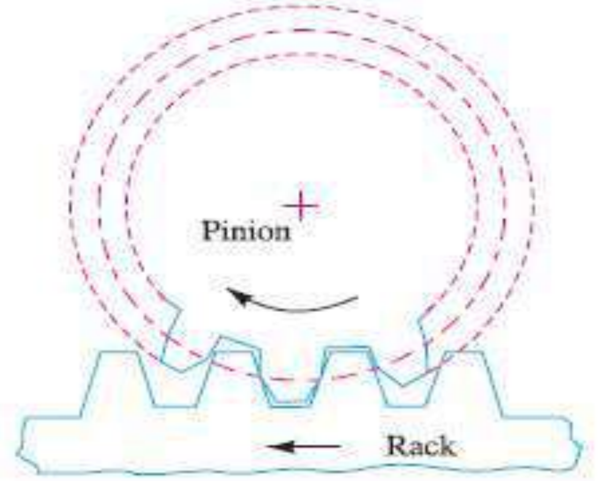


Fig. 28.5. Rack and pinion.

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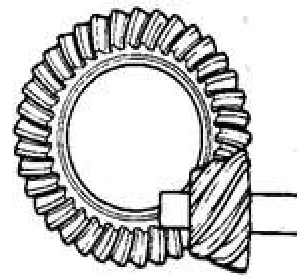
- **Hypoid gears:** A hypoid gear is a type of spiral bevel gears whose axis does not intersect with the axis of the meshing gear.



HYPOID GEARS

- **Spiral gear:** The two non-intersecting and non-parallel i.e. non-coplanar shaft connected by gears is shown in Fig These gears are called **skew bevel gears** or **spiral gears**

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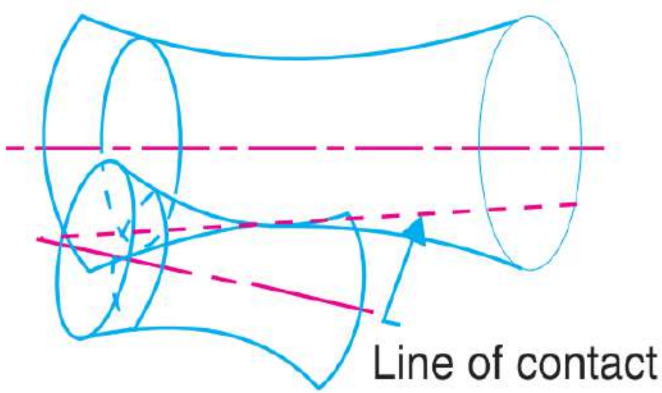

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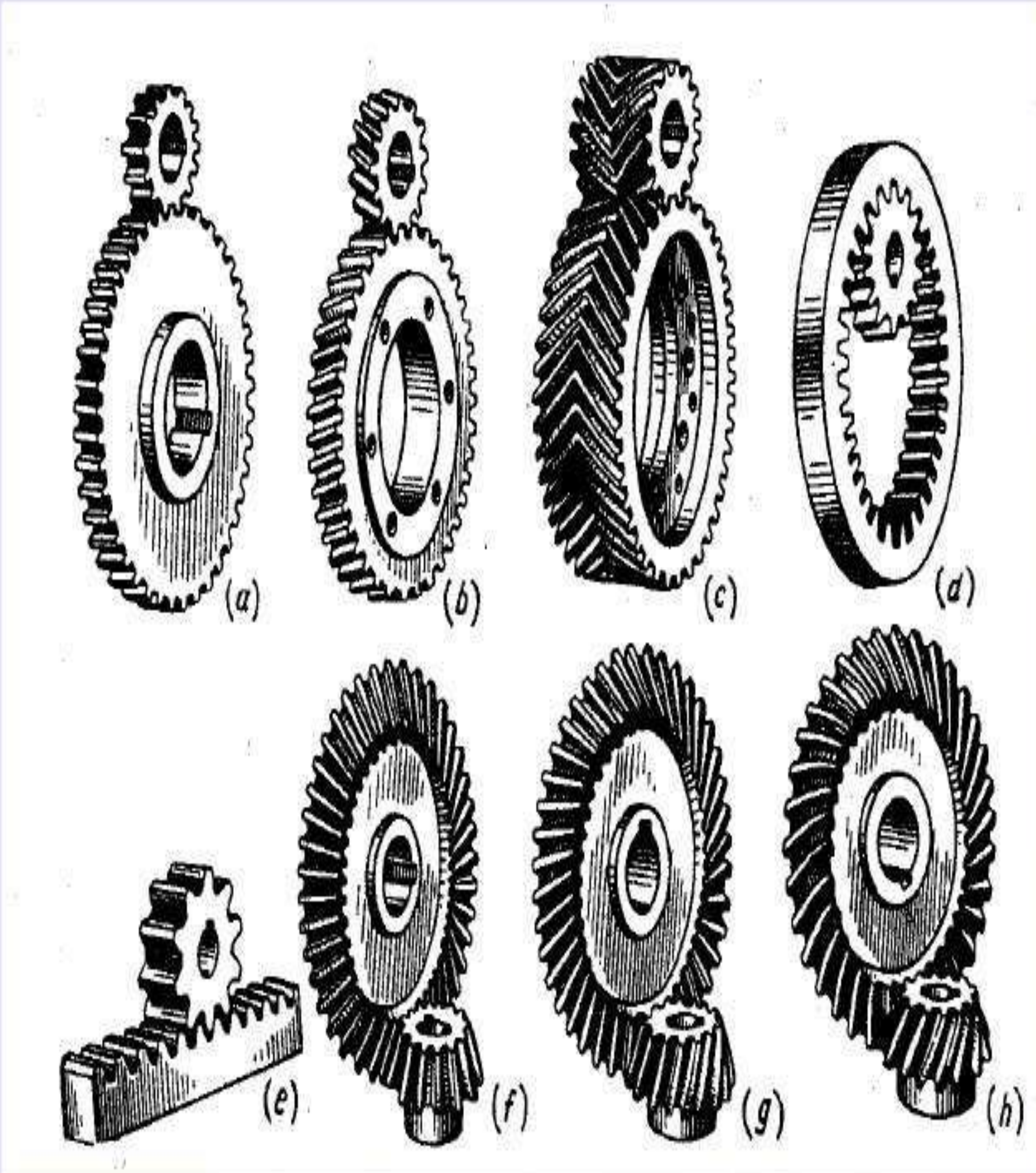
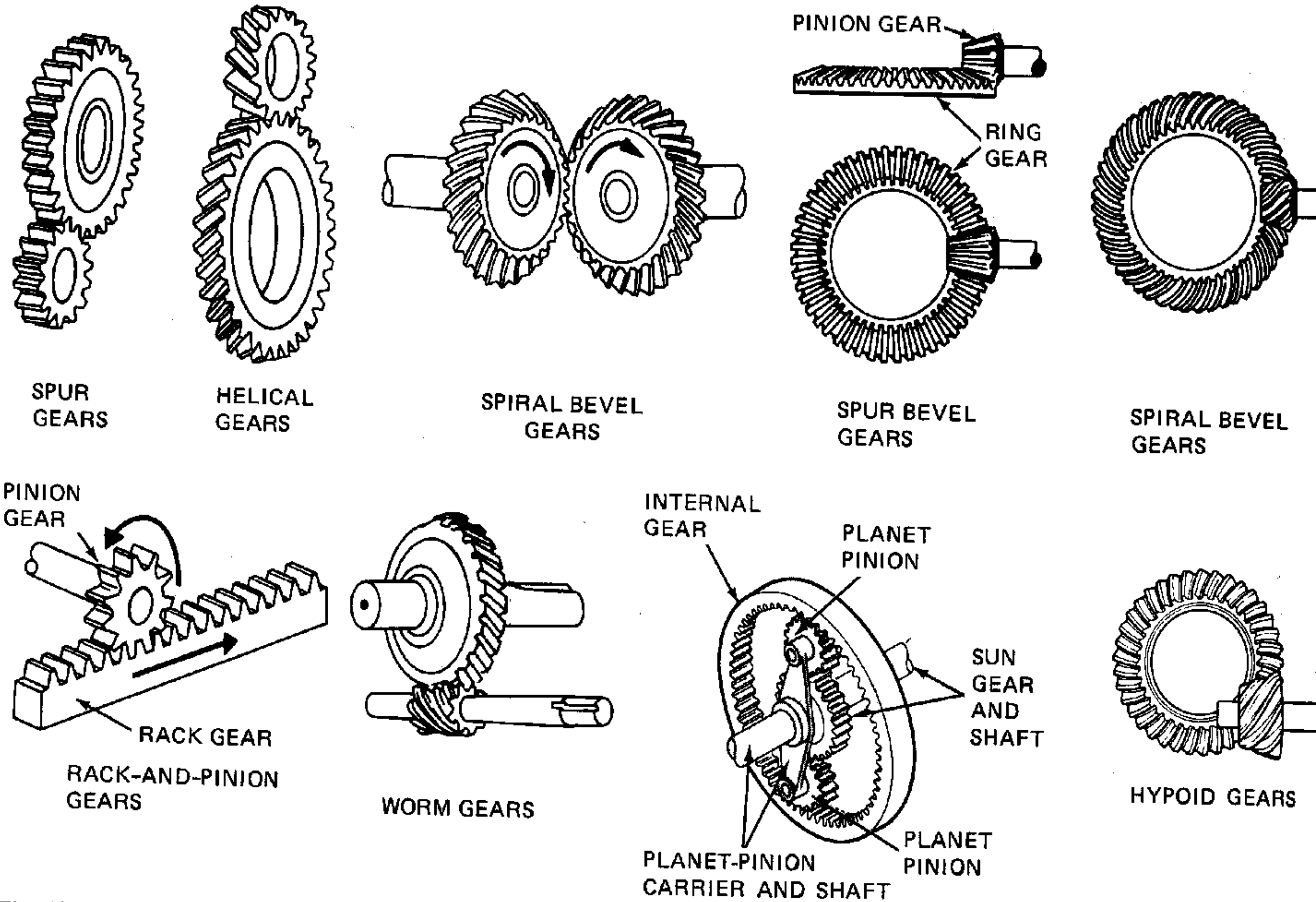



(i)


(j)

Fig. 1.6 (a) Spur gear, (b) helical gear, (c) Double helical gear or herringbone gear, (d) Internal gear , (e) Rack and pinion, (f) Straight bevel gear, (g) Spiral bevel gear, (h) Hypoid bevel gear , (i) worm gear and (j) Spiral gear



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2. According to the peripheral velocity of the gears

- (a) Low velocity, (b) Medium velocity, and (c) High velocity

(a)The gears having velocity less than 3m/s are termed as **low velocity**

(b)gears and gears having velocity between 3 and 15 m/s are known as **medium velocity gears.**

(c)If the velocity of gears is more than 15 m/s, then these are called **high speed gears.**

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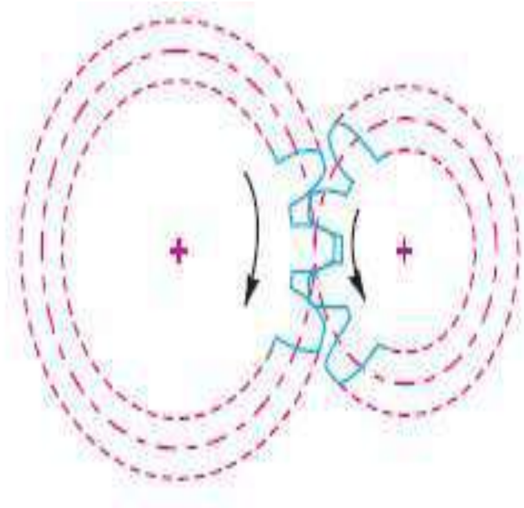
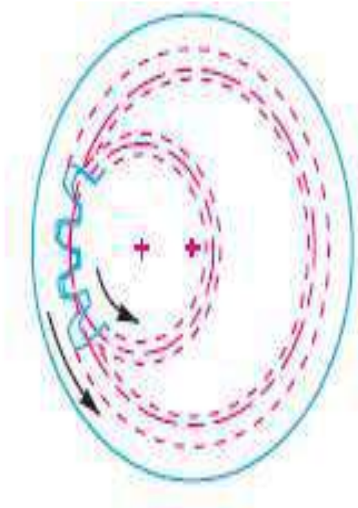
3. According to the type of gearing

(a) External gearing, (b) Internal gearing, and (c) Rack and pinion



(*a*) External gearing.



(*b*) Internal gearing.

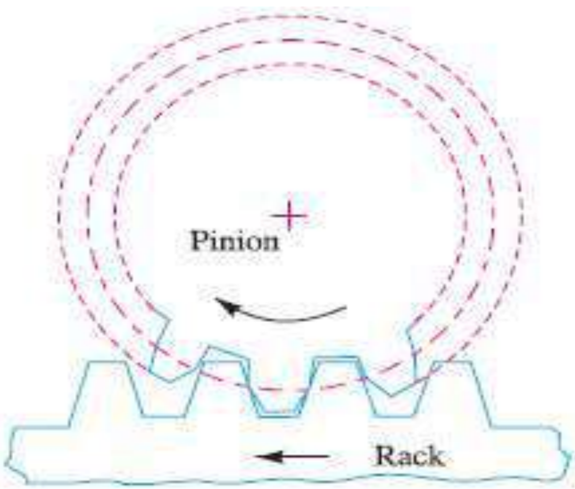


Fig. 28.5. Rack and pinion.

Internal gear

4. According to the position of teeth on the gear surface

(a) Straight- spur gear, (b) Inclined- helical gear, and (c) Curved-spiral gear.

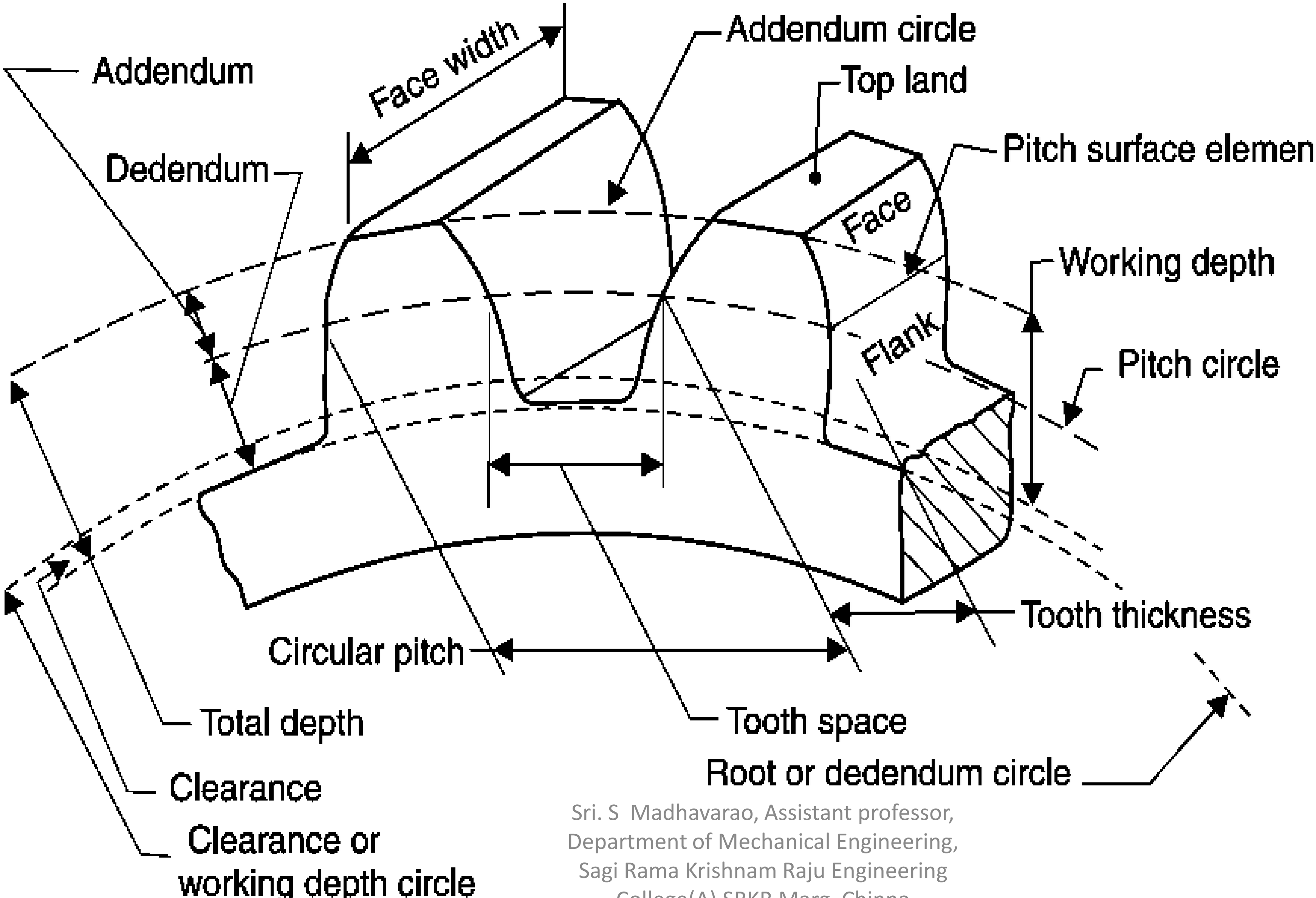
TYPES OF SPUR GEARS

EXTERNAL SPUR GEAR

RACK & PINION

INTERNAL SPUR GEAR

Nomenclature of Spur Gear Teeth/ Terms Used in Gears



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Terms Used in Gears

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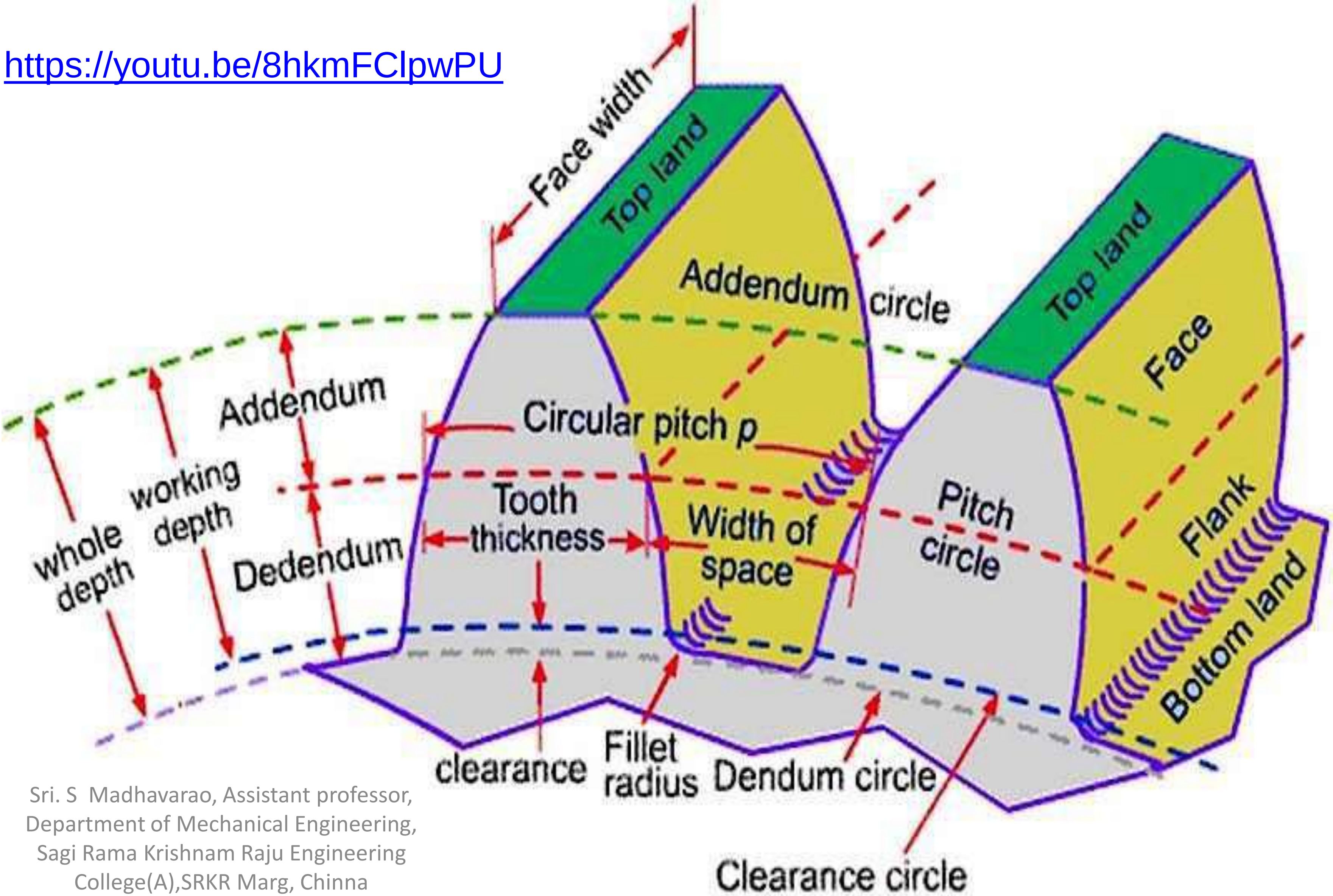

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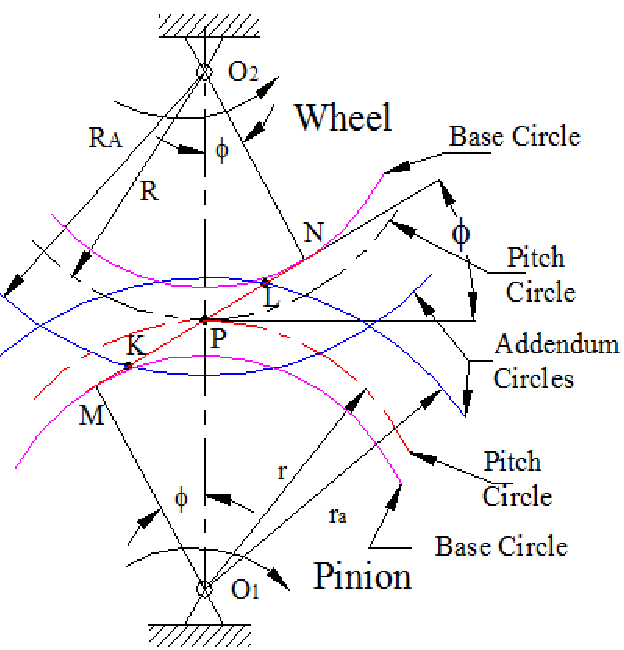
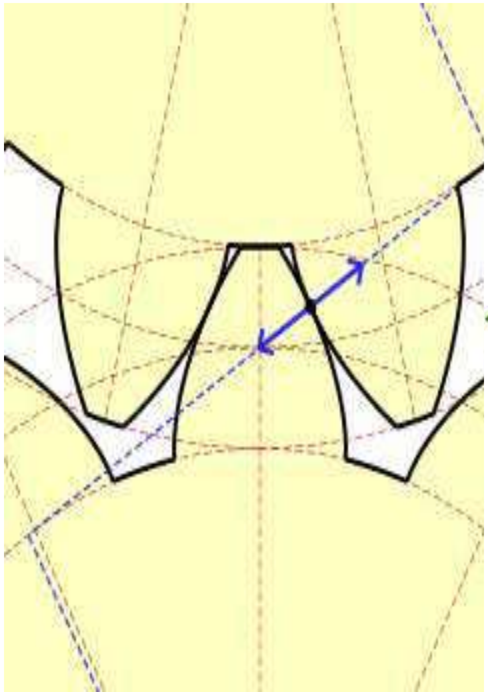
Pitch circle: It is an imaginary circle which by pure rolling action would give the same motion as the actual gear.

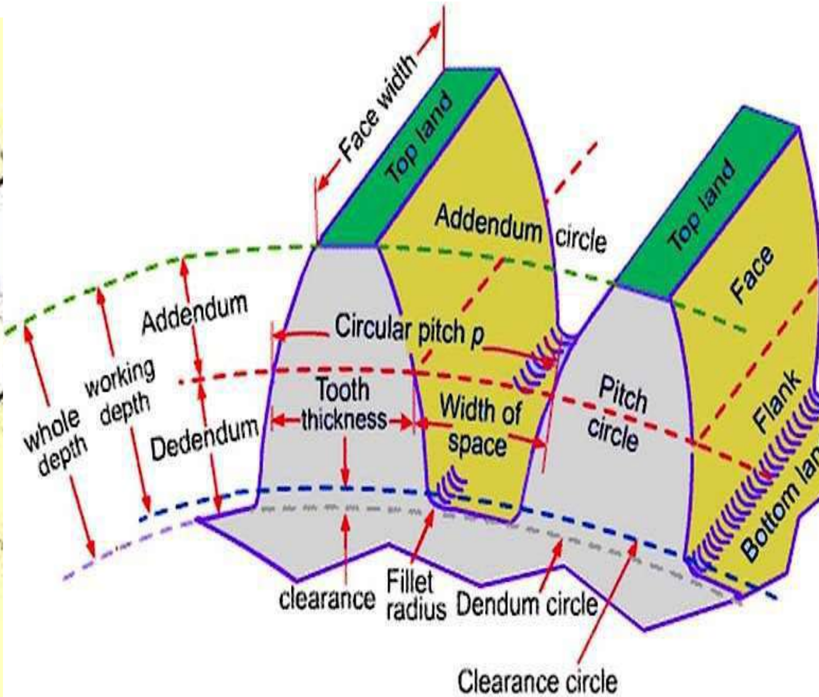
Pitch circle diameter: It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as pitch diameter.

Pitch point: It is a common point of contact between two pitch circles.

Pressure angle or angle of obliquity: It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by ϕ. The standard pressure angles are 14 1/2 ° and 20°.







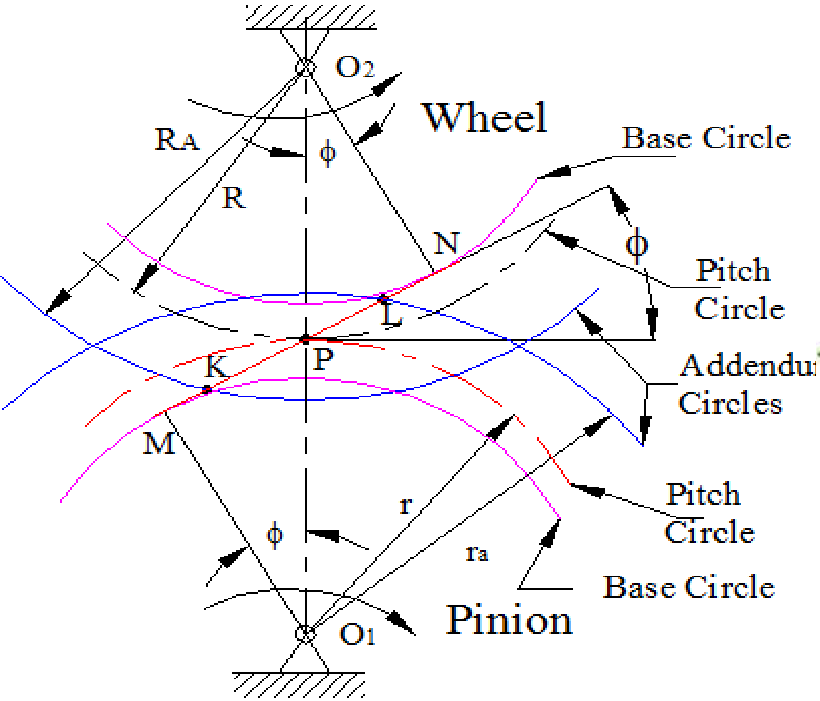
Addendum: It is the radial distance of a tooth from the pitch circle to the top of the tooth.

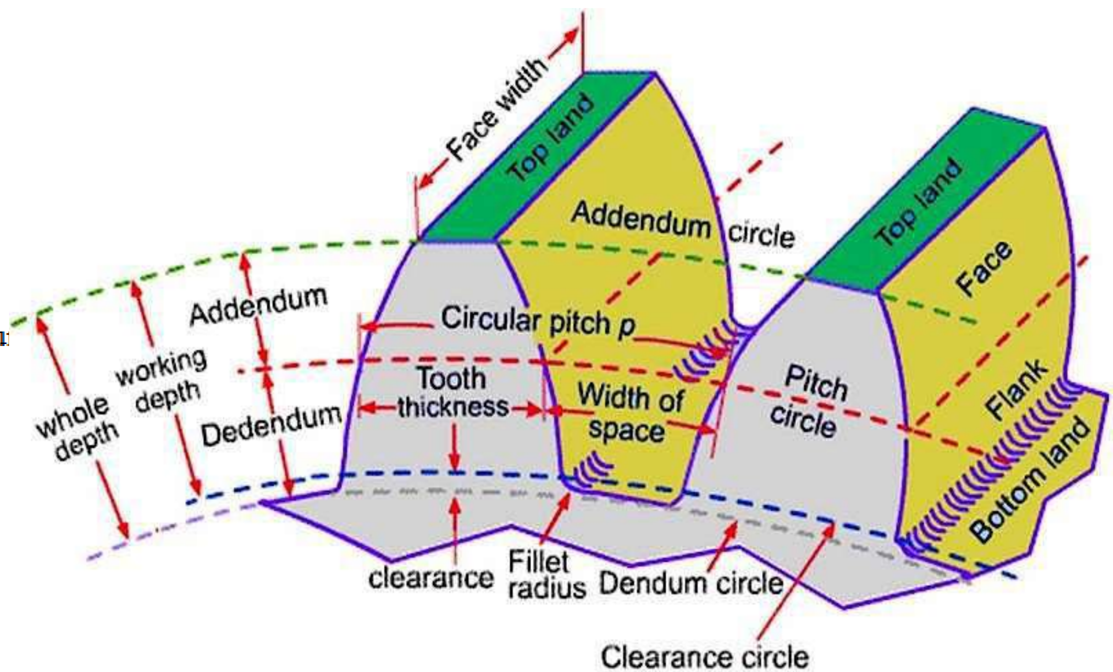
Dedendum: It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

Addendum circle: It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

Dedendum circle: It is the circle drawn through the bottom of the teeth. It is also called root circle.

Note : Root circle diameter = Pitch circle diameter × cos ϕwhere ϕ is the pressure





Circular pitch: It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by Pc ,Mathematically

Circular pitch, $p_c = \pi D/T$

where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

little consideration will show that the **two gears will mesh together correctly, if the two wheels have the same circular pitch.**

Note : If D1 and D2 are the diameters of the two meshing gears having the teeth T1 and T2 respectively, then for them to mesh correctly,

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

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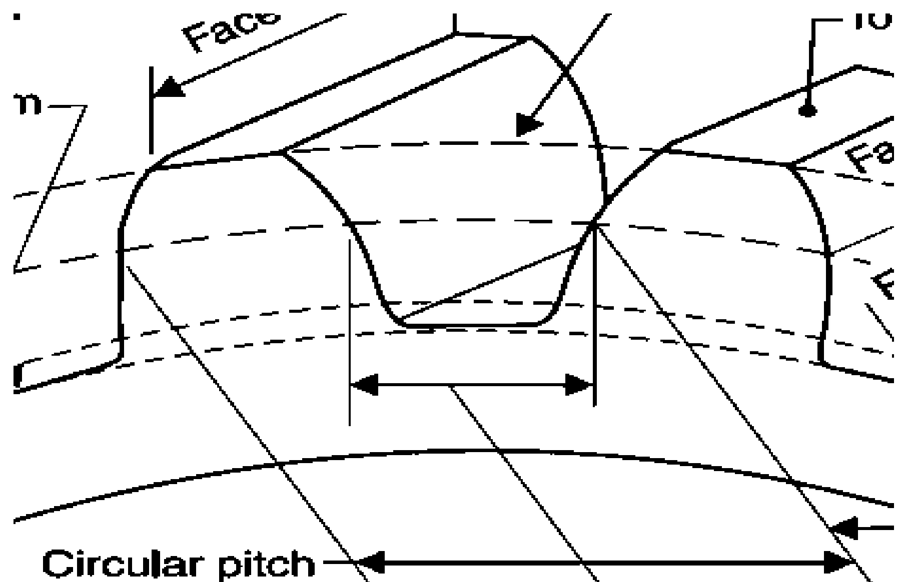


Diametral pitch: It is the ratio of number of teeth to the pitch circle diameter in millimetres. It is denoted by pd. Mathematically,

$$p_d = \frac{T}{D} = \frac{\pi}{p_c} \qquad \cdots\left(\because\ p_c = \frac{\pi D}{T}\right)$$

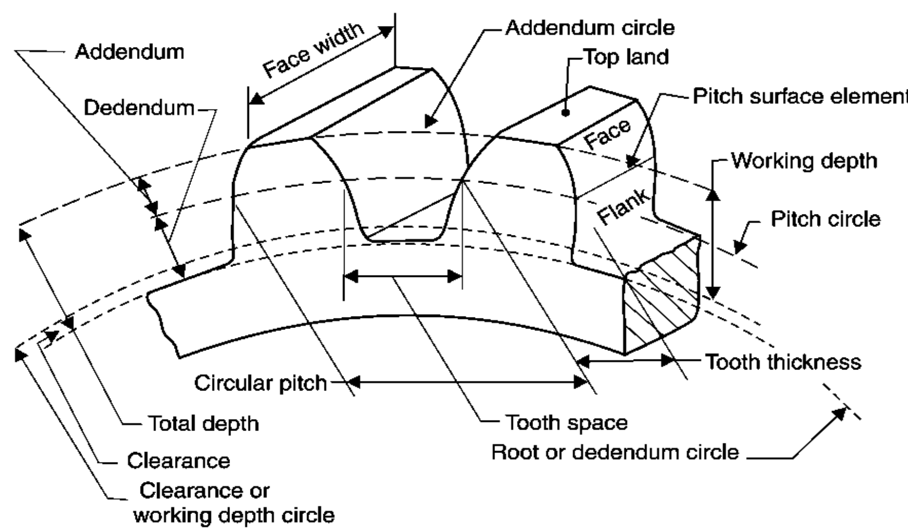
Module: It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m.

Mathematically, $\text{Module, } m = D/T$

Clearance: It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as **clearance circle.**

Total depth: It is the radial distance between the addendum and the dedendum circles of a gear.

It is equal to the sum of the addendum and dedendum.



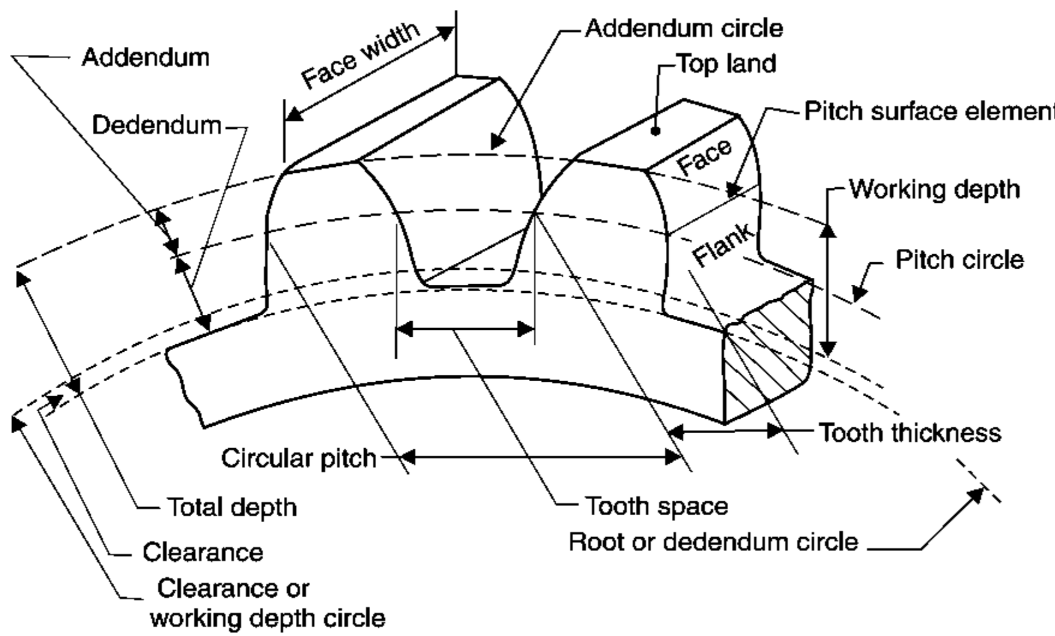
Working depth: It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

Tooth thickness: It is the width of the tooth measured along the pitch circle.

Tooth space: It is the width of space between the two adjacent teeth measured along the pitch circle.

Backlash: It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero,

but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.



Face of tooth: **It is the surface of the gear tooth above the pitch** surface.

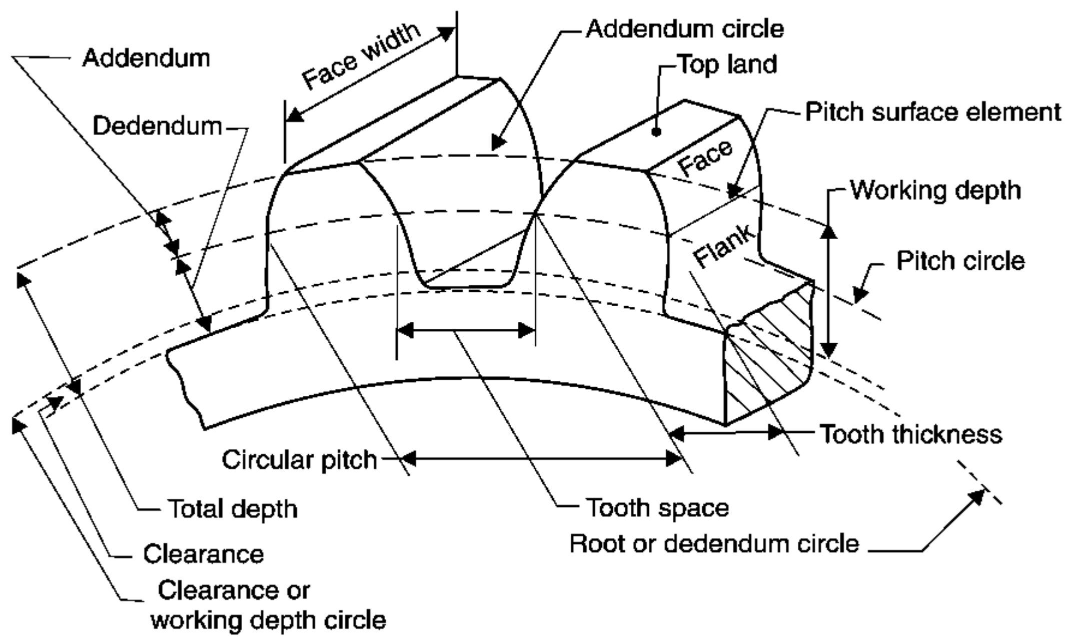
Flank of tooth: **It is the surface of the gear tooth below the pitch** surface.

Top land: **It is the surface of the top of the tooth.**

Face width: **It is the width of the gear tooth measured parallel to** its axis.

Profile: **It is the curve formed by the face and flank of the tooth.**

Fillet radius: **It is the radius that connects the root circle to the** profile of the tooth.



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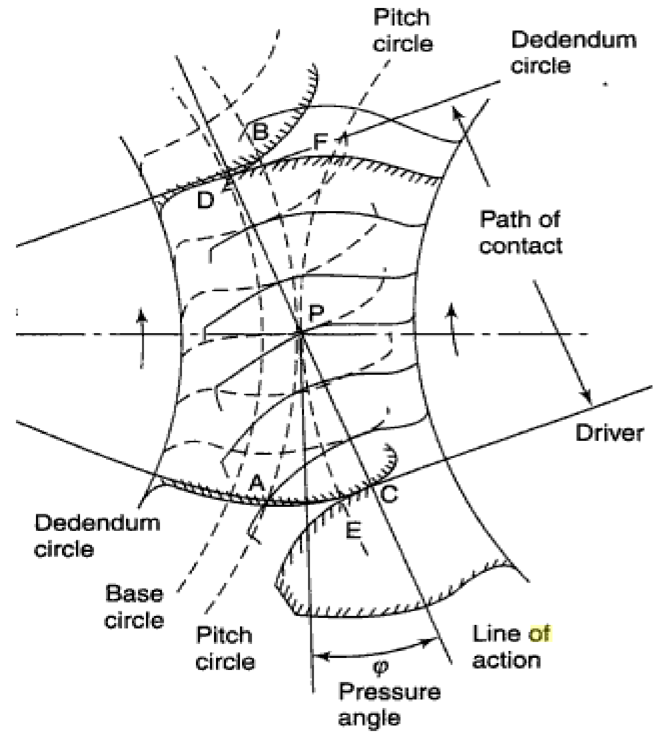
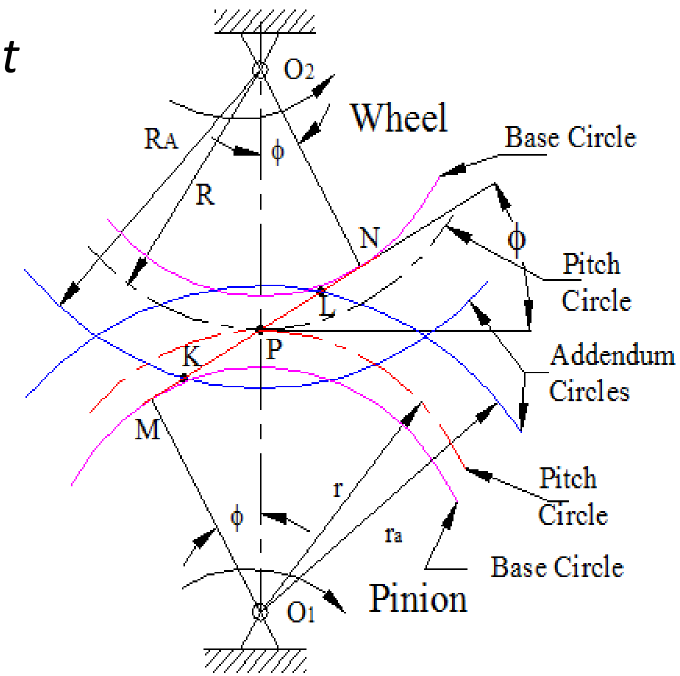
Path of contact: It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

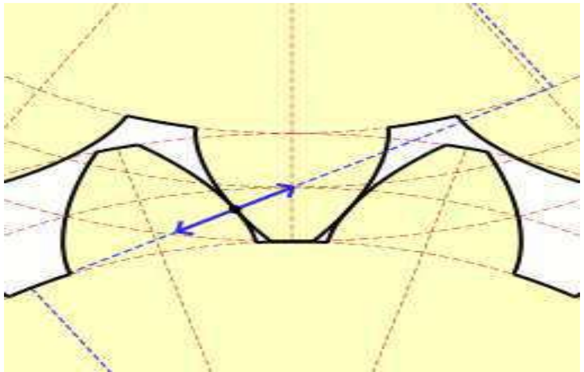
Length of the path of contact: *It is the length of the common normal cut-off by the addendum circles of the* wheel and pinion.

Arc of contact: *It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a* given pair of teeth. The arc of contact consists of two parts, *i.e.***(*a*) *Arc of approach.*** *It is the portion of the arc of contact from the beginning of the engagement the* pitch point.

(*b*) *Arc of recess:* *It is the portion of the arc of Contact from the pitch point to the end of the engagement of a* pair of teeth.







Condition for Constant Velocity Ratio of Toothed Wheels– Law of Gearing

The common normal at the point of contact between a pair of teeth must always pass through the pitch point.

Let T T be the common tangent and MN be the common normal to the curves at the point of contact Q. From the Centres O1 and O2 , draw O1M and O2 N perpendicular to MN.

A little consideration will show that the point Q moves in the direction QC, when considered as a point on wheel 1, and in the direction QD when considered as a point on wheel2.

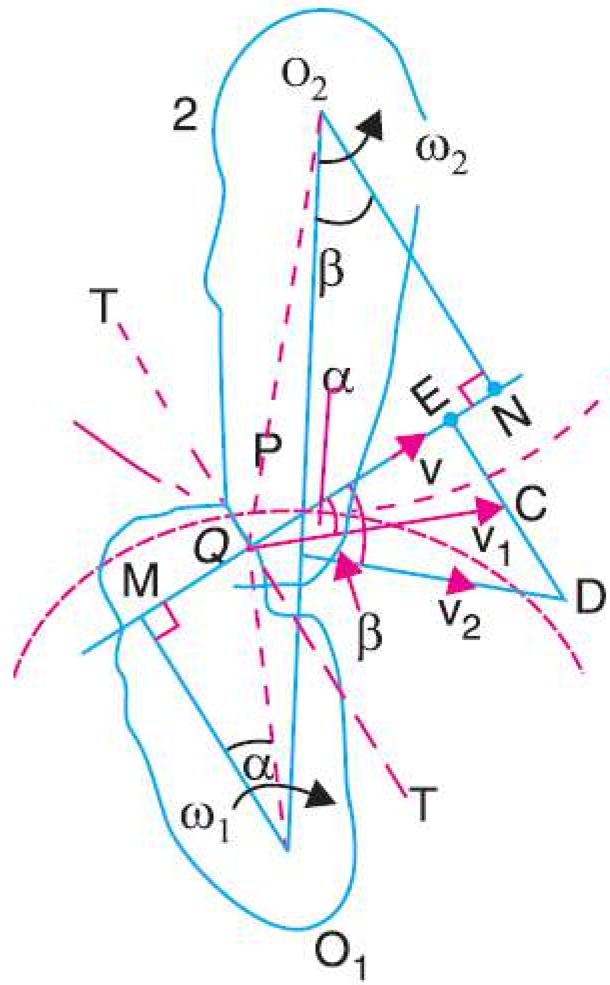


Fig. 12.6. Law of gearing.

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Let v1 and v2 be the velocities of the point Q on the wheels1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal MN must be equal.

V=rω

$$v_1 \cos\alpha = v_2 \cos\beta$$

$$(\omega_1 \times O_1 Q)\cos\alpha = (\omega_2 \times O_2 Q)\cos\beta$$

$$(\omega_1 \times O_1 Q)\frac{O_1 M}{O_1 Q} = (\omega_2 \times O_2 Q)\frac{O_2 N}{O_2 Q}$$

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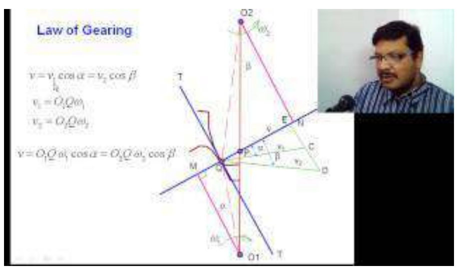
$$\text{or} \quad \omega_1 \times O_1 M = \omega_2 \times O_2 N$$



$$\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M} \quad \text{...}(i)$$

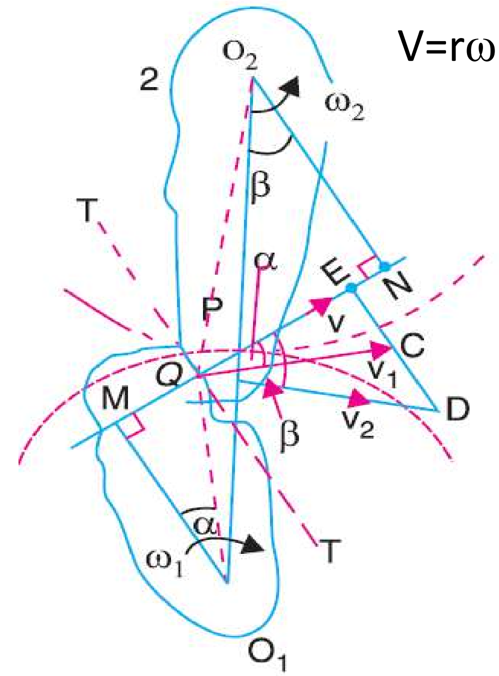


Fig. 12.6. Law of gearing.

From above, we see that the angular velocity ratio is inversely proportional to the ratio of the distances of the point P from the centres O1 and O2, or the common normal to the two surfaces at the point of contact Q intersects the line of centres at point P which divides the centre distance inversely as the ratio of angular velocities

Therefore in order to have a constant angular velocity ratio for all positions of the wheels, the point P must be the fixed point (called pitch point) for the two wheels.

Also from similar triangles O_1MP and O_2NP,

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \quad \dots(ii)$$

Combining equations (*i*) and (*ii*), we have

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \quad \dots(iii)$$

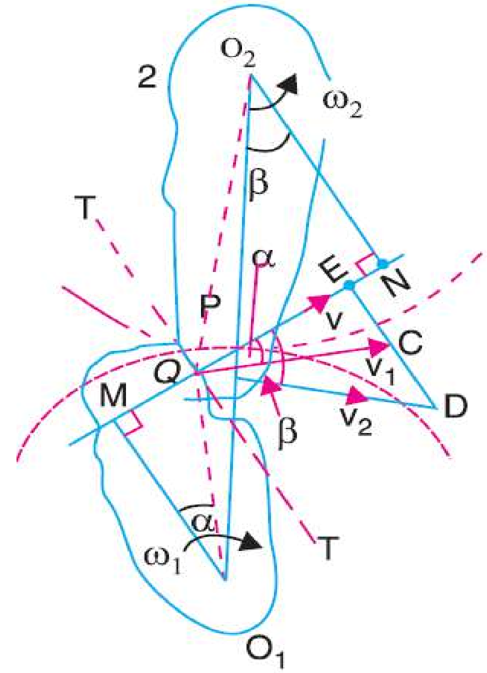


Fig. 12.6. Law of gearing.

This is the fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as **law of gearing**

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$$

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Velocity of Sliding of Teeth

The sliding between a pair of teeth in contact at Q occurs along the common tangent T T to the tooth curves as shown in Fig. **The velocity of sliding is the velocity of one tooth relative to its mating tooth along the common tangent at the point of contact**.

The velocity of point Q, considered as a point on wheel 1, along the common tangent T T is represented by EC. From similar triangles QEC and O1MQ,

$$\frac{EC}{MQ} = \frac{v}{O_1Q} = \omega_1 \quad \text{or} \quad EC = \omega_1 . MQ$$

Similarly, the velocity of point Q, considered as a point on wheel 2, along the common tangent T T is represented by ED. From similar triangles QCD and O2 NQ,

$$\frac{ED}{QN} = \frac{v_2}{O_2Q} = \omega_2 \quad \text{or} \quad ED = \omega_2 . QN$$

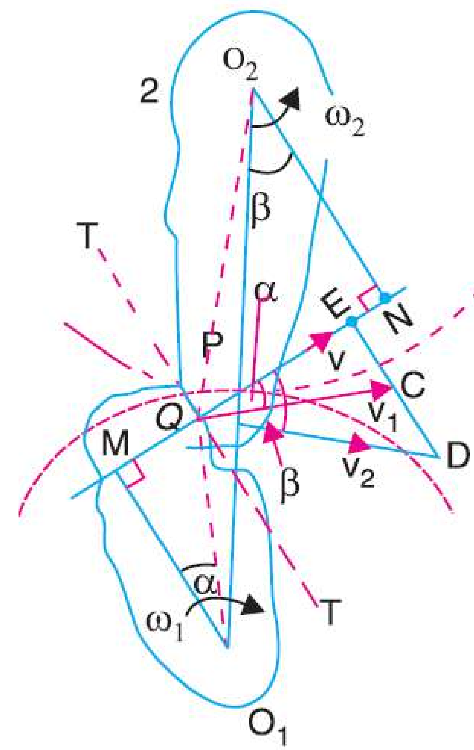

Fig. 12.6. Law of gearing.

v_S = Velocity of sliding at Q.

$$v_S = ED - EC = \omega_2 . QN - \omega_1 . MQ$$

$$= \omega_2 (QP + PN) - \omega_1 (MP - QP)$$

$$= (\omega_1 + \omega_2)\, QP + \omega_2 . PN - \omega_1 . MP \quad .(i)$$

Since $\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{PN}{MP}$ or $\omega_1 . MP = \omega_2 . PN$, therefore equation (*i*) becomes

$$v_S = (\omega_1 + \omega_2)\, QP \quad ..(ii)$$

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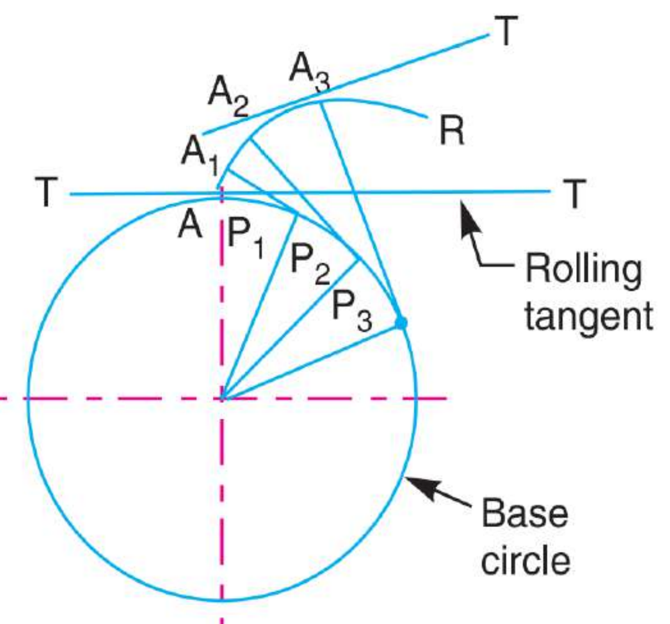
Involute Teeth

An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut string which in unwrapped from a reel as shown in Fig.. In connection with toothed wheels, the circle is known as base circle. The involute is traced as follows :

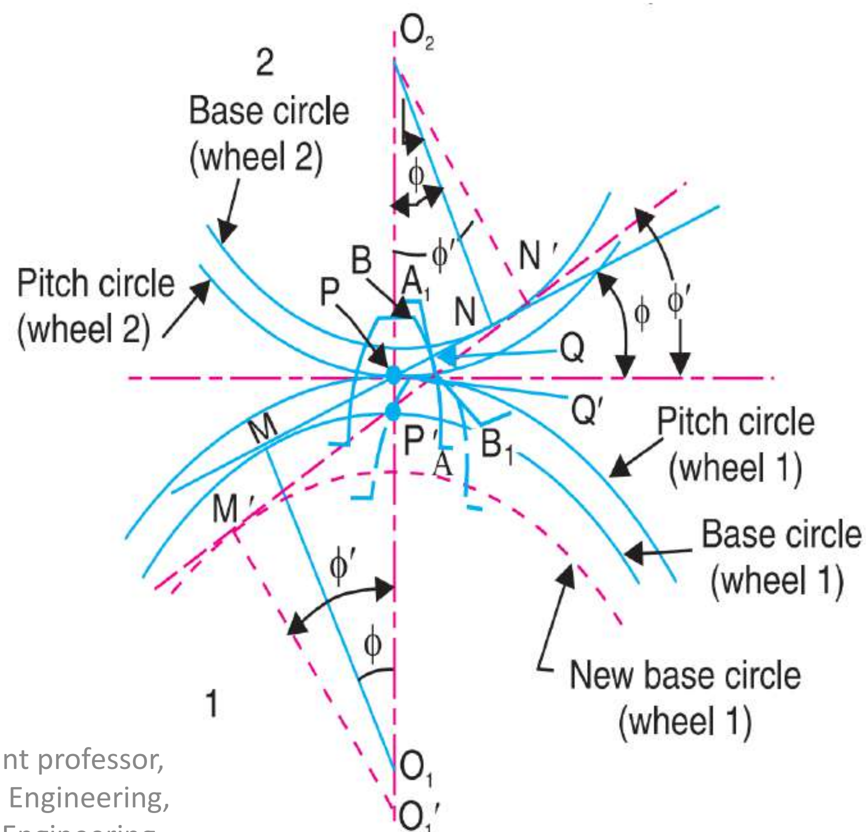
- Let A be the starting point of the involute. The base circle is divided into equal number of parts e.g. AP1, P1P2,P2P3 etc

The tangents at P1, P2, P3 etc. are drawn and the length P1A1, P2A2, P3A3 equal to the arcs AP1, AP2 and AP3 are set off. Joining the points A, A1, A2, A3 etc. we obtain the involute curve A R. A little consideration will show that at any instant A3, the tangent A3T to the involute is perpendicular to P3A3 and P3A3 is the normal to the involute. In other words, normal at any point of an involute is a tangent to the circle.



Now, let *O*1 and *O*2 be the fixed centres of the two base circles as shown in Fig. 12.10 (*a*). Let the corresponding involutes *A B* and *A*1*B*1 be in contact at point *Q*. *MQ* and *NQ* are normals to the involutes at *Q* and are tangents to base circles. Since the normal of an involute at a given point is the tangent drawn from that point to the base circle,

therefore the common normal *MN* at *Q* is also the common tangent to the two base circles. We see that the common normal *MN* intersects the line of centres *O*1*O*2 at the fixed point *P* (called pitch point). Therefore the involute teeth satisfy the fundamental condition of constant velocity ratio.



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From similar triangles O_2NP and O_1MP,

$$\frac{O_1M}{O_2N} = \frac{O_1P}{O_2P} = \frac{\omega_2}{\omega_1}$$

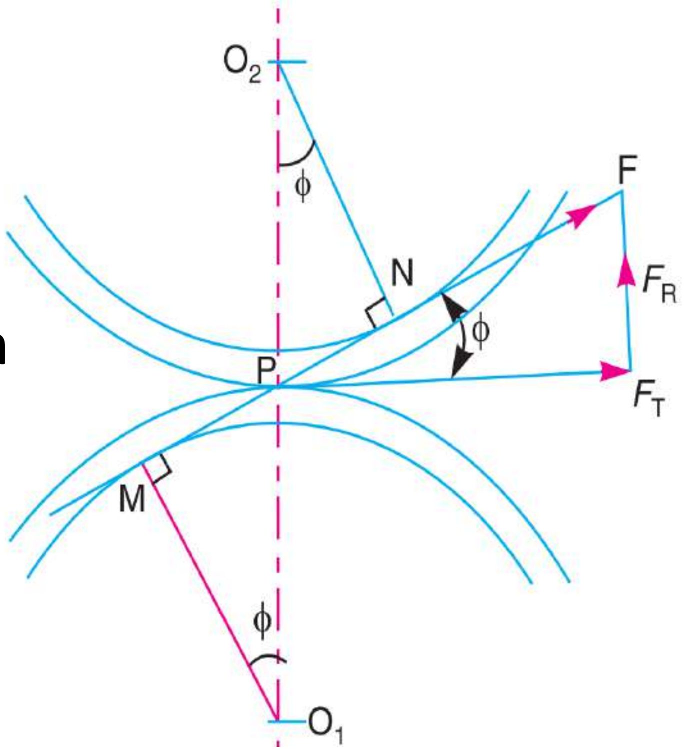
which determines the ratio of the radii of the two base circles. The radii of the base circles is given by

$$O_1M = O_1P\cos\phi, \quad \text{and} \quad O_2N = O_2P\cos\phi$$

Also the centre distance between the base circles,

$$O_1O_2 = O_1P + O_2P = \frac{O_1M}{\cos\phi} + \frac{O_2N}{\cos\phi} = \frac{O_1M + O_2N}{\cos\phi}$$

where ϕ is the pressure angle or the angle of obliquity. It is the angle which the common normal to the base circles (i.e. MN) makes with the common tangent to the pitch circles.



If F is the maximum tooth pressure as shown in Fig. (b), then

Tangential force, $F_T = F\cos\phi$

and radial or normal force, $F_R = F\sin\phi$.

∴ Torque exerted on the gear shaft = $F_T \times r$,

where r is the pitch circle radius of the gear.

Length of Path of Contact

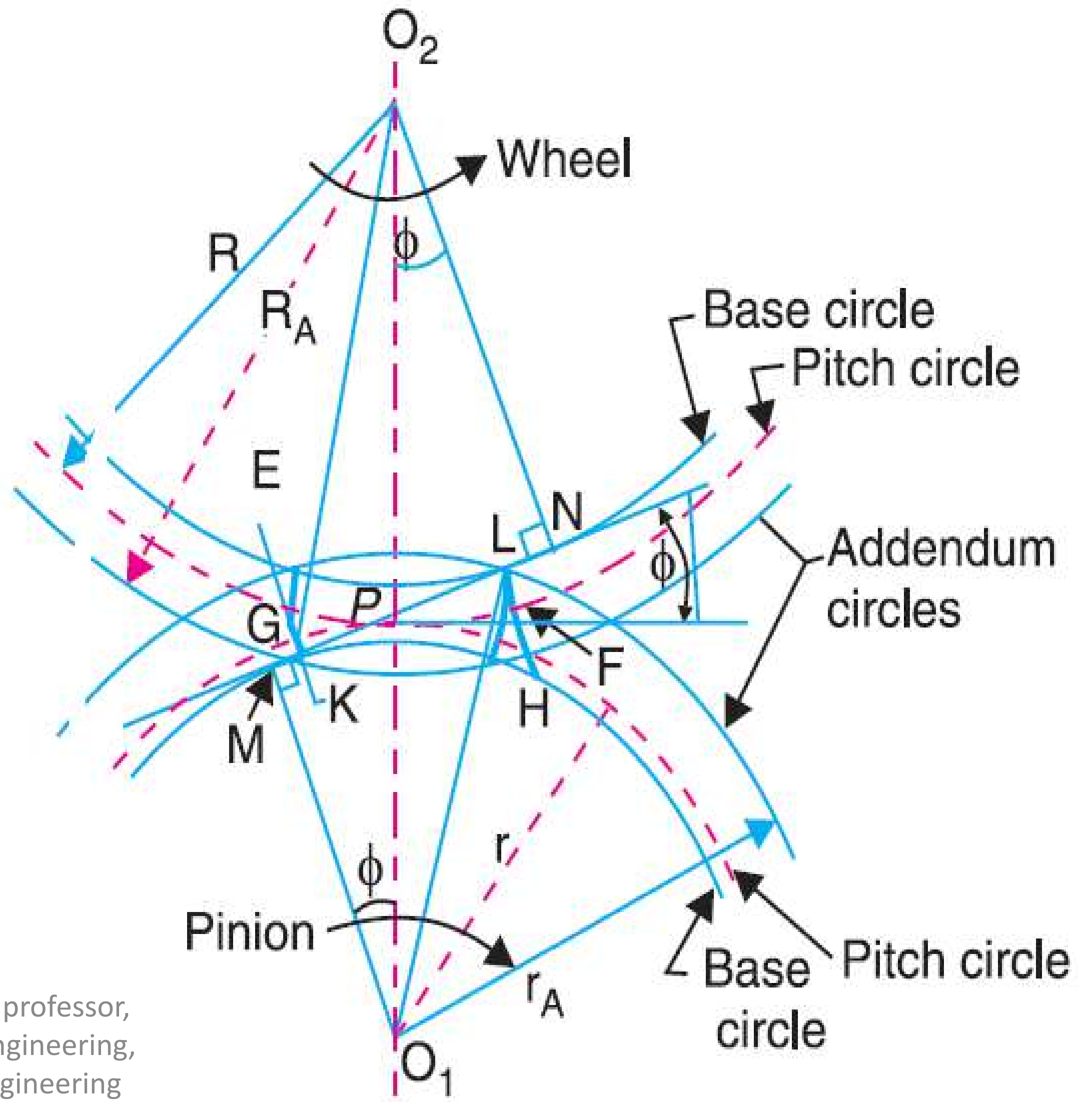


Fig. Length of path of contact.

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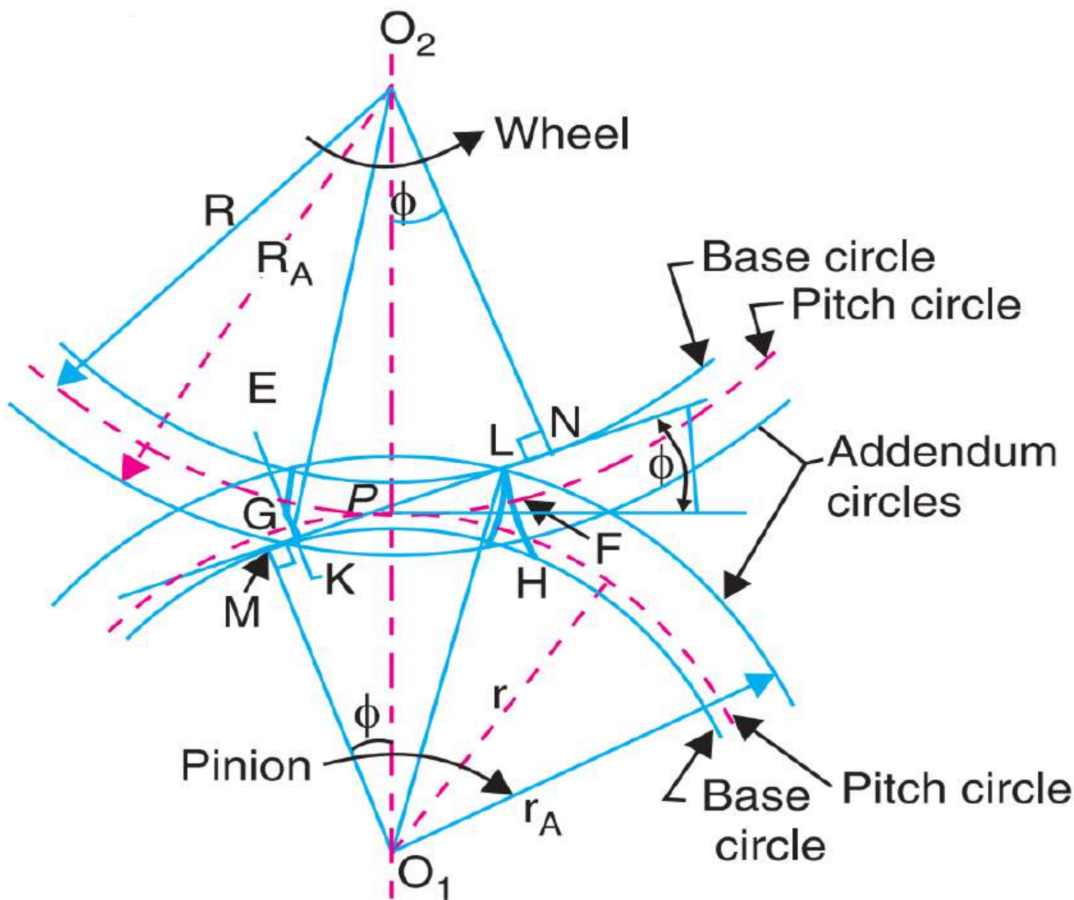
Length of Path of Contact

Consider a pinion driving the wheel as shown in Fig. When the pinion rotates in clockwise direction, the contact between a pair of involute teeth begins at K (on the flank near the base circle of pinion or the outer end of the tooth face on the wheel) and* ends at L (outer end of the tooth face on the pinion or on the flank near the base circle of wheel)..

MN is the common normal at the point of contacts and the common tangent to the base circles. The point K is the intersection of the addendum circle of wheel and the common tangent

The point L is the intersection of the addendum circle of pinion and common tangent



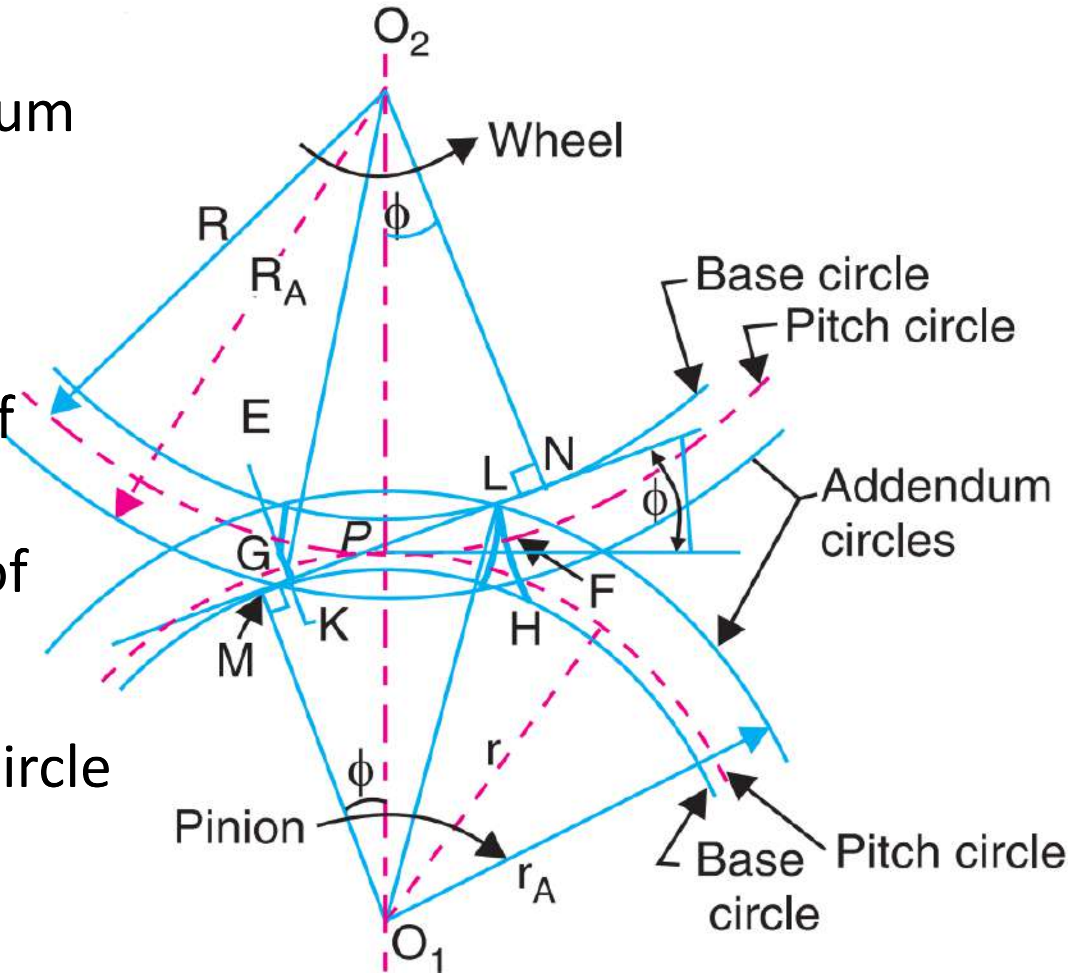
The length of path of contact is the length of common normal cutoff by the addendum circles of the wheel and the pinion. Thus the length of path of contact is KL which is the sum of the parts of the path of contacts KP and PL. The part of the path of contact KP is known as **path of approach** and the part of the path of contact PL is known as **path of recess.**

Let $r_A = O_1L$ = Radius of addendum circle of pinion,

$R_A = O_2K$ = Radius of addendum circle of wheel,

$r = O_1P$ = Radius of pitch circle of pinion, and

$R = O_2P$ = Radius of pitch circle of wheel.



we find that radius of the base circle of pinion

$$O_1M = O_1P \cos \phi = r \cos \phi$$

and radius of the base circle of wheel,

$$O_2N = O_2P \cos \phi = R \cos \phi$$

Now from right angled triangle *O2KN,*

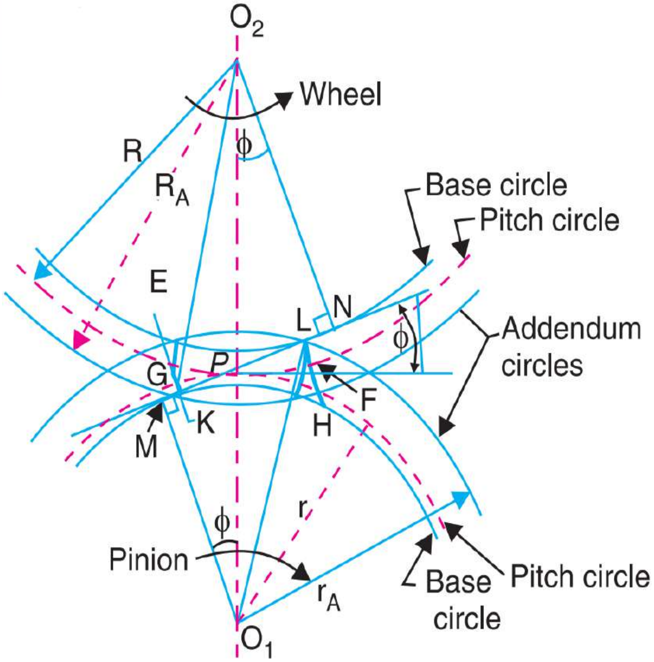
$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2 \cos^2 \phi}$$

,

$$PN = O_2P \sin \phi = R \sin \phi$$

Length of the part of the path of contact, or the path of approach,

$$KP = KN - PN = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi$$



Similarly from right angled triangle O1ML

$$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2 \cos^2 \phi}$$

$$MP = O_1P \sin \phi = r \sin \phi$$

Length of the part of the path of contact, or path of recess,

$$PL = ML - MP = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

Length of the path of contact

$$KL = KP + PL = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} + \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - (R + r) \sin \phi$$

Length of Arc of Contact

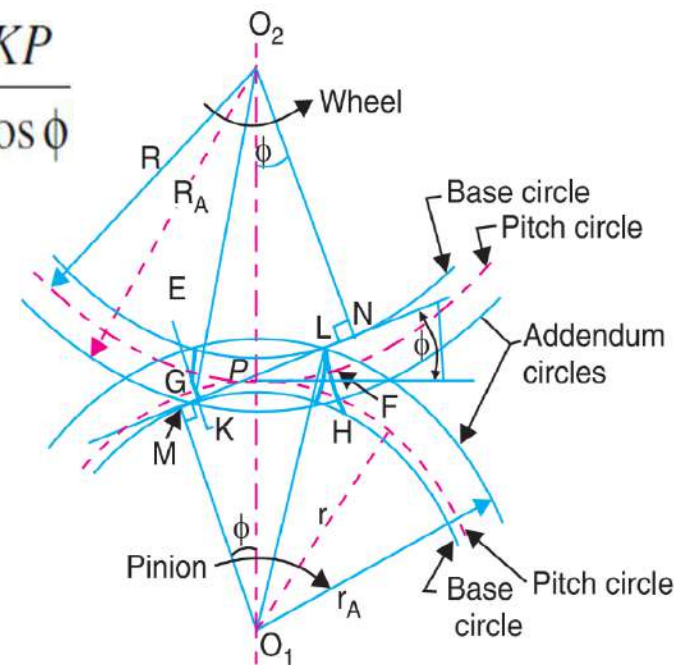
The arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. In Fig., the arc of contact is EPF or GPH. Considering the arc of contact GPH, it is divided into two parts i.e. arc GP and arc PH. The arc GP is known as **arc of approach** and the arc PH is called **arc of recess**. The angles subtended by these arcs at O1 are called **angle of approach** and **angle of recess** respectively

We know that the length of the arc of approach (arc GP)

$$= \frac{\text{Length of path of approach}}{\cos\phi} = \frac{KP}{\cos\phi}$$

and the length of the arc of recess (arc PH)

$$= \frac{\text{Length of path of recess}}{\cos\phi} = \frac{PL}{\cos\phi}$$



Since the length of the arc of contact GPH is equal to the sum of the length of arc of approach and arc of recess, therefore,

Length of the arc of contact

$$= \text{arc } GP + \text{arc } PH = \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KL}{\cos\phi}$$

$$= \frac{\text{Length of path of contact}}{\cos\phi}$$

Contact Ratio (or Number of Pairs of Teeth in Contact)

The contact ratio or the number of pairs of teeth in contact is defined as the **ratio of the length of the arc of contact** to **the circular pitch.**

Mathematically,

Contact ratio or number of pairs of teeth in contact

$$= \frac{\text{Length of the arc of contact}}{p_c}$$

p_c = Circular pitch = πm, and

m = Module.

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Number of pair of teeth in contact(contact ratio)

The arc of contact is the length of the pitch circle traversed by a point on it during the mating of a pair of teeth.

- Thus, all the teeth lying in between the arc of contact will be meshing with the teeth on the other wheel. Therefore, the number of teeth in contact =

$$\frac{\text{Arc of contact}}{\text{Circular pitch}} = \frac{\text{path of contact}}{\cos\phi} \times \frac{1}{p_c} \quad \text{Where } P_c = \pi D/T$$

- As the ratio of the arc of contact to the circular pitch is also the contact ratio, the number of teeth is also expressed in terms of contact ratio.
- For continuous transmission of motion, at least one tooth of one wheel must be in contact with another tooth of the second wheel.
- Therefore, n must be greater than unity.

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Each of two gears in a mesh has 48 teeth and a module of 8 mm. The teeth are of 20° involute profile. The arc of contact is 2.25 times the circular pitch. Determine the addendum.

Solution $\varphi = 20°; t = T = 48; m = 8$ mm;

$$R = r = \frac{mT}{2} = \frac{8 \times 48}{2} = 192 \text{ mm}; R_a = r_a$$

Arc of contact = 2.25 × Circular pitch = 2.25π m $= 2.25\pi \times 8 = 56.55$ mm

$$p_c = \pi D/T$$

Path of contact = 56.55 × cos 20° = 53.14 mm

or $$(\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi) + (\sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi) = 53.14$$

or $$2(\sqrt{R_a^2 - 192^2\cos^2 20°} - 192\sin 20°) = 53.14 \quad \text{or} \quad R_a = 202.6 \text{ mm}$$

Addendum $= R_a - R = 202.6 - 192 = 10.6$ mm

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A pinion having 30 teeth drives a gear having 80 teeth. The profile of the gears is involute with 20° pressure angle, 12 mm module and 10 mm addendum. Find the length of path of contact, arc of contact and the contact ratio.

Solution. Given : $t = 30$; $T = 80$; $\phi = 20°$; $m = 12$ mm ; Addendum = 10 mm

Length of path of contact

We know that pitch circle radius of pinion,

$$r = m.t/2 = 12 \times 30/2 = 180 \text{ mm}$$

and pitch circle radius of gear,

$$R = m.T/2 = 12 \times 80/2 = 480 \text{ mm}$$

∴ Radius of addendum circle of pinion,

$$r_A = r + \text{Addendum} = 180 + 10 = 190 \text{ mm}$$

and radius of addendum circle of gear,

$$R_A = R + \text{Addendum} = 480 + 10 = 490 \text{ mm}$$

We know that length of the path of approach,

$$KP = \sqrt{(R_A)^2 - R^2 \cos^2\phi} - R\sin\phi$$

$$= \sqrt{(490)^2 - (480)^2 \cos^2 20°} - 480 \sin 20° = 191.5 - 164.2 = 27.3 \text{ mm}$$

A pinion having 30 teeth drives a gear having 80 teeth. The profile of the gears is involute with 20° pressure angle, 12 mm module and 10 mm addendum. Find the length of path of contact, arc of contact and the contact ratio.

and length of the path of recess,

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

$$= \sqrt{(190)^2 - (180)^2 \cos^2 20°} - 180 \sin 20° = 86.6 - 61.6 = 25 \text{ mm}$$

We know that length of path of contact,

$$KL = KP + PL = 27.3 + 25 = 52.3 \text{ mm} \quad \textbf{Ans.}$$

Length of arc of contact

We know that length of arc of contact

$$= \frac{\text{Length of path of contact}}{\cos \phi} = \frac{52.3}{\cos 20°} = 55.66 \text{ mm} \quad \textbf{Ans.}$$

Contact ratio

We know that circular pitch,

$$p_c = \pi . m = \pi \times 12 = 37.7 \text{ mm}$$

$$\therefore \quad \text{Contact ratio} = \frac{\text{Length of arc of contact}}{p_c} = \frac{55.66}{37.7} = 1.5$$

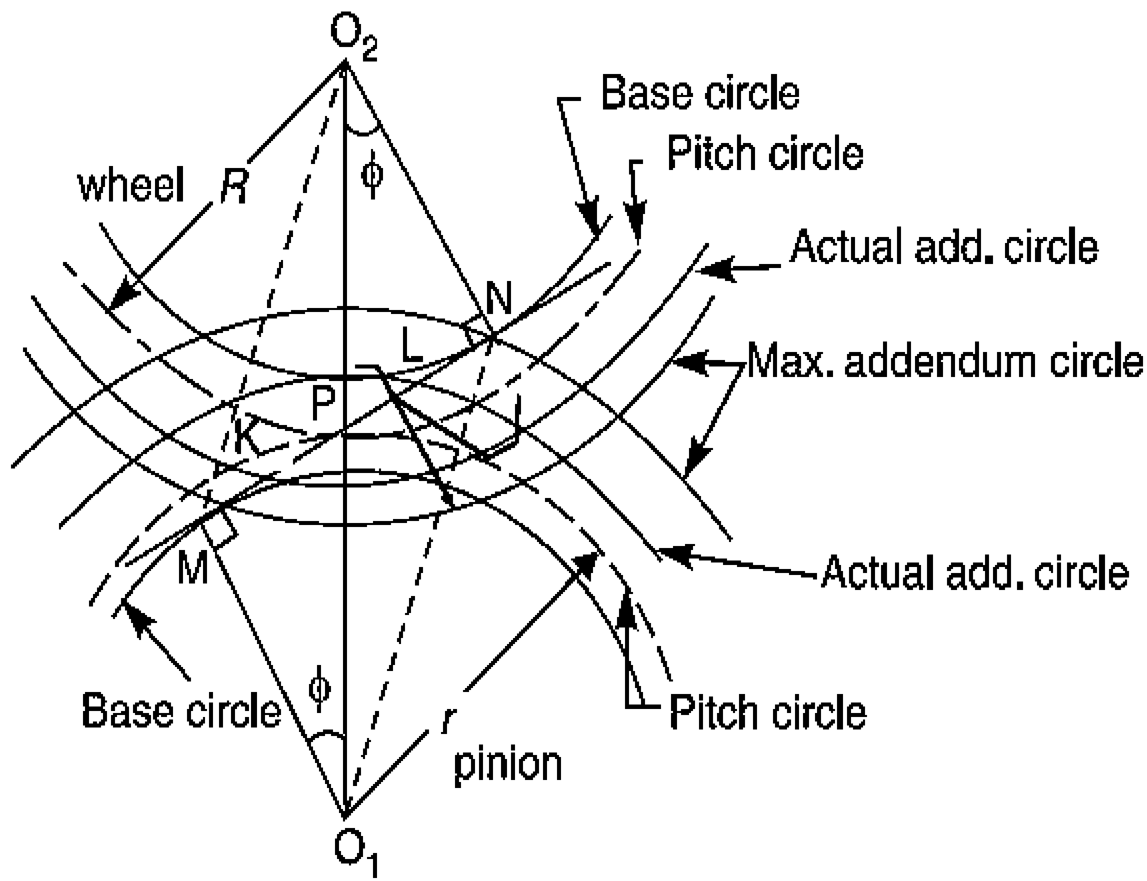
Interference in involute gears

Fig. shows a pinion with centre O1, in mesh with wheel or gear with centre O2.

- MN is the common tangent to the base circles(common normal at point of contacts) and KL is the path of contact between the two mating teeth.
- The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel. This effect is known as interference, and occurs when the teeth are being cut.
- In brief, the phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference.

- A little consideration will show, that if the radius of the addendum circle of pinion is increased to O 1 N, the point of contact L will move from L to N.

- When this radius is further increased, the point of contact L will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel.



https://youtu.be/LDhZJ5Ya5YI

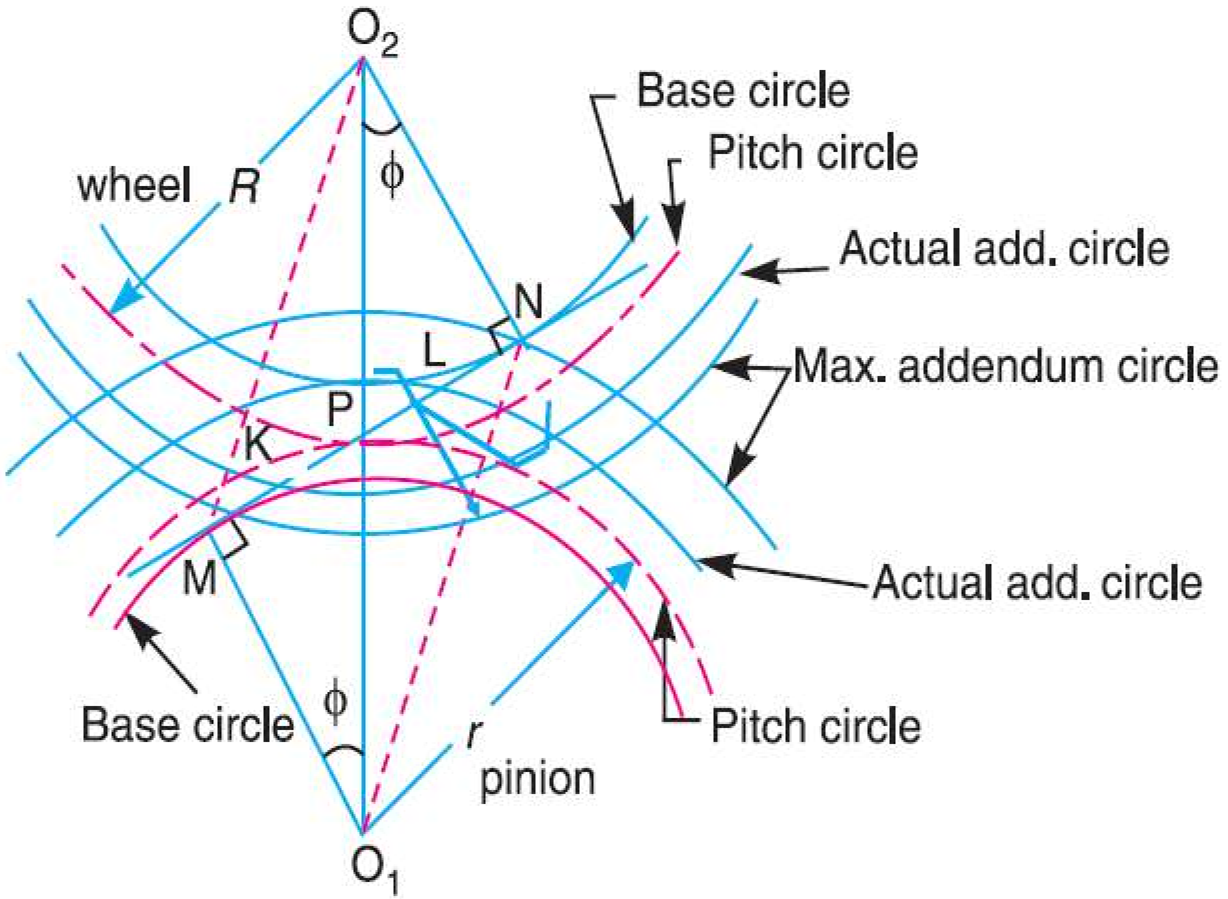


Fig. 12.13. Interference in involute gears.

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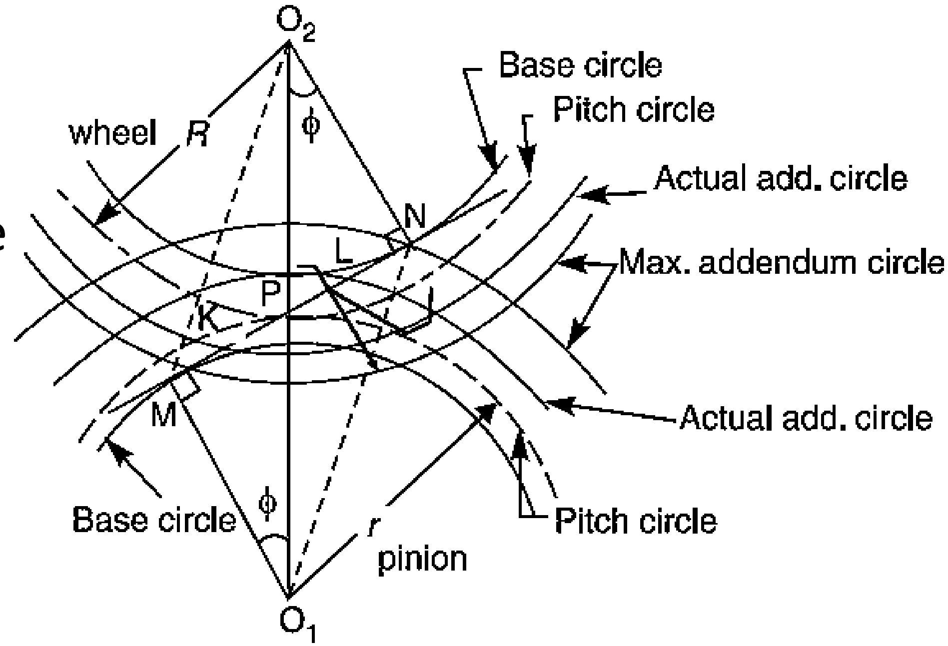
- Similarly, if the radius of the addendum circle of the wheel increases beyond O2M,then the tip of tooth on wheel will cause interference with the tooth on pinion. The points M and N are called interference points.



- Obviously, interference may be avoided if the path of contact does not extend beyond interference points. The limiting value of the radius of the addendum circle of The pinion is O1N and of the wheel is O2M.

- From the above discussion, we conclude that the interference may only be avoided, if the point of contact between the two teeth is always on the involute profiles of both the teeth. In other words, interference may only be prevented, if the addendum circles of the two mating gears cut the common tangent to the base circles between the points of tangency.
- When interference is just avoided, the maximum length of path of contact is MN when the maximum addendum circles for pinion and wheel pass through the points of tangency N and M respectively as shown in Fig.

Maximum length of path of approach,

$$MP = r \sin \phi$$

and maximum length of path of recess,

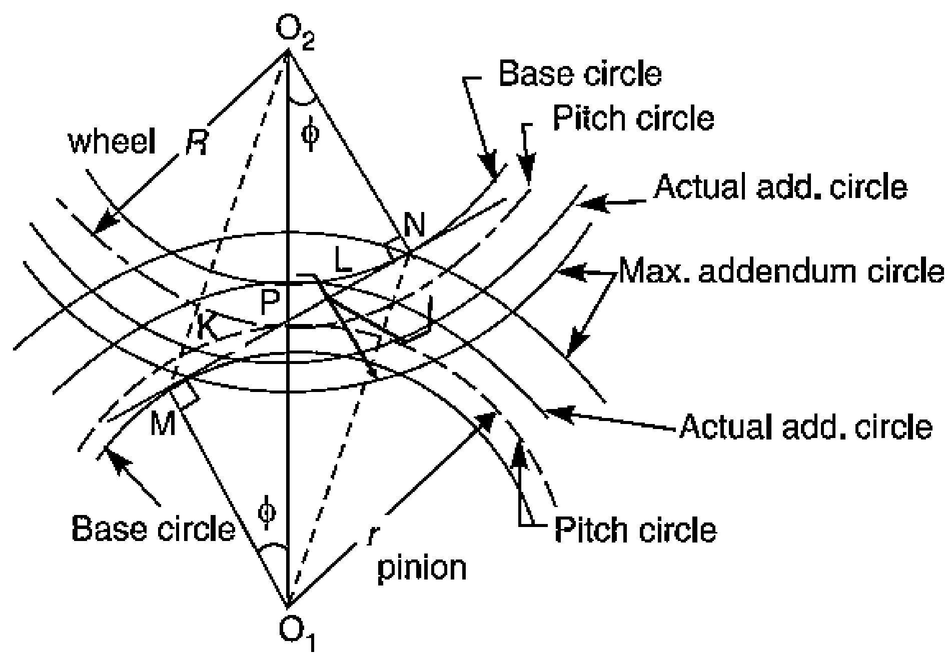
$$PN = R \sin \phi$$

$\therefore$ Maximum length of path of contact,

$$MN = MP + PN = r \sin \phi + R \sin \phi = (r + R) \sin \phi$$

and maximum length of arc of contact

$$= \frac{(r + R) \sin \phi}{\cos \phi} = (r + R) \tan \phi$$



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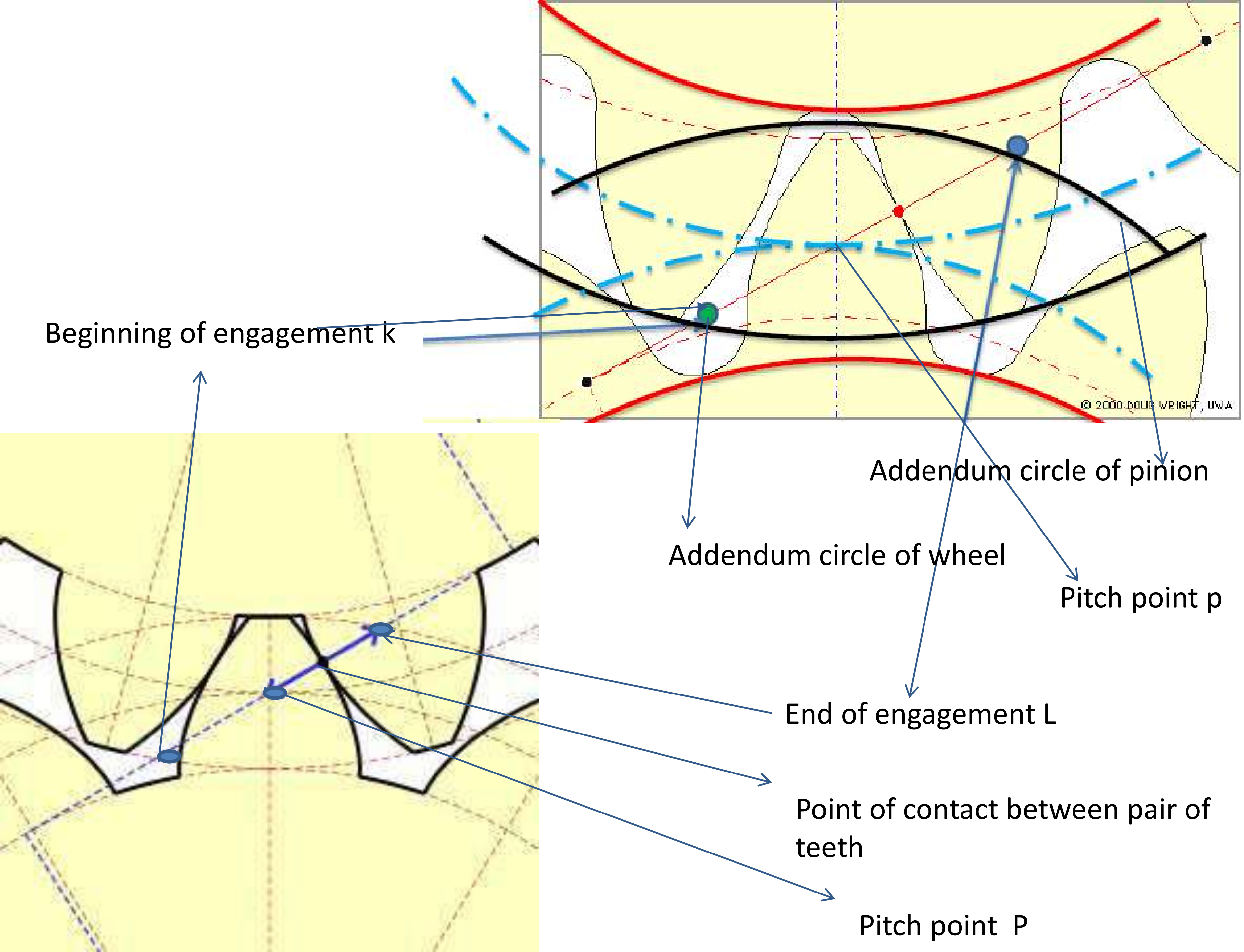
Beginning of engagement k
Addendum circle of pinion
Addendum circle of wheel
Pitch point p
End of engagement L
Point of contact between pair of teeth
Pitch point P
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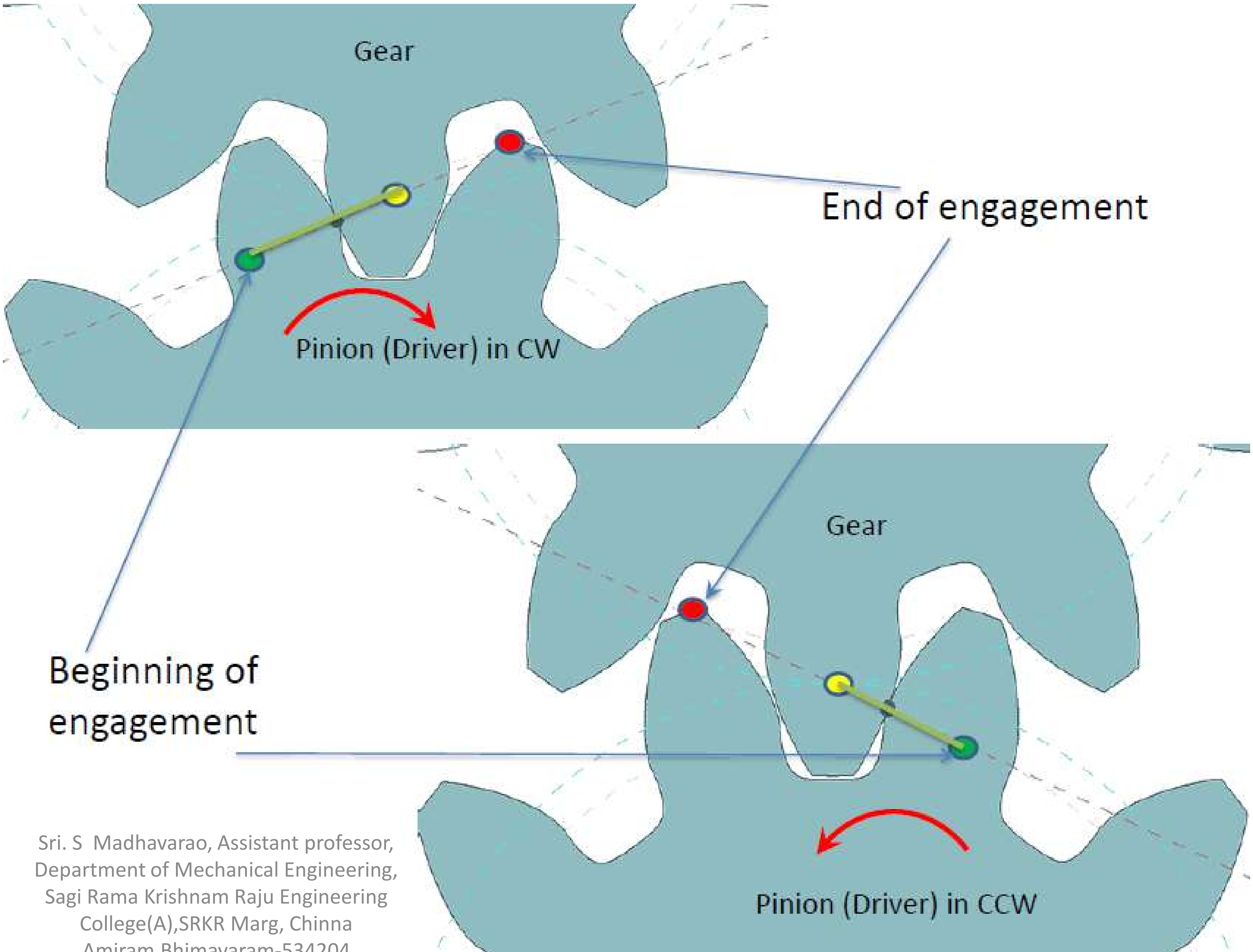

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Methods of elimination of Gear tooth Interference

In certain spur designs if interference exists, it can be overcome by:

1. Removing the cross hatched tooth tips i.e., using stub teeth.
2. Increasing the number of teeth on the mating pinion.
3. Increasing the pressure angle
4. Tooth profile modification or profile shifting
5. Increasing the centre distance.

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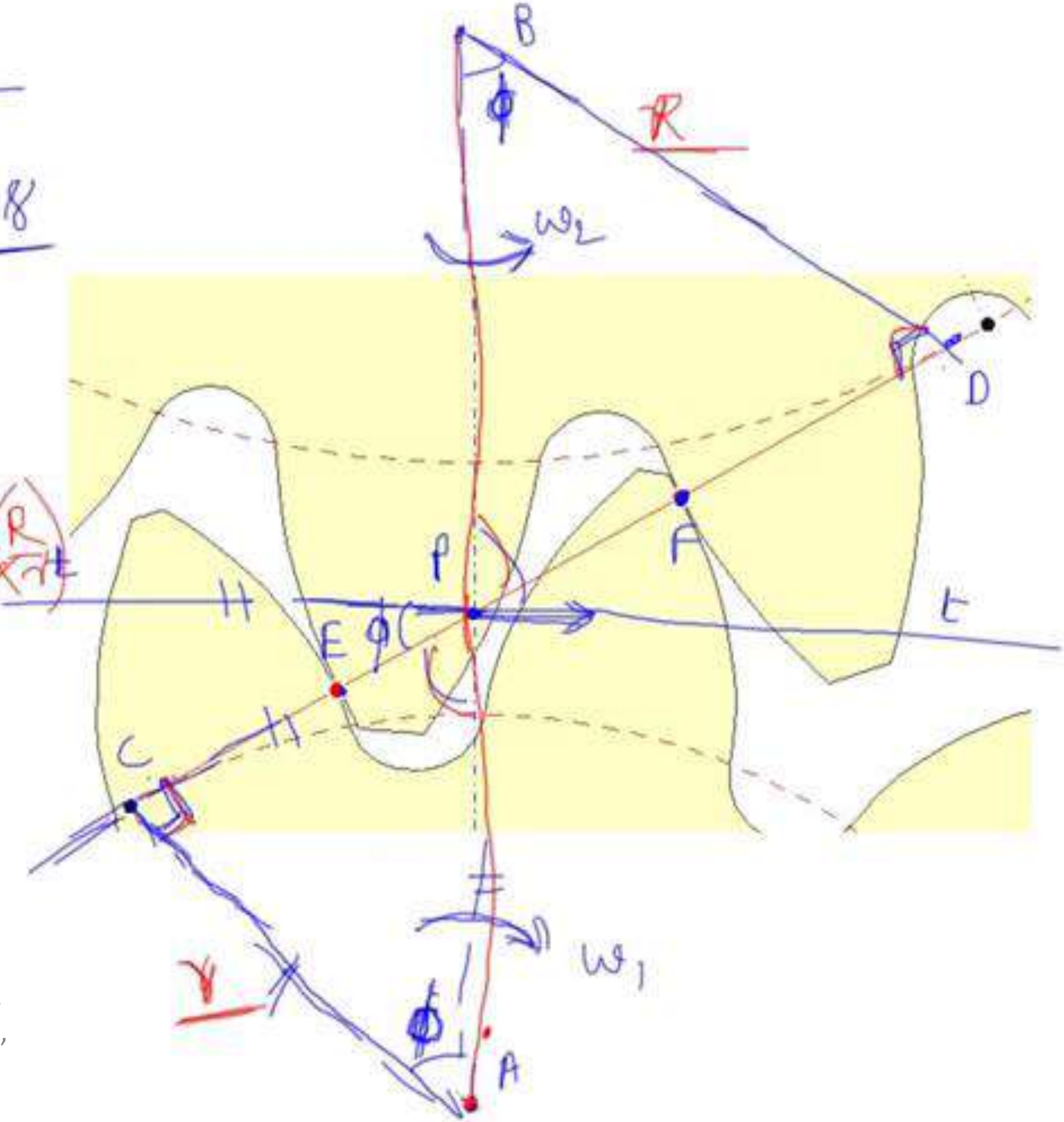
Forms of teeth

Involute Profiles

$V_{P_1} = \omega_1 \times AP$

$V_{P_2} = \omega_2 BP$

$$\frac{\omega_1}{\omega_2} = \frac{BP}{AP} = \frac{BD}{AC} = \frac{R}{r}$$



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Minimum Number of Teeth on the wheel in Order to Avoid Interference

We have already discussed that in order to avoid interference, the addendum circles for the two mating gears must cut the common tangent to the base circles between the points of tangency. The limiting condition reaches, when the addendum circles of pinion and wheel pass through points *N and M*

t = Number of teeth on the pinion,,

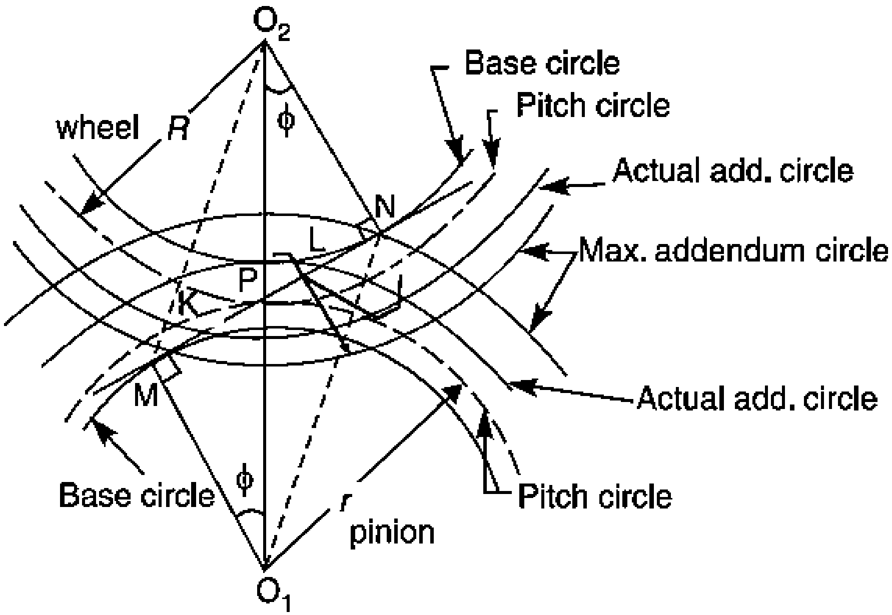
T = Number of teeth on the wheel,

m = Module of the teeth,

r = Pitch circle radius of pinion = $m.t / 2$

G = Gear ratio = $T / t = R / r$

ϕ = Pressure angle or angle of obliquity.



we have from triangle O_2MP

$$(O_2M)^2 = (O_2P)^2 + (PM)^2 - 2 \times O_2P \times PM \cos O_2PM$$

$$= R^2 + r^2 \sin^2\phi - 2\, R.r \sin\phi \cos(90° + \phi)$$

https://youtu.be/ixuEOizkhVk

$$...(\because PM = O_1P \sin\phi = r)\sin\phi$$

$$= R^2 + r^2 \sin^2\phi + 2R.r \sin^2\phi$$

$$= R^2\left[1 + \frac{r^2 \sin^2\phi}{R^2} + \frac{2r \sin^2\phi}{R}\right] = R^2\left[1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi\right]$$

∴ Limiting radius of wheel addendum circle,

$$O_2M = R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi} = \frac{m.T}{2}\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi}$$

$A_W m$ = Addendum of the wheel, where A_W is a fraction by which the standard addendum for the wheel should be multiplied.

We know that the addendum of the wheel

$$= O_2M - O_2P$$

$$\therefore \quad A_W m = \frac{m.T}{2}\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - \frac{m.T}{2} \quad \ldots(\because O_2P = R = m.T/2)$$

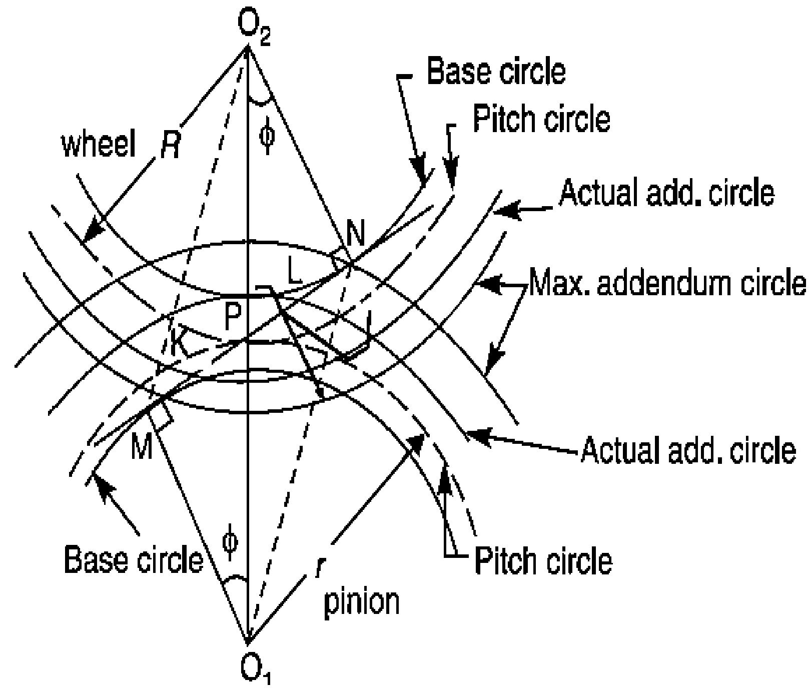
$$= \frac{m.T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$A_W = \frac{T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$T = \frac{2A_W}{\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1} = \frac{2A_W}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

Minimum Number of Teeth on the Pinion in Order to Avoid Interference

- We have already discussed that in order to avoid interference, the addendum circles for the two mating gears must cut the common tangent to the base circles between the points of tangency. The limiting condition reaches, when the addendum circles of pinion and wheel pass through points *N and M*



t = Number of teeth on the pinion,,

T = Number of teeth on the wheel,

m = Module of the teeth,

r = Pitch circle radius of pinion = $m.t / 2$

G = Gear ratio = $T / t = R / r$

ϕ = Pressure angle or angle of obliquity.

From triangle O_1NP,

$$(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2 \times O_1P \times PN \cos O_1PN$$

$$= r^2 + R^2 \sin^2\phi - 2r.R \sin\phi \cos(90° + \phi)$$

$$...(\because \; PN = O_2P \sin\phi = R\sin\phi)$$

$$(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2 \times O_1P \times PN \cos O_1PN$$

$$= r^2 + R^2 \sin^2\phi - 2r.R \sin\phi \cos(90° + \phi)$$

$$= r^2 + R^2 \sin^2\phi + 2r.R \sin^2\phi$$

$$= r^2\left[1 + \frac{R^2 \sin^2\phi}{r^2} + \frac{2R\sin^2\phi}{r}\right] = r^2\left[1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi\right]$$

∴ Limiting radius of the pinion addendum circle,

$$O_1N = r\sqrt{1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi} = \frac{m.t}{2}\sqrt{1 + \frac{T}{t}\left[\frac{T}{t} + 2\right]\sin^2\phi}$$

Let $A_P.m$ = Addendum of the pinion, where A_P is a fraction by which the standard addendum of one module for the pinion should be multiplied in order to avoid interference.

We know that the addendum of the pinion

$$= O_1N - O_1P$$

$$A_P.m = \frac{m.t}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - \frac{m.t}{2}$$

$$= \frac{m.t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$A_P = \frac{t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$t = \frac{2A_P}{\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1}$$

$$= \frac{2A_P}{\sqrt{1 + G(G + 2)\sin^2\phi} - 1}$$

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Two 20° involute spur gears mesh externally and give a velocity ratio of3. Module is 3 mm and the addendum is equal to 1.1 module. If the pinion rotates at 120 rpm, determine (i) the minimum number of teeth on each wheel to avoid interference (ii) the number of pairs of teeth in contact.

Solution $\varphi = 20°$ $N_p = 120$ rpm
$VR = 3$ Addendum = 1.1 m
$m = 3$ mm $\alpha_w = 1.1$

(i) $$T = \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi} - 1}$$

$$= \frac{2\times 1.1}{\sqrt{1+\frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20°} - 1} = 49.44$$

Taking the higher whole number divisible by the velocity ratio,

i.e., $T = 51$ and $t = \frac{51}{3} = \underline{17}$

(ii) Contact ratio or number of pairs of teeth in contact,

$$n = \frac{\text{Arc of contact}}{\text{Circular pitch}}$$

$$= \left(\frac{\text{Path of contact}}{\cos\varphi}\right) \times \frac{1}{\pi m}$$

or

$$n = \frac{\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi + \sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi}{\cos\varphi \times \pi m}$$

We have, $R = \frac{mT}{2} = \frac{3\times 51}{2} = 76.5$ mm

$R_a = R + 1.1\,m = 76.5 + 1.1\times 3 = 79.8$ mm

$r = \frac{mt}{2} = \frac{3\times 17}{2} = 25.5$ mm

$r_a = 25.5 + 1.1\times 3 = 28.8$ mm

$$n = \frac{\left[\begin{array}{l}\sqrt{(79.8)^2 - (76.5\cos 20°)^2} - 76.5\sin 20° \\ +\sqrt{(28.8)^2 - (25.5\cos 20°)^2} - 25.5\sin 20°\end{array}\right]}{\cos 20° \times \pi \times 3}$$

$$= \frac{34.646 - 26.165 + 15.977 - 8.720}{\cos 20° \times \pi \times 3}$$

$$= 1.78$$

Thus, 1 pair of teeth will always remain in contact whereas for 78% of the time, 2 pairs of teeth will be in contact

A pair of spur gears with involute teeth is to give a gear ratio of 4: 1. The arc of approach is not to be less than the circular pitch and smaller wheel is the driver. The angle of pressure is 14.5°. Find : 1. the least number of teeth that can be used on each wheel, and 2. the addendum of the wheel in terms of the circular pitch.

Solution. *Given* : $G = T/t = R/r = 4 \,;\, \phi = 14.5°$

1. ***Least number of teeth on each wheel***

We know that the maximum length of the arc of approach

$$= \frac{\text{Maximum length of the path of approach}}{\cos\phi} = \frac{r\sin\phi}{\cos\phi} = r\tan\phi$$

circular pitch, $$p_c = \pi m = \frac{2\pi r}{t}$$

Since the arc of approach is not to be less than the circular pitch, therefore

$$r\tan\phi = \frac{2\pi r}{t} \quad \text{or} \quad t = \frac{2\pi}{\tan\phi} = \frac{2\pi}{\tan 14.5°} = 24.3 \text{ say } 25$$

$$T = G.t = 4 \times 25 = 100 \textbf{ Ans.}$$

2. *Addendum of the wheel*

We know that addendum of the wheel

$$= \frac{mT}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2 \phi} - 1\right]$$

$$= \frac{m \times 100}{2}\left[\sqrt{1 + \frac{25}{100}\left(\frac{25}{100} + 2\right)\sin^2 14.5^\circ} - 1\right]$$

$$= 50\,m \times 0.017 = 0.85\ m = 0.85 \times p_c / \pi = 0.27\ p_c$$

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The following data relate to two meshing involute gears: Number of teeth on the gear wheel = 60 ; pressure angle = 20 degrees ; Gear ratio = 1.5; Speed of the gear wheel =100 rpm; Module = 8 mm; The addendum on each wheel is such that the path of approach and the path of recess on each side are 40% of the maximum possible length each. Determine the addendum for the pinion and the gear and the length of the arc of contact.

Solution $R = \frac{mT}{2} = \frac{8 \times 60}{2} = 240$ mm;

$$r = \frac{mT}{2} = \frac{8 \times (60/1.5)}{2} = 160 \text{ mm}$$

Refer Fig. 10.24 and let the pinion be the driver.

Maximum possible length of path of approach = $r \sin \varphi$

Actual length of path of approach = $0.4 \times r \sin \varphi$

Similarly, actual length of path of recess = $0.4 R \sin \varphi$

Thus, we have

$$0.4 r \sin \varphi = \sqrt{R_a^2 - (R \cos \varphi)^2} - R \sin \varphi$$

$$0.4 \times 160 \sin 20° = \sqrt{R_a^2 - (240 \cos 20°)^2} - 240 \sin 20°$$

$$R_a^2 - 50\,862 = 10\,809.8$$

$$R_a^2 = 61671.8$$

$$R_a = 248.3 \text{ mm}$$

Addendum of the wheel = 248.3 – 240 = 8.3 mm

Also, $0.4R \sin \varphi = \sqrt{r_a^2 - (r \cos \varphi)^2} - r \sin \varphi$

$$0.4 \times 240 \sin 20° = \sqrt{r_a^2 - (160 \cos 20°)^2} - 160 \sin 20°$$

or $r_a^2 - 22\,605 = 7666$

or $r_a^2 = 30\,271$

or $r_a = 174$ mm

Addendum of the pinion = 174 – 160 = 14 mm

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UNIT V BIT BANK

1.The two parallel and coplanar shafts are connected by gears having teeth parallel to the axis of the shaft. This arrangement is called (a)

(a) spur gearing (b) helical gearing (c) bevel gearing (d) spiral gearing

2. The type of gears used to connect two non-parallel non-intersecting shafts are (c)

(a) spur gears (b) helical gears (c) spiral gears (d) none of these

3. An imaginary circle which by pure rolling action, gives the same motion as the actual gear, is called (c)

(a) addendum circle (b) dedendum circle

(c) pitch circle (d) clearance circle

4. The size of a gear is usually specified by (d)

(a) pressure angle (b) circular pitch

(c) diametral pitch (d) pitch circle diameter

5. The radial distance of a tooth from the pitch circle to the bottom of the tooth, is called (a)

(a) Dedendum (b) addendum (c) clearance (d) working depth

6. The product of the diametral pitch and circular pitch is equal to (c)

(a) 1 (b) $1/\pi$ (c) π (d) 2π

7. The module is the reciprocal of (a)

(a) diametral pitch (b) circular pitch

(c) pitch diameter (d) none of these

8. Which is the incorrect relationship p of gears? (d)

(a) Circular pitch × Diametral pitch = π

(b) Module = P.C.D/No.of teeth

(c) Dedendum = 1.157 module (d) Addendum = 2.157 module

9. If the module of a gear be m, the number of teeth T and pitch circle diameter D, then (a)

(a) $m = D/T$ (b) $D = T/m$ (c) $m = D/2T$ (d) none of these

10. Mitre gears are used for (b)

(a) great speed reduction (b) equal speed

(c) Minimum axial thrust (d) minimum backlash

11.The condition of correct gearing is (c)

(a) pitch line velocities of teeth be same

(b) radius of curvature of two profiles be same

(c) common normal to the pitch surface cuts the line of centres at a fixed point

(d) none of the above

12. Law of gearing is satisfied if (b)

(a) two surfaces slide smoothly

(b) common normal at the point of contact passes through the pitch point on the line joining the centres of rotation

(c) number of teeth = P.C.D. / module

(d) addendum is greater than dedendum

13. Involute profile is preferred to cyloidal because (b)

(a) the profile is easy to cut

(b) only one curve is required to cut

(c) the rack has straight line profile and hence can be cut accurately

(d) none of the above

14. The contact ratio for gears is (c)

(a) zero (b) less than one (c) greater than one

15. The maximum length of arc of contact for two mating gears, in order to avoid interference, is (c)

(a) (r + R) sin ϕ (b) (r + R) cos ϕ

(c) (r + R) tan ϕ (d) none of these

where r = Pitch circle radius of pinion,

R = Pitch circle radius of driver, and ϕ = Pressure angle.

16. When the addenda on pinion and wheel is such that the path of approach and path of recess are half of their maximum possible values, then the length of the path of contact is given by (a)

(a) $\frac{(r + R)\sin\phi}{2}$ (b) $\frac{(r + R)\cos\phi}{2}$

(c) $\frac{(r + R)\tan\phi}{2}$ (d) none of these

17.Interference can be avoided in involute gears with 20° pressure angle by (c)

(a) cutting involute correctly

(b) using as small number of teeth as possible

(c) using more than 20 teeth(d) using more than 8 teeth

18. The ratio of face width to transverse pitch of a helical gear with α as the helix angle is normally (a)

(a) more than 1.15/tan α (b) more than 1.05/tan α

(c) more than 1/tan α (d) none of these

19. For a speed ratio of 100, smallest gear box is obtained by using (d)

(a) a pair of spur gears

(b) a pair of helical and a pair of spur gear compounded

(c) a pair of bevel and a pair of spur gear compounded

(d) a pair of helical and a pair of worm gear compounded

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20. The maximum efficiency for spiral gears is (c)

(a) $\dfrac{\sin(\theta + \phi) + 1}{\cos(\theta - \phi) + 1}$

(b) $\dfrac{\cos(\theta - \phi) + 1}{\sin(\theta + \phi) + 1}$

(c) $\dfrac{\cos(\theta + \phi) + 1}{\cos(\theta - \phi) + 1}$

(d) $\dfrac{\cos(\theta - \phi) + 1}{\cos(\theta + \phi) + 1}$

where θ = Shaft angle, and ϕ = Friction angle

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1.Tooth interference in an external involute spur gear pair can be reduced by (D) **GATE-ME-2010**

(A) Decreasing centre distance between gear pair(B) Decreasing module

(C) Decreasing pressure angle

(D) Increasing number of gear teeth

2. Data for two crossed helical gears used for speed reduction: **GATE-ME-2012**

Gear I: pitch circle diameter in the plane of rotation 80 mm and helix angle 30

Gear II: pitch circle diameter in the plane of rotation 120 mm and helix angle 22.5

If the input speed is 1440 rpm. The output speed in rpm is (B)

(A) 1200 (B) 900 (C) 875 (D) 720

Gear 1 : $Ø80, \alpha_1 = 30$

Gear 2 : $Ø120, \alpha_2 = 22.5$

$d_1^1 = d_1 \cos \alpha_1 = 80 \times \cos 30° = 40\sqrt{3}\ mm$

$d_2^1 = d_2 \cos \alpha_2 = 120 \times \cos 22.5° = 110.87\ mm$

$$\frac{N_1}{N_2} = \frac{d_2^1}{d_1^1}$$

$$\frac{1440}{N_2} = \frac{110.87}{40\sqrt{3}}$$

$$\therefore\ N_2 = 900\ rpm$$

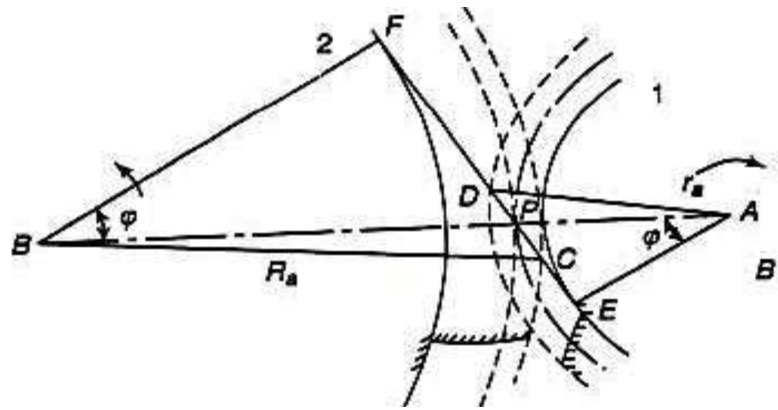
References:

1. Theory of Machines, Rattan, Tata McGraw-Hill Education, 2009.

2. Theory of Machines, R S Kurmi, Eurasia Publishing House, 2005

Path of contact

The pinion 1 is the driver and is rotating clockwise. The wheel 2 is driven in the counter-clockwise direction. *EF* is their common tangent to the base circles. Contact of the two teeth is made where the Addendum circle of the wheel meets the line of action EF, i.e., at *C* and is broken where the addendum circle of the pinion meets the line of action, i.e., at *D*.*CD is then the path of contact.*

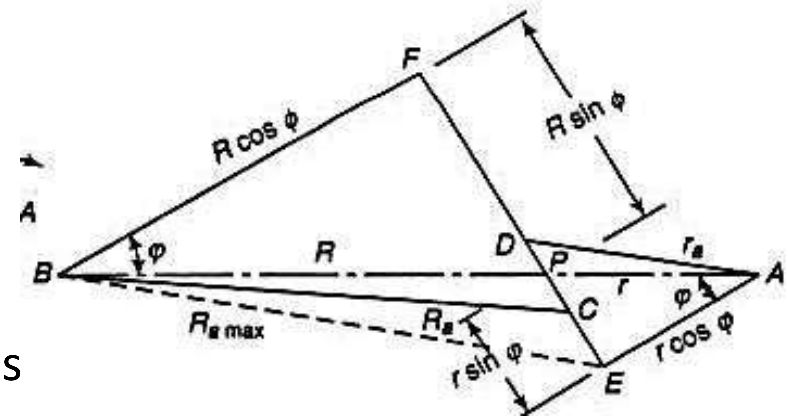


Let *r = pitch circle radius of pinion*

R = pitch circle radius of wheel

ra = addendum circle radius of pinion

Ra = addendum circle radius of wheel.



Path of contact = path of approach + path of recess

CD = CP + PD= (CF-PF)+(DE-PE)

$$= \left(\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi\right) + \left(\sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi\right)$$

$$= \left(\sqrt{R_a^2 - R^2\cos^2\phi}\right) + \left(\sqrt{r_a^2 - r^2\cos^2\phi} + (R + r)\sin\phi\right)$$

.

It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

Observe that the path of approach can be found if the dimensions of the driven wheel are known. Similarly, the path of recess is known from the dimensions of the driving wheel

F

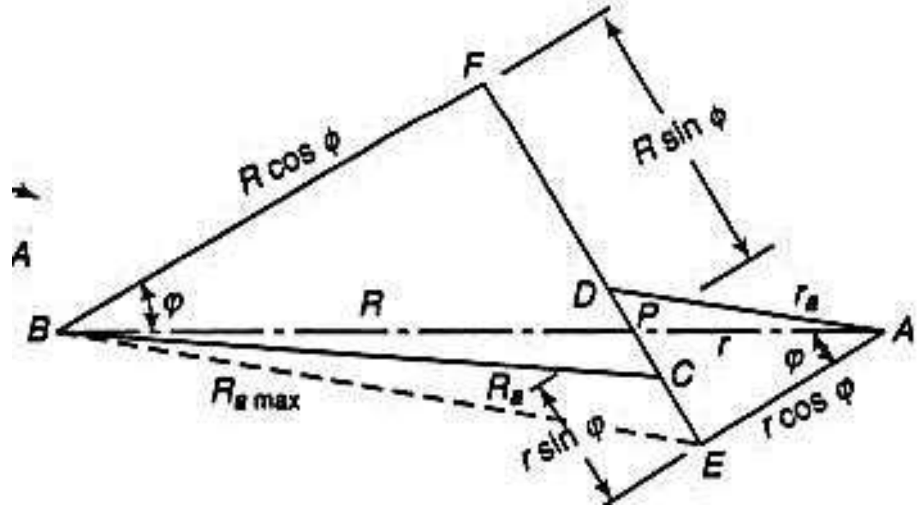
R Cosφ

$\sqrt{R_a^2 - R^2\cos^2\phi}$

B

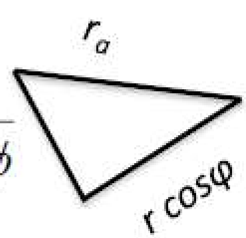
R_a

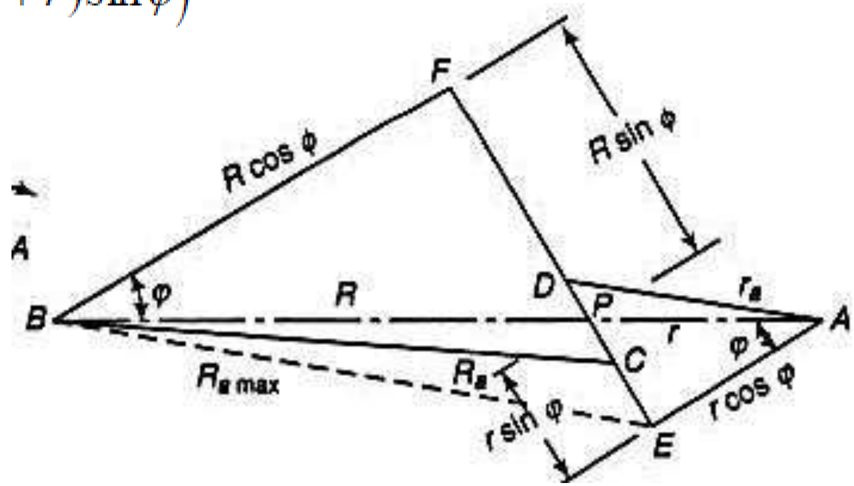
C



$$CD = CP + PD$$
$$= (CF\text{-}PF) + (DE\text{-}PE)$$
$$= \left(\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi\right) + \left(\sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi\right)$$
$$= \left(\sqrt{R_a^2 - R^2\cos^2\phi}\right) + \left(\sqrt{r_a^2 - r^2\cos^2\phi} + (R+r)\sin\phi\right)$$

$\sqrt{r_a^2 - r^2\cos^2\phi}$





Arc of contact

The arc of contact is the distance travelled by a point on either pitch circle of the two wheels during the period of contact of a pair of teeth.

- In Fig., at the beginning of engagement, the driving involute is shown as *GH; when the point of contact is at P it is shown as JK and when at* the end of engagement, it is *DL.*
- The arc of contact is *P'P" and it consists of the arc of approach P'P and* the arc of recess *PP".*

Let the time to traverse the arc of approach is *ta.*

Then, Arc of approach = *P'P = Tangential velocity of P' x Time of approach*

$$= \omega_a r \times t_a$$

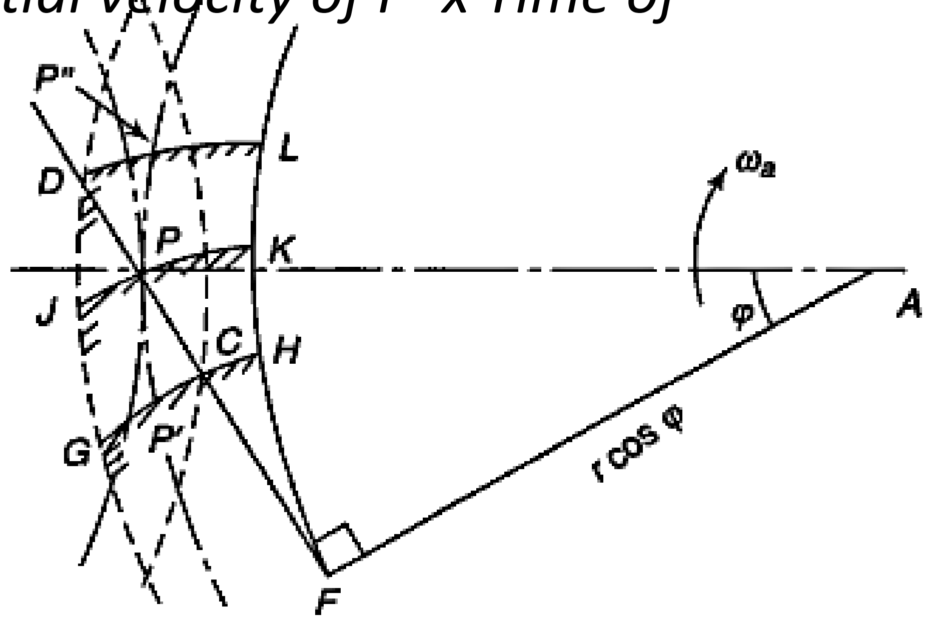
$$= \omega_a (r\cos\varphi)\frac{1}{\cos\varphi} t_a$$

$$= (\text{Tang. vel. of } H) t_a \frac{1}{\cos\varphi}$$

$$= \frac{\text{Arc } HK}{\cos\varphi}$$

$$= \frac{\text{Arc } FK - \text{Arc } FH}{\cos\varphi}$$

$$= \frac{FP - FC}{\cos\varphi} = \frac{CP}{\cos\varphi}$$



Arc of contact

Arc *FK is equal to the path FP as the point P is on the generator FP that* rolls on the base circle *FHK to generate the involute PK.*

Similarly, arc *FH = Path FC.*

Arc of recess = *PP" = Tang. vel. of P x Time of recess*

$$= \omega_a r \times t_r$$

$$= \omega_a (r\cos\varphi)\frac{1}{\cos\varphi}t_r$$

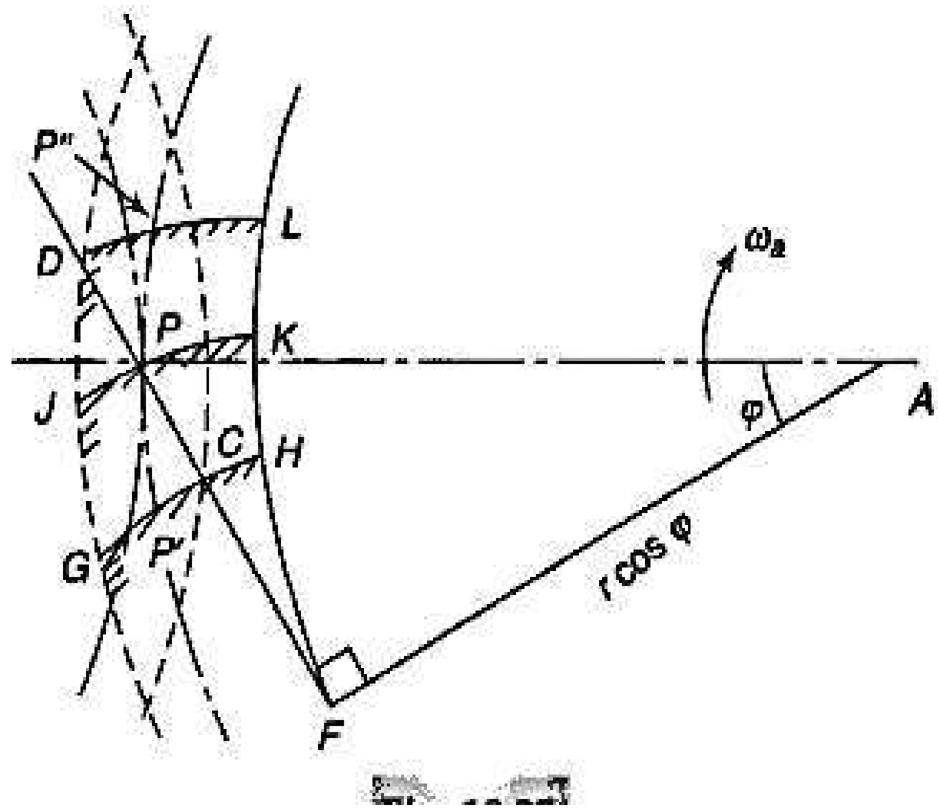
$$= (\text{Tang. vel. of } K)\, t_r \frac{1}{\cos\varphi}$$

$$= \frac{\text{Arc } KL}{\cos\varphi} = \frac{\text{Arc } FL - \text{Arc } FK}{\cos\varphi}$$

$$PP'' = \frac{FD - FP}{\cos\varphi} = \frac{PD}{\cos\varphi}$$

$$\text{Arc of contact} = \frac{CP}{\cos\varphi} + \frac{PD}{\cos\varphi} = \frac{CP + PD}{\cos\varphi} = \frac{CD}{\cos\varphi}$$

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\varphi}$$



UNIT V
GEAR TRAINS

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Gear Trains:

Introduction – Types – Simple – compound and reverted gear trains – Epicyclic gear train.

Methods of finding train value or velocity ratio of Epicyclic gear trains. Torques in epicyclic gear trains

Introduction

- A gear train is a combination of gears used to transmit motion from one shaft to another. It becomes necessary when it is required to obtain large speed reduction with in a small space .

- The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.

Module: It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m.

Mathematically, Module, $m = D/T$

Pitch point: It is a common point of contact between two pitch circles. A point at which the speed of the driver and driven are equal

- The gears are represented by their pitch circles. Module is constant for all the gear trains

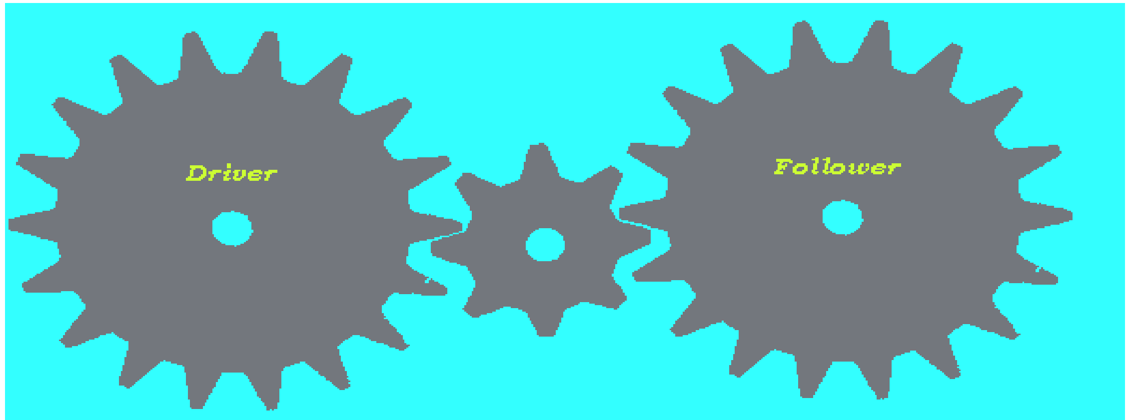
Types of Gear Trains

Following are the different types of gear trains, depending upon the arrangement of wheels :

1. Simple gear train
2. Compound gear train
3. Reverted gear train
4. Epicyclic gear train

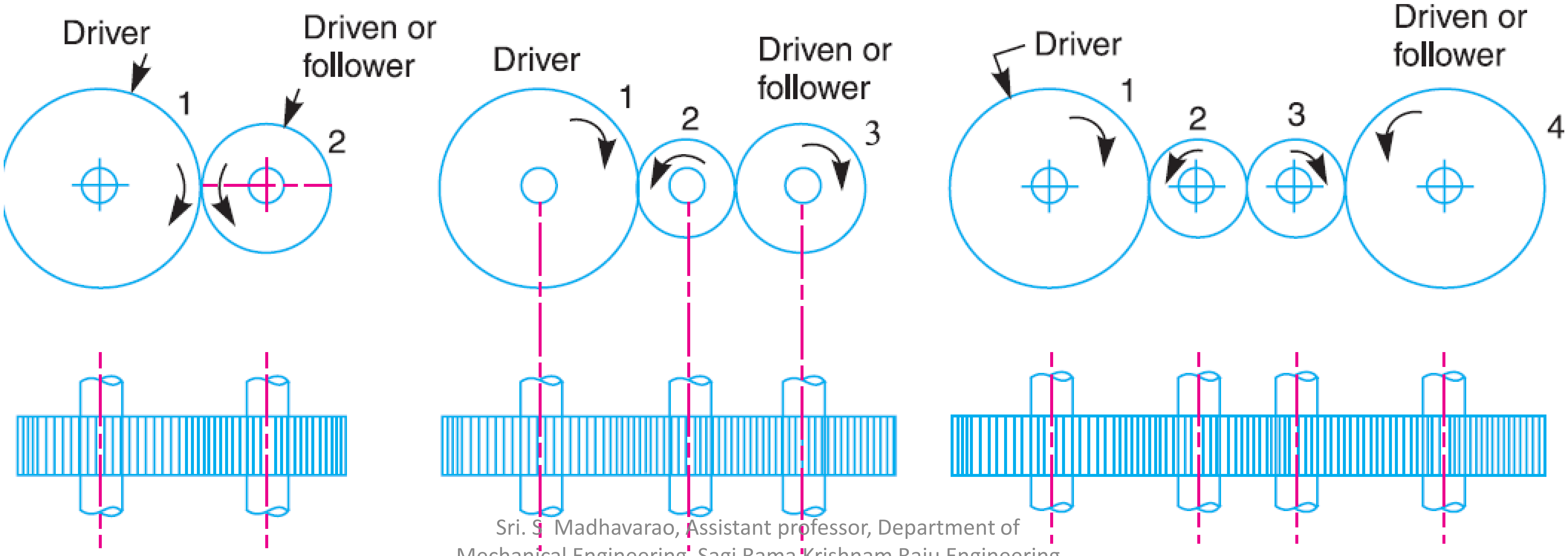
In the first three types of gear trains, the axes of rotation of the wheels are fixed in position and the gears rotates about their respective axes

• But in case of epicyclic gear trains, the axes of some of wheels are not fixed but rotate around the axes of other wheels with which they mesh. Epicyclic gear trains are useful to transmit very high velocity ratios with gears of smaller sizes in a lesser space



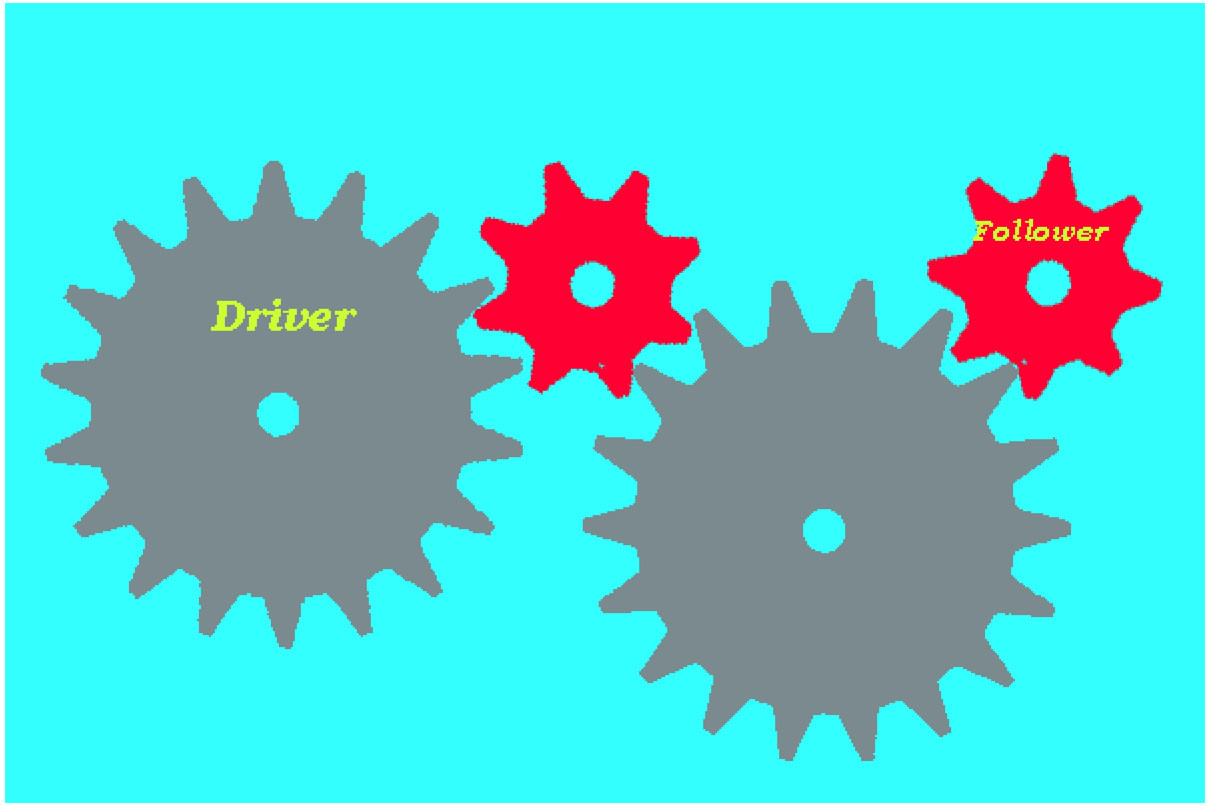
Simple Gear Train

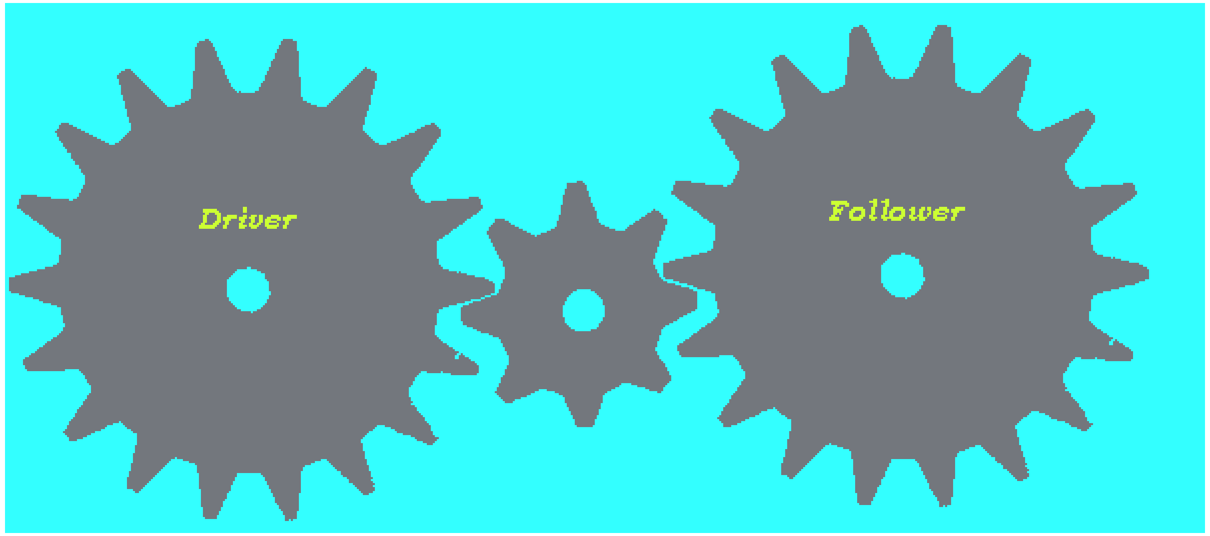
- When there is only one gear on each shaft, as shown in Fig., it is known as simple gear train. The gears are represented by their pitch circles.
- When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Fig. (a). Since the gear 1 drives the gear 2, therefore gear 1 is called the driver and the gear 2 is called the driven or follower.



(a) (b) (c)

- It may be noted that the motion of the driven gear is opposite to the motion of driving gear.





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N_1 = Speed of gear 1(or driver) in r.p.m.,

N_2 = Speed of gear 2 (or driven or follower) in r.p.m.,

T_1 = Number of teeth on gear 1, and

T_2 = Number of teeth on gear 2.

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as ***train value*** of the gear train. Mathematically,

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

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From above, we see that the train value is the reciprocal of speed ratio.

- Sometimes, the distance between the two gears is large. The motion from one gear to another, in such a case, may be transmitted by either of the following two methods :

  1. By providing the large sized gear, or 2. By providing one or more intermediate gears.

- A little consideration will show that the former method (i.e. providing large sized gears) is very inconvenient and uneconomical method ; whereas the latter method (i.e. providing one or more intermediate gear) is very convenient and economical.
- It may be noted that when the number of intermediate gears are odd, the motion of both the gears (i.e. driver and driven or follower) is like as shown in Fig. (b).
- But if the number of intermediate gears are even, the motion of the driven or follower will be in the opposite direction of the driver as shown in Fig. (c).
- Now consider a simple train of gears with one intermediate gear as shown in Fig. (b).

Let N1 = Speed of driver in r.p.m.,

N 2 = Speed of intermediate gear in r.p.m.,

N3 = Speed of driven or follower in r.p.m.,
T1 = Number of teeth on driver,
T2 = Number of teeth on intermediate gear, and
T3 = Number of teeth on driven or follower.

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Since the driving gear 1 is in mesh with the intermediate gear 2, therefore speed ratio for these two gears is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \text{...}(i)$$

Similarly, as the intermediate gear 2 is in mesh with the driven gear 3, therefore speed ratio for these two gears is

$$\frac{N_2}{N_3} = \frac{T_3}{T_2} \quad \text{...}(ii)$$

The speed ratio of the gear train as shown in Fig. 13.1 (*b*) is obtained by multiplying the equations **(*i*)** and **(*ii*)**.

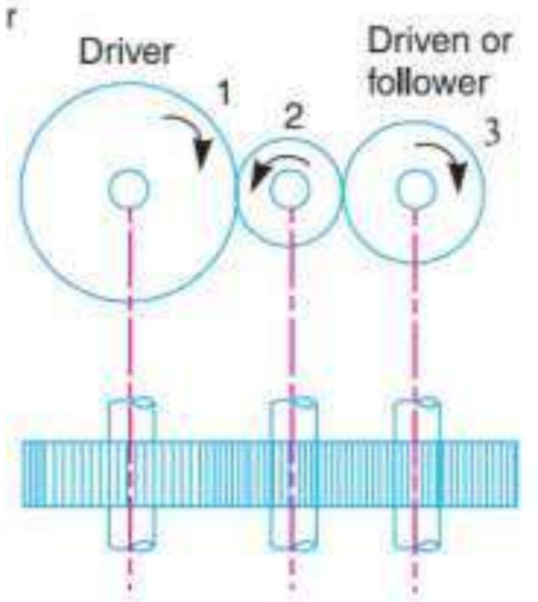
$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2} \quad \text{or} \quad \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

i.e.

$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

and

$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$



(*b*)

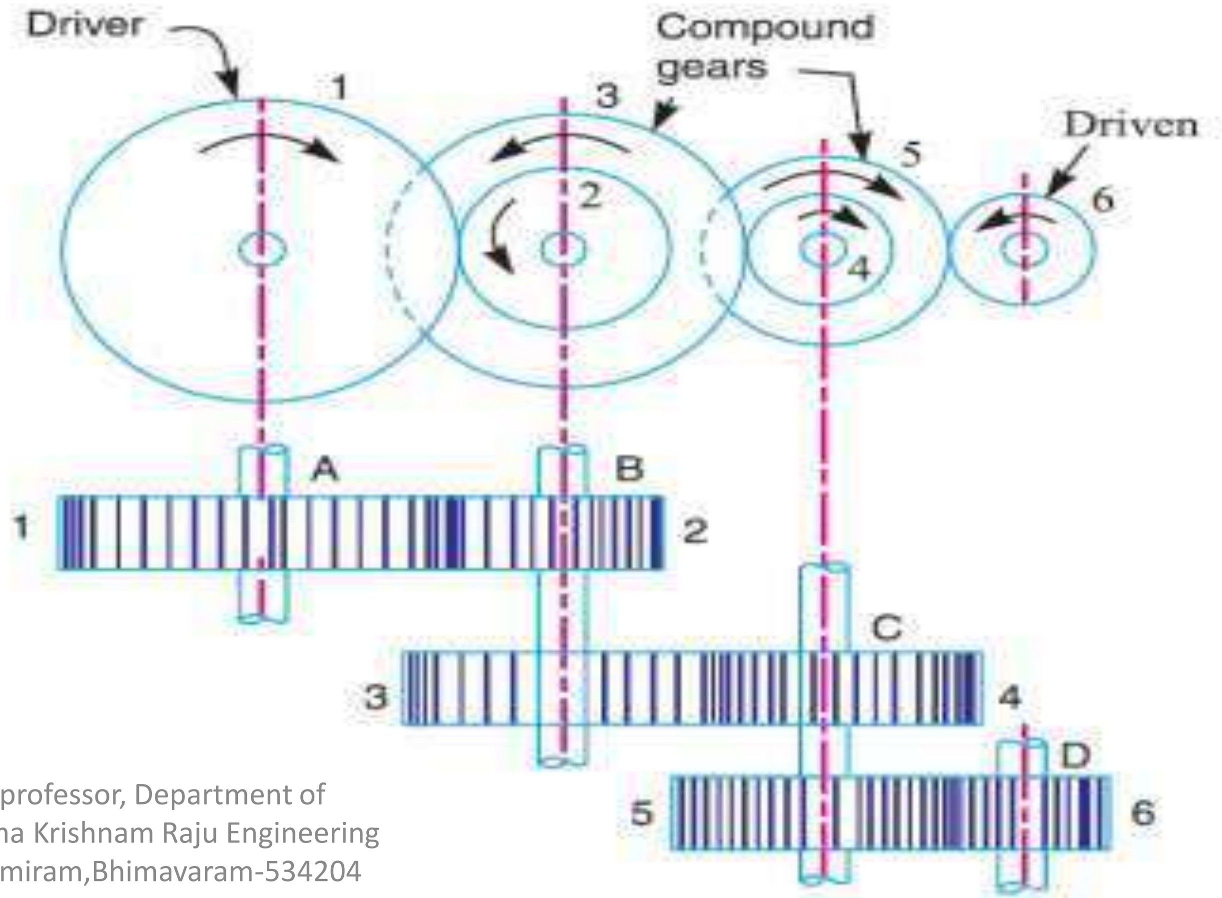
- Similarly, it can be proved that the above equation holds good even if there are any number of intermediate gears. From above, we see that the speed ratio and the train value, in a simple train of gears, is independent of the size and number of intermediate gears. These intermediate gears are called idle gears, as they do not effect the speed ratio or train value of the system. The idle gears are used for the following two purposes :

1. To connect gears where a large centre distance is required, and

2. To obtain the desired direction of motion of the driven gear (i.e. clockwise or anticlockwise).

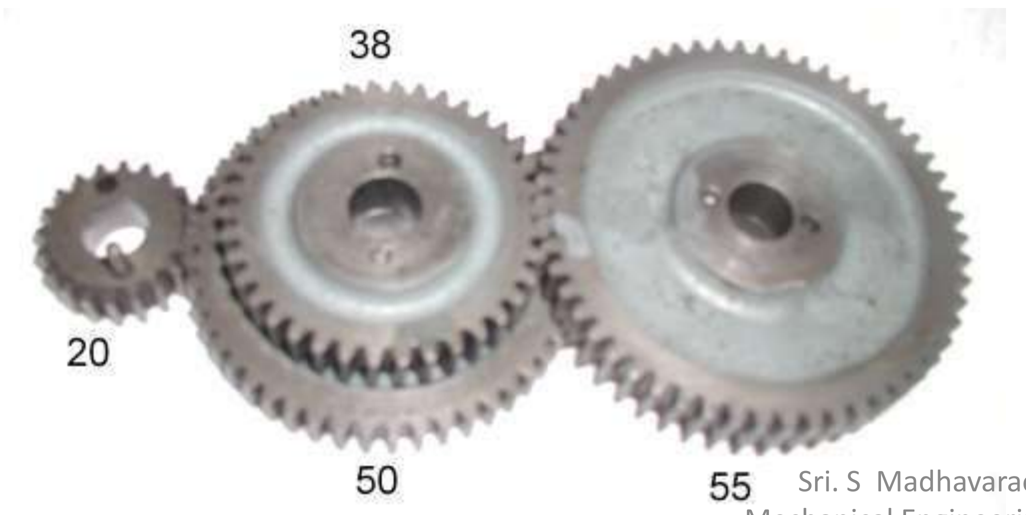
Compound Gear Train

- When there are more than one gear on a shaft, as shown in Fig. , it is called a compound train of gear.
- We have seen that the idle gears, in a simple train of gears do not effect the speed ratio of the system. But these gears are useful in bridging over the space between the driver and the driven



Compound Gear Train:

- But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is increased by providing compound gears on intermediate shafts.
- In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed.
- One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft as shown in Fig.



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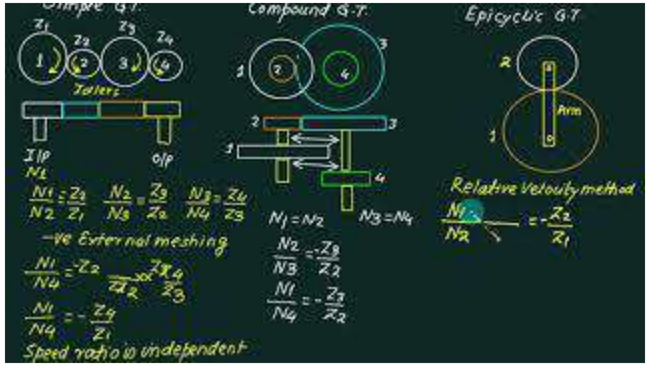


Gear Train 01

In a compound train of gears, as shown in Fig. 13.2, the gear 1 is the driving gear mounted on shaft A, gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let N_1 = Speed of driving gear 1,

T_1 = Number of teeth on driving gear 1,

$N_2, N_3 ..., N_6$ = Speed of respective gears in r.p.m., and

$T_2, T_3..., T_6$ = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

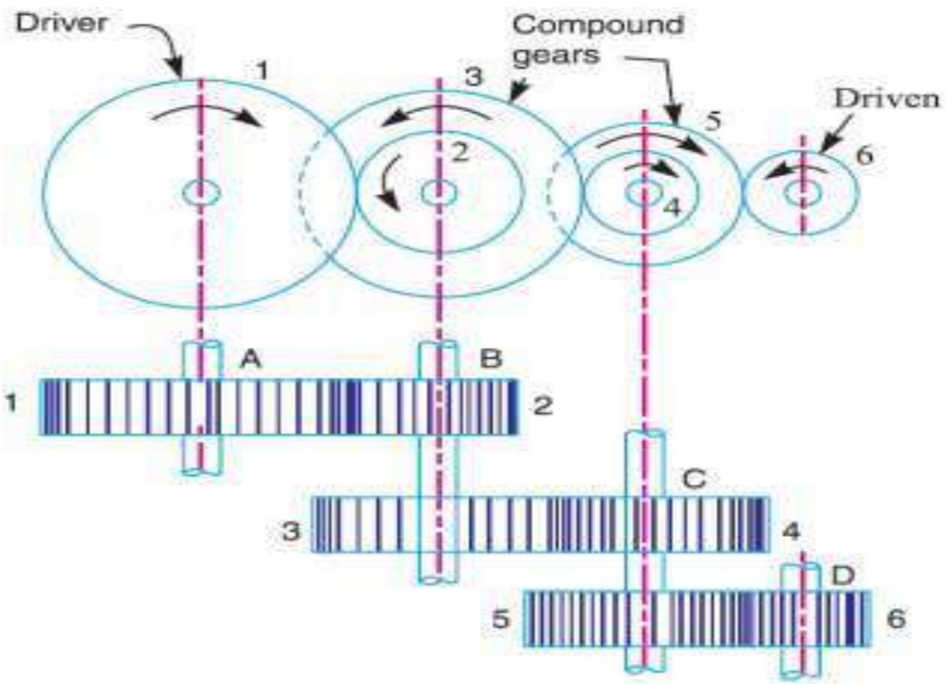
$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad ...(i)$$

Similarly, for gears 3 and 4, speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad ...(ii)$$

and for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \quad ...(iii)$$



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The speed ratio of compound gear train is obtained by multiplying the equations (*i*), (*ii*) and (*iii*),

$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5} \quad \text{or} \quad {}^{*}\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

Since gears 2 and 3 are mounted on one shaft *B*, therefore $N_2 = N_3$. Similarly gears 4 and 5 are mounted on shaft *C*, therefore $N_4 = N_5$.

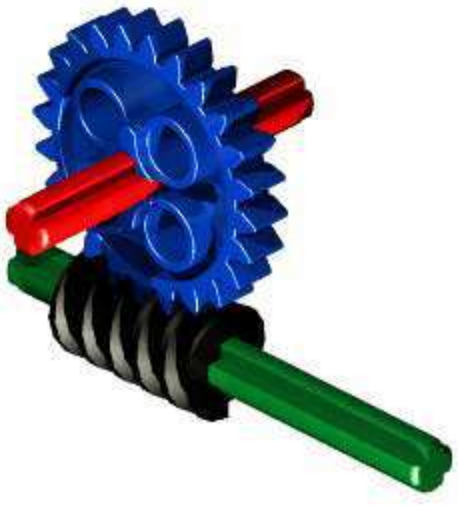
$$\text{Speed ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}}$$

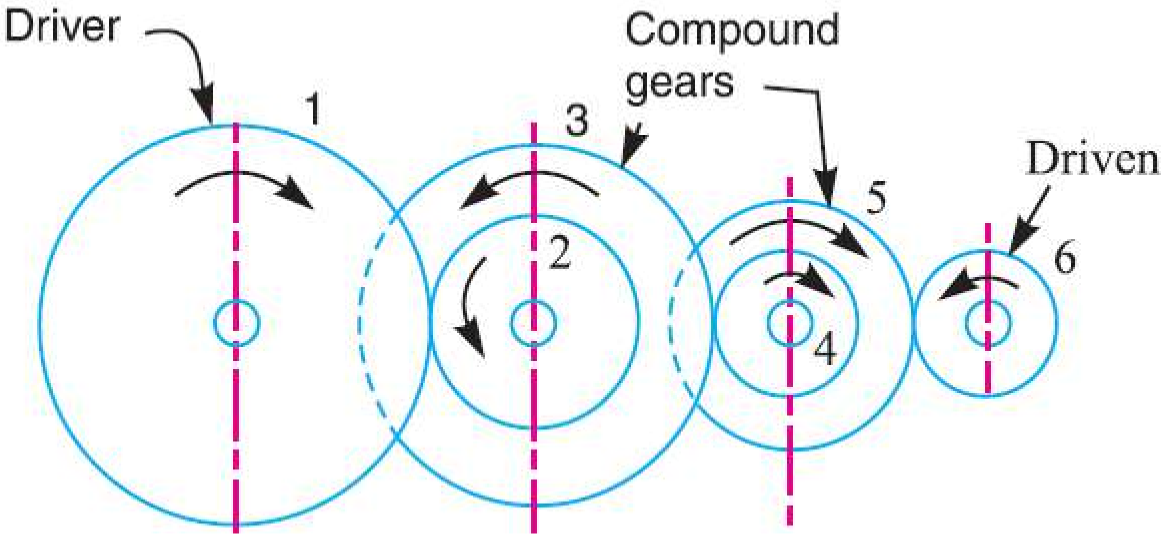
$$= \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

$$\text{Train value} = \frac{\text{Speed of the last driven or follower}}{\text{Speed of the first driver}}$$

$$= \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the drivens}}$$

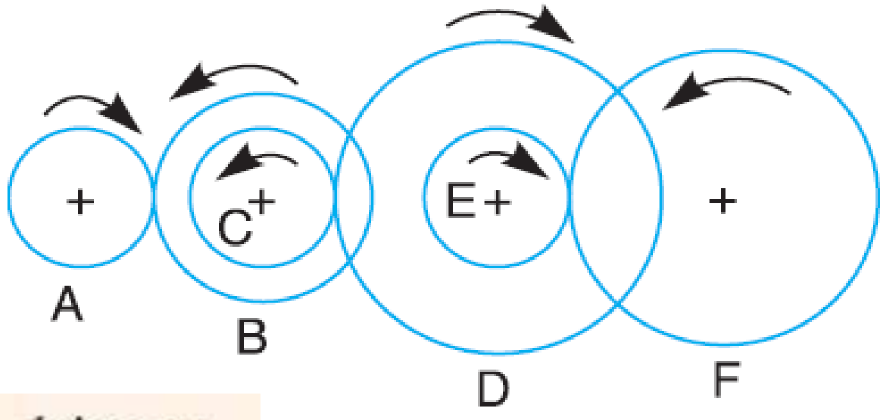
- The advantage of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears. If a simple gear train is used to give a large speed reduction, the last gear has to be very large. Usually for a speed reduction in excess of 7 to 1, a simple train is not used and a compound train or worm gearing is employed





Q1. The gearing of a machine tool is shown in Fig. 13.3. The motor shaft is connected to gear A and rotates at 975 r.p.m. The gear wheels B, C, D and E are fixed to parallel shafts rotating together. The final gear F is fixed on the output shaft. What is the speed of gear F ? The number of teeth on each gear are as given below :

Gear	A	B	C	D	E	F
No. of teeth	20	50	25	75	26	65



$$\frac{\text{Speed of the first driver}}{\text{Speed of the last driven}} = \frac{\text{Product of no. of teeth on drivens}}{\text{Product of no. of teeth on drivers}}$$

$$\frac{N_A}{N_F} = \frac{T_B \times T_D \times T_F}{T_A \times T_C \times T_E} = \frac{50 \times 75 \times 65}{20 \times 25 \times 26} = 18.75$$

$$N_F = \frac{N_A}{18.75} = \frac{975}{18.75} = 52 \text{ r. p. m.}$$ **Ans.**

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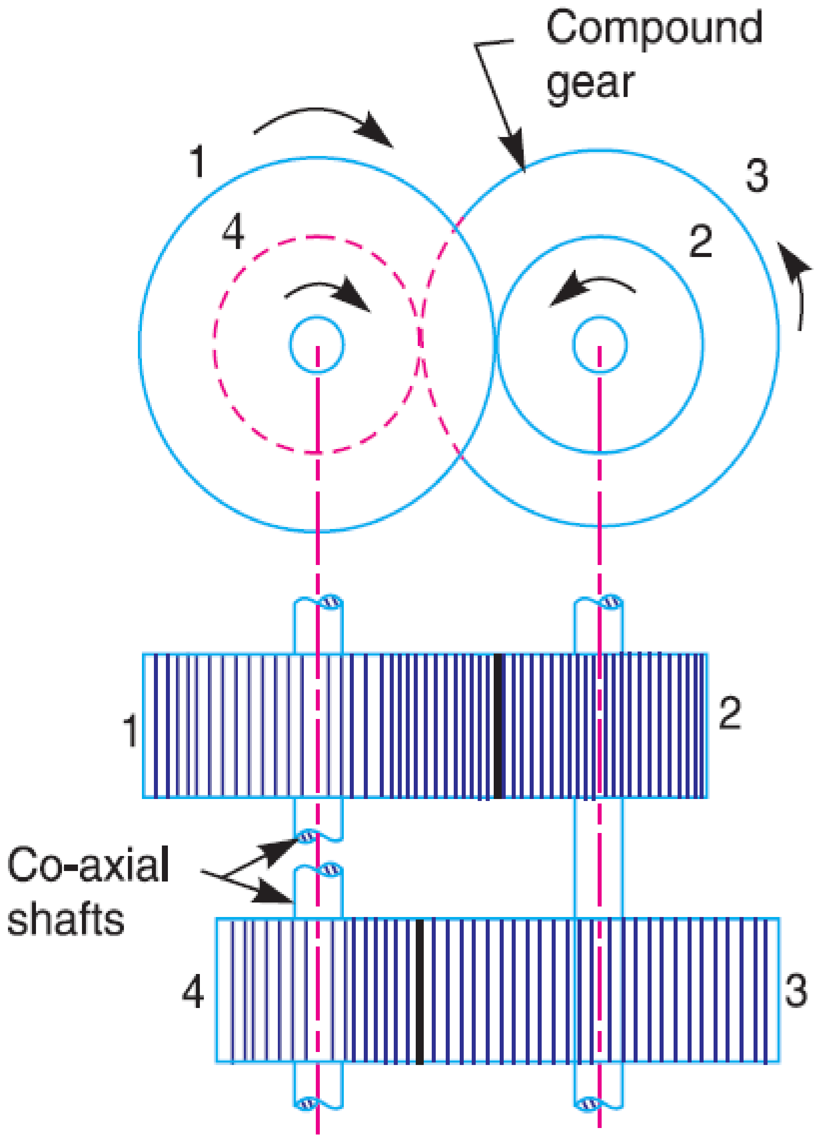
Reverted Gear Train :

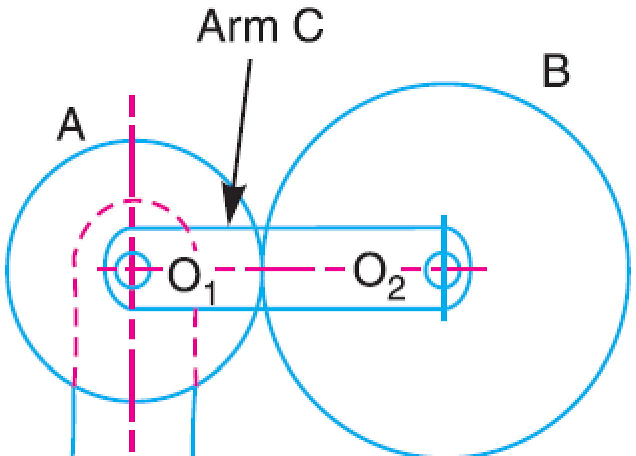
- When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as **reverted gear train as** shown in Fig.

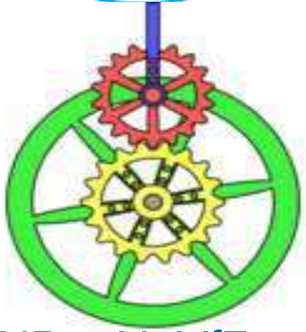


- We see that gear 1 (i.e. first driver) drives the gear 2 (i.e. first driven or follower) in the opposite direction.
- Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2.
- The gear 3 (which is now the second driver) drives the gear 4 (i.e. the last driven or follower) in the same Direction as that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is **like** (i.e same direction)

Planetary or Epicyclic Gear Train

- In an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis.
- A simple epicyclic gear train is shown in Fig., where a gear A and the arm C have a common axis at O1 About which they can rotate. The gear B meshes with gear A and has its axis on the arm at O2, about which the gear B can rotate. If the arm is fixed, the gear train is simple and gear A can drive gear B or vice versa,





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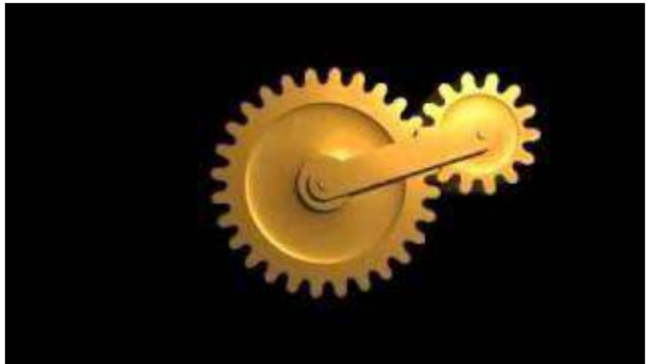

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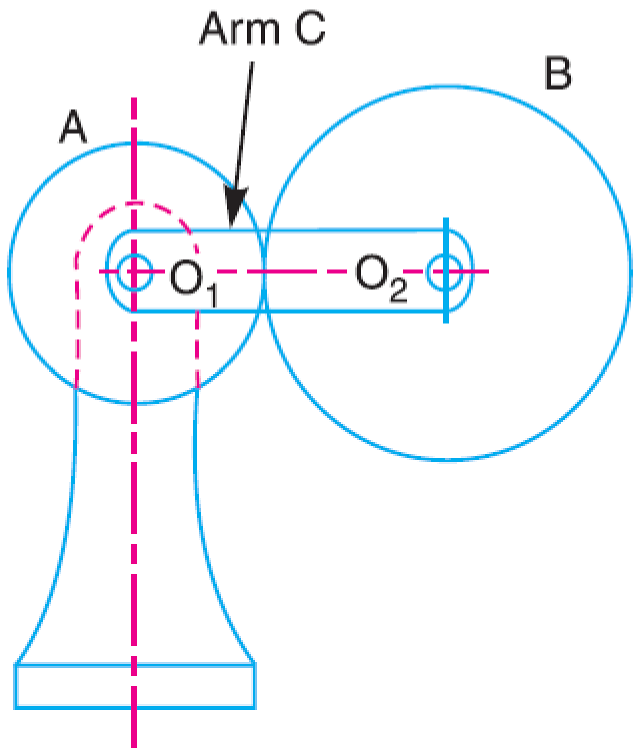
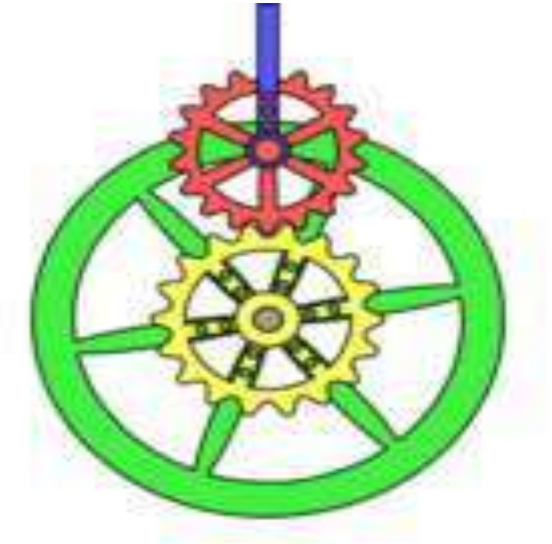
Planetary or Epicyclic Gear Train



• but if gear A is fixed and the arm is rotated about the axis of gear A (i.e. O1), then the gear B is forced to Rotate upon and around gear A Such a motion is called epicyclic and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as epicyclic gear trains (epi. Means upon and cyclic means around).

• The epicyclic gear trains may be simple or compound. The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space.

• The epicyclic gear trains are used in the back gear of lathe, Differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.



Algebraic method

Let the arm C be fixed in an epicyclic gear train as shown in Fig. 13.6. Therefore speed of the gear A relative to the arm C

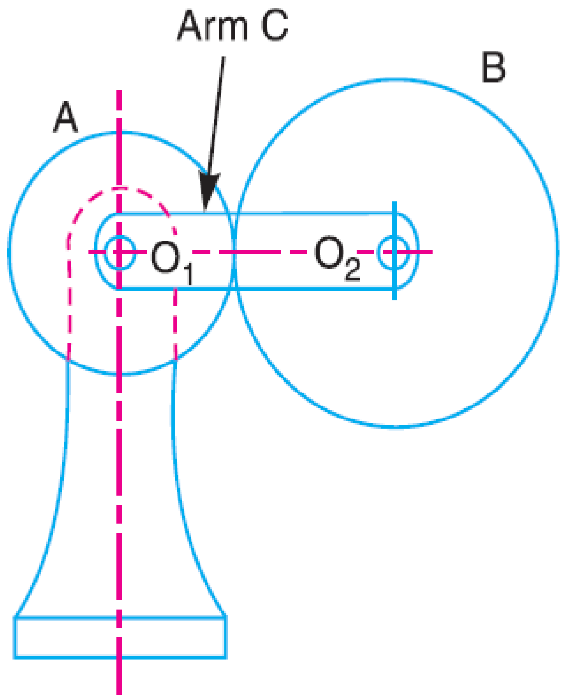
$$= N_A - N_C$$

and speed of the gear B relative to the arm C,

$$= N_B - N_C$$

Since the gears A and B are meshing directly, therefore they will revolve in ***opposite*** directions.

$$\therefore \quad \frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$



Since the arm C is fixed, therefore its speed, $N_C = 0$.

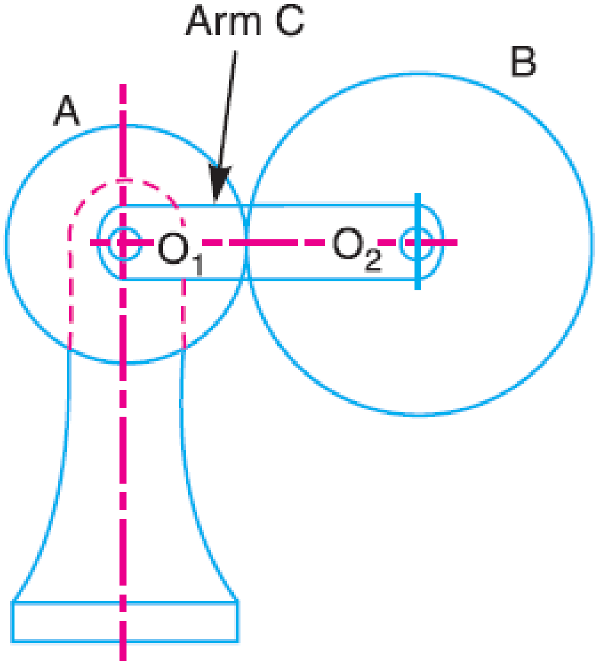
$$\therefore \quad \frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

If the gear A is fixed, then $N_A = 0$.

$$\frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B} \quad \text{or} \quad \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}$$

Tabular method

Consider an epicyclic gear train as shown in Fig. Let T_A = Number of teeth on gear A, and T_B = Number of teeth on gear B.



- First of all, let us suppose that the arm is fixed. Therefore the axes of both the gears are also fixed relative to each other. When the gear A makes one revolution anticlockwise(+ve), the gear B will make

T A / TB revolutions, clockwise.

We know that $N_B / N_A = T_A / T_B$. Since $N_A = 1$ revolution, therefore $N_B = T_A / T_B$.

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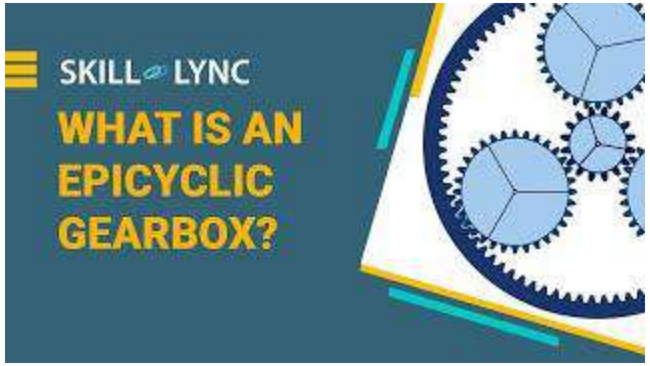


What is an Epicyclic Gearbox?

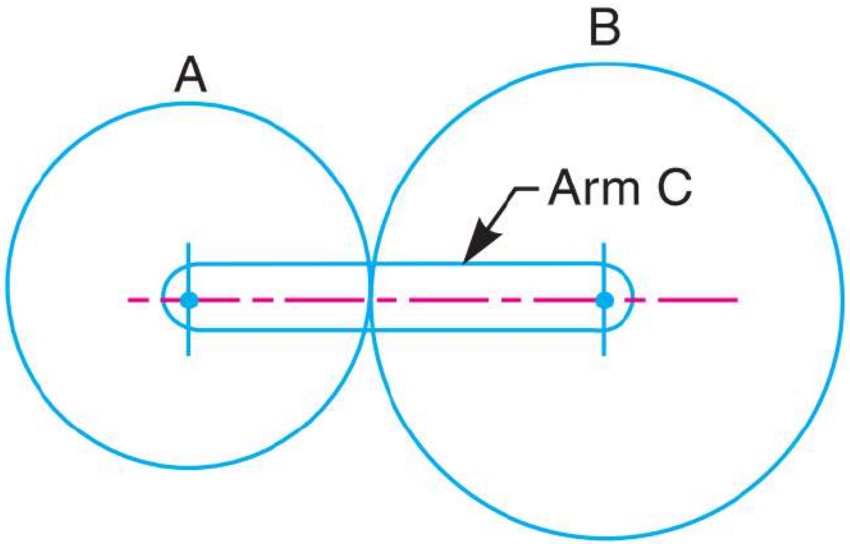
Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear *A* rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions (i.e **Multiplied the 1st row by +x)**	0	+ *x*	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \frac{T_A}{T_B}$

Tabular method.

- when gear A makes + 1 revolution, then the gear B will make (– TA / TB) revolutions. This statement of relative motion is entered in the first row of the table.
- Secondly, if the gear A makes + x revolutions, then the gear B will make – x × TA /TB revolutions. This statement is entered in the second row of the table. In other words, multiply the each motion (entered in the first row) by x.
- Thirdly, each element of an epicyclic train is given + y revolutions and entered in the third row.
- Finally, the motion of each element of the gear train is added up (i.e row2+row3)and entered in the fourth row.

Step No.	*Conditions of motion*	*Revolutions of elements*		
		Arm C	*Gear A*	*Gear B*
1.	Arm fixed-gear A rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through + x revolutions (i.e **Multiplied the 1st row by +x)**	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

Example 13.4. *In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed, makes 300 r.p.m. in the clockwise direction, what will be the speed of gear B ?*



Solution. Given : $T_A = 36$; $T_B = 45$; $N_C = 150$ r.p.m. (anticlockwise)

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear *A* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions ((i.e **Multiplied the 1st row by +x)**	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x + y$	$y - x \times \frac{T_A}{T_B}$

Speed of gear B when gear A is fixed

Since the speed of arm is 150 r.p.m. anticlockwise, therefore from the fourth row of the table,

$$y = +\ 150 \text{ r.p.m.}$$

Also the gear A is fixed, therefore

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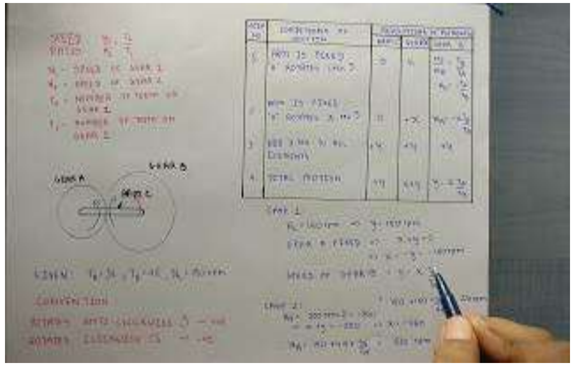
$$x + y = 0 \qquad \text{or} \qquad x = -\,y = -\,150 \text{ r.p.m.}$$



$\therefore$ Speed of gear B, $\quad N_B = y - x \times \dfrac{T_A}{T_B} = 150 + 150 \times \dfrac{36}{45} = +\ 270$ r.p.m.

$= 270$ r.p.m. (anticlockwise) **Ans.**

Speed of gear B when gear A makes 300 r.p.m. clockwise

Since the gear A makes 300 r.p.m.clockwise, therefore from the fourth row of the table,

$$x + y = -\,300 \qquad \text{or} \qquad x = -\,300 - y = -\,300 - 150 = -\,450 \text{ r.p.m.}$$

$\therefore$ Speed of gear B,

$$N_B = y - x \times \frac{T_A}{T_B} = 150 + 450 \times \frac{36}{45} = +\ 510 \text{ r.p.m.}$$

$= 510$ r.p.m. (anticlockwise) **Ans.**

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Q2.An epicyclic gear consists of three gears A, B and C as shown in Fig. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the centre of A at 18 r.p.m.. If the gear A is fixed, determine the speed of gears B and C.

Solution. Given : $T_A = 72$; $T_C = 32$; Speed of arm EF = 18 r.p.m.

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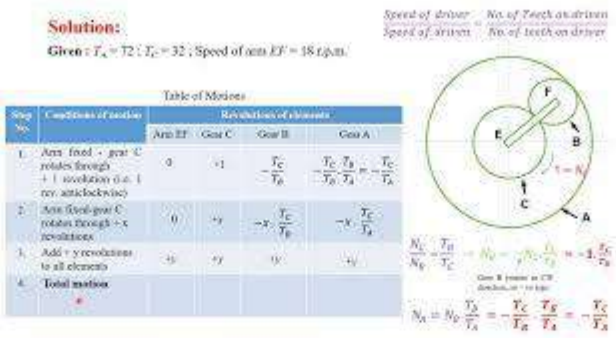

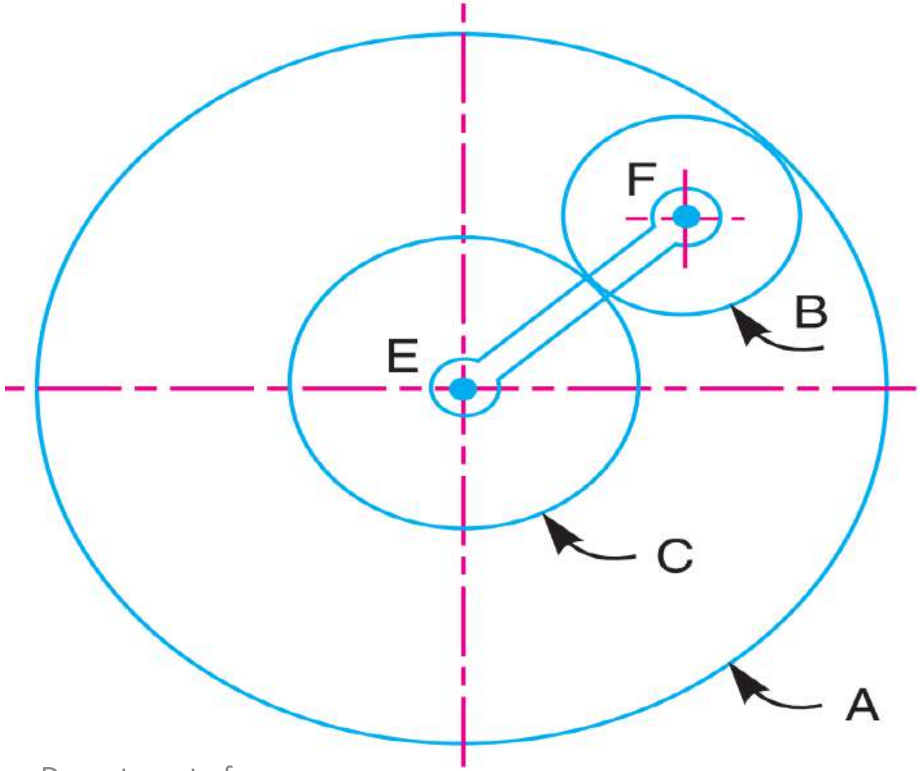

Gear Trains - Epicyclic Gear Problem

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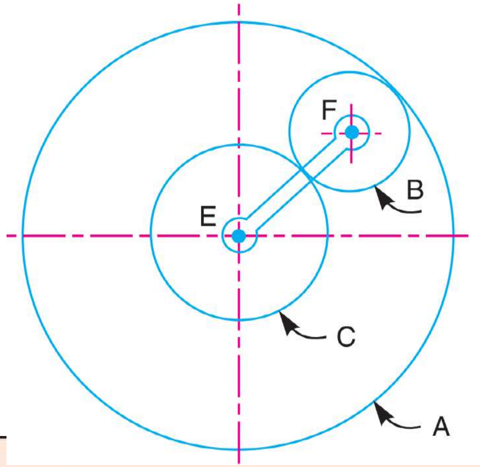


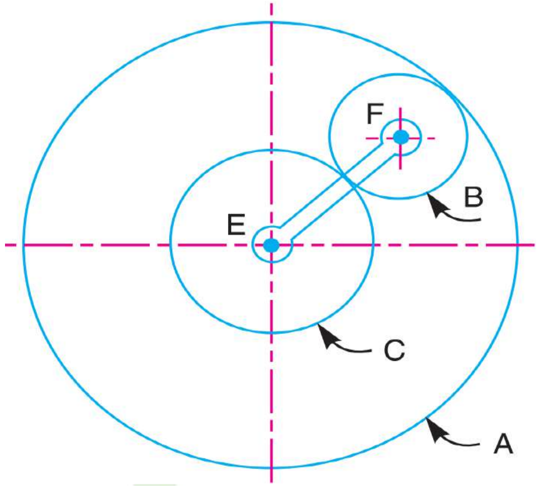
Step No.	Conditions of motion	Revolutions of elements			
		Arm EF	Gear C	Gear B	Gear A
1.	Arm fixed-gear *C* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_B} \times \frac{T_B}{T_A} = -\frac{T_C}{T_A}$
2.	Arm fixed-gear *C* rotates through $+x$ revolutions (i.e **Multiplied the 1st row by +x)**	0	$+x$	$-x \times \frac{T_C}{T_B}$	$-x \times \frac{T_C}{T_A}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_C}{T_B}$	$y - x \times \frac{T_C}{T_A}$

We know that the speed of the arm is 18 r.p.m. therefore

$$y = 18 \text{ r.p.m.}$$

and the gear A is fixed, therefore

$$y - x \times \frac{T_C}{T_A} = 0 \quad \text{or} \quad 18 - x \times \frac{32}{72} = 0$$



$$\therefore \qquad x = 18 \times 72 / 32 = 40.5$$

$$\therefore \text{ Speed of gear } C = x + y = 40.5 + 18$$

$$= +58.5 \text{ r.p.m.}$$

= 58.5 r.p.m. in the direction of arm. **Ans.**

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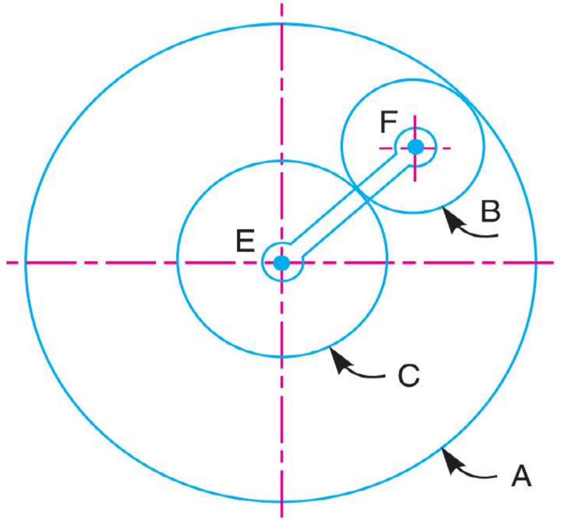
Speed of gear B

Let d_A, d_B and d_C be the pitch circle diameters of gears A, B and C respectively. Therefore, from the geometry of Fig. 13.10,



$$d_B + \frac{d_C}{2} = \frac{d_A}{2} \quad \text{or} \quad 2\,d_B + d_C = d_A$$

Since the number of teeth are proportional to their pitch circle diameters, therefore

$$2\,T_B + T_C = T_A \quad \text{or} \quad 2\,T_B + 32 = 72 \quad \text{or} \quad T_B = 20$$

$\therefore$ Speed of gear B $= y - x \times \frac{T_C}{T_B} = 18 - 40.5 \times \frac{32}{20} = -46.8$ r.p.m.

$= 46.8$ r.p.m. in the opposite direction of arm. **Ans.**

Example 13.7. *An epicyclic train of gears is arranged as shown in Fig.13.11. How many revolutions does the arm, to which the pinions B and C are attached, make :*

1. *when A makes one revolution clockwise and D makes half a revolution anticlockwise, and*

2. *when A makes one revolution clockwise and D is stationary ?*

The number of teeth on the gears A and D are 40 and 90 respectively.

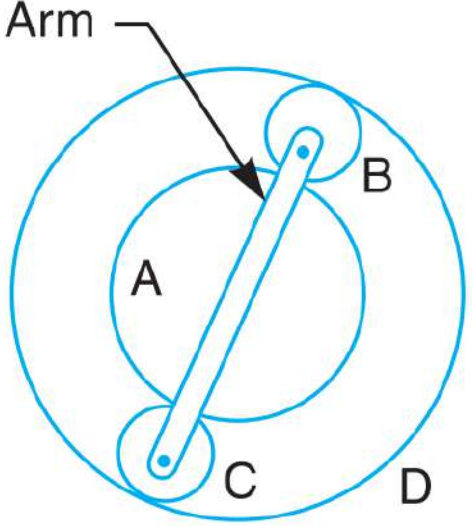


Fig. 13.11

Solution. Given : $T_A = 40$; $T_D = 90$

First of all, let us find the number of teeth on gears *B* and *C* (*i.e.* T_B and T_C). Let d_A, d_B, d_C and d_D be the pitch circle diameters of gears *A* , *B*, *C* and *D* respectively. Therefore from the geometry of the figure,

$$d_A + d_B + d_C = d_D \quad \text{or} \quad d_A + 2\,d_B = d_D \qquad ...(\because\ d_B = d_C)$$

Since the number of teeth are proportional to their pitch circle diameters, therefore,

$$T_A + 2\,T_B = T_D \quad \text{or} \quad 40 + 2\,T_B = 90$$

$$\therefore \quad T_B = 25, \quad \text{and} \quad T_C = 25 \qquad ...(\because\ T_B = T_C)$$

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Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Compound gear B-C	Gear D
1.	Arm fixed, gear *A* rotates through – 1 revolution (*i.e.* 1 rev. clockwise)	0	– 1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_D}=+\frac{T_A}{T_D}$
2.	Arm fixed, gear *A* rotates through – *x* revolutions ((i.e **Multiplied the 1st row by +x)**	0	$-x$	$+x\times\frac{T_A}{T_B}$	$+x\times\frac{T_A}{T_D}$
3.	Add – *y* revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$x\times\frac{T_A}{T_B}-y$	$x\times\frac{T_A}{T_D}-y$

1. *Speed of arm when A makes 1 revolution clockwise and D makes half revolution anticlockwise*

Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x-y=-1 \quad \text{or} \quad x+y=1 \qquad ...(i)$$

Also, the gear D makes half revolution anticlockwise, therefore

$$x\times\frac{T_A}{T_D}-y=\frac{1}{2} \quad \text{or} \quad x\times\frac{40}{90}-y=\frac{1}{2}$$

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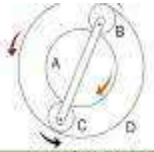
$$\therefore \quad 40x-90y=45 \quad \text{or} \quad x-2.25y=1.125 \qquad ...(ii)$$

From equations (*i*) and (*ii*), $x = 1.04$ and $y = -0.04$

$$\therefore \quad \text{Speed of arm} = -y = -(-0.04) = +0.04$$

$$= 0.04 \text{ revolution anticlockwise}$$ **Ans.**



Step	Conditions of motion	Arm	Gear A	Gear B & C	Gear D
1	Arm fixed, gear A rotates through −1 revolution (i.e. 1 rev. clockwise)	0	-1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_D}=+\frac{T_A}{T_D}$
2	Arm fixed, gear A rotates through −x revolutions	0	$-x$	$+x\times\frac{T_A}{T_B}$	$+x\times\frac{T_A}{T_D}$
3	Add −y revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4	Total motion	$-y$	$-x-y$	$x\times\frac{T_A}{T_B}-y$	

Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Compound gear B-C	Gear D
1.	Arm fixed, gear A rotates through – 1 revolution (*i.e.* 1 rev. clockwise)	0	– 1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_D}=+\frac{T_A}{T_D}$
2.	Arm fixed, gear A rotates through $-x$ revolutions ((i.e **Multiplied the 1st row by +x)**	0	$-x$	$+x\times\frac{T_A}{T_B}$	$+x\times\frac{T_A}{T_D}$
3.	Add $-y$ revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$x\times\frac{T_A}{T_B}-y$	$x\times\frac{T_A}{T_D}-y$

2. *Speed of arm when A makes 1 revolution clockwise and D is stationary*

Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x - y = -1 \quad \text{or} \quad x + y = 1 \qquad \text{...}(iii)$$

Also the gear D is stationary, therefore

$$x \times \frac{T_A}{T_D} - y = 0 \quad \text{or} \quad x \times \frac{40}{90} - y = 0$$

$$\therefore \quad 40\,x - 90\,y = 0 \quad \text{or} \quad x - 2.25\,y = 0 \qquad \text{...}(iv)$$

From equations (*iii*) and (*iv*),

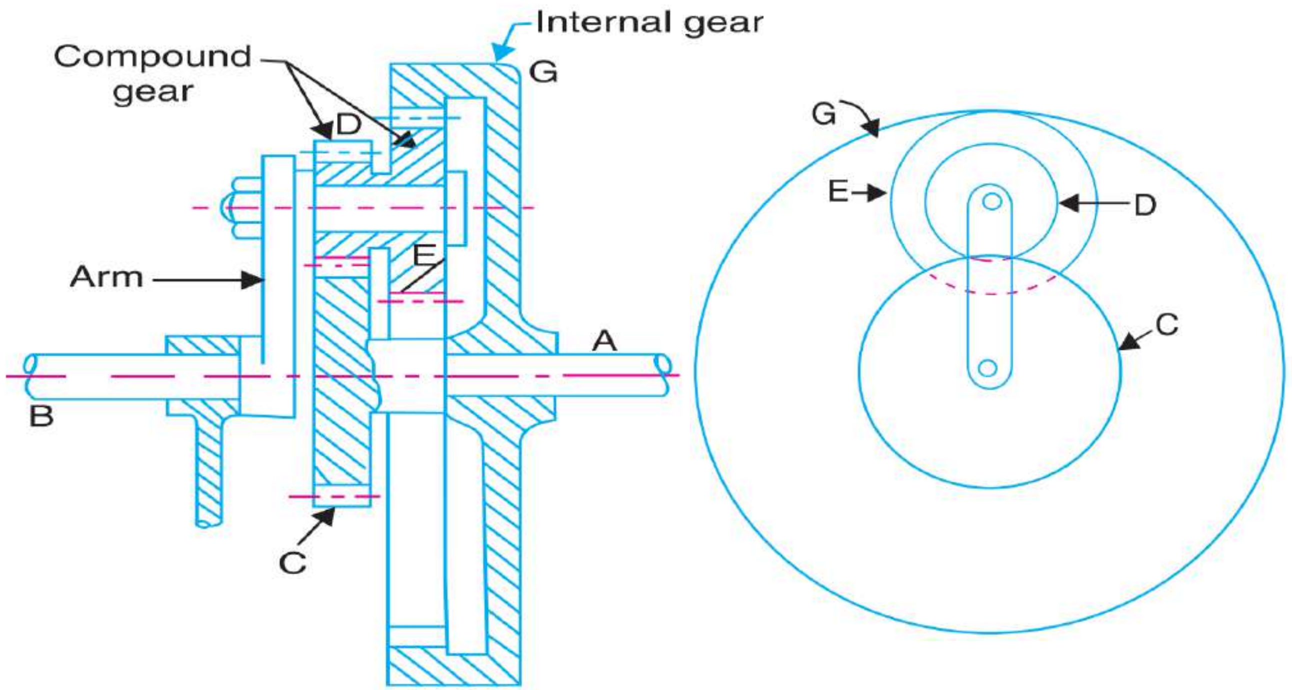
$$x = 0.692 \quad \text{and} \quad y = 0.308$$

$\therefore$ Speed of arm $= -y = -0.308 = 0.308$ revolution clockwise **Ans.**

Q4.Two shafts A and B are co-axial. A gear C (50 teeth) is rigidly mounted on shaft A. A compound gear D-E gears with C and an internal gear G. D has 20 teeth and gears with C and E has 35 teeth and gears with an internal gear G. The gear G is fixed and is concentric with the shaft axis. The compound gear D-E is mounted on a pin which projects from an arm keyed to the shaft B. Sketch the arrangement and find the number of teeth on internal gear G. assuming that all gears have the same module. If the shaft A rotates at 110 r.p.m., find the speed of shaft B.

Solution. Given : $T_C = 50$; $T_D = 20$; $T_E = 35$; $N_A = 110$ r.p.m.

The arrangement is shown in Fig.



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Number of teeth on internal gear G

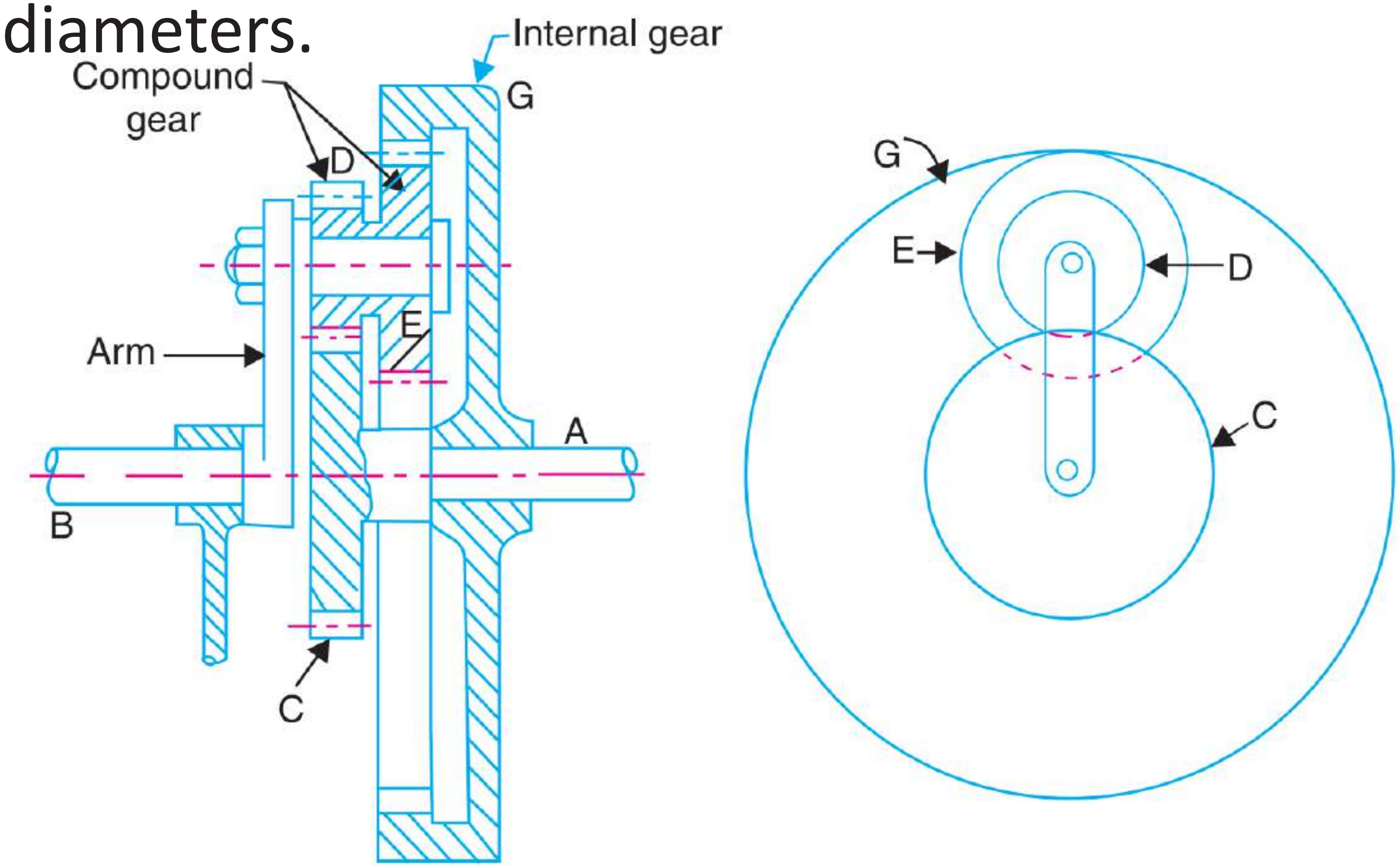
Let d_C, d_D, d_E and d_G be the pitch circle diameters of gears C, D, E and G respectively. From the geometry of the figure,

$$\frac{d_G}{2} = \frac{d_C}{2} + \frac{d_D}{2} + \frac{d_E}{2}$$

$$d_G = d_C + d_D + d_E$$

Let T_C, T_D, T_E and T_G be the number of teeth on gears C, D, E and G respectively. Since all the gears have the same module, therefore number of teeth are proportional to their pitch circle diameters.

$$T_G = T_C + T_D + T_E = 50 + 20 + 35 = 105 \text{ Ans.}$$



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Speed of shaft B : The table of motions is given below :

Step No.	Conditions of motion	Revolutions of elements: Arm	Gear C (or shaft A)	Compound gear D-E	Gear G
1.	Arm fixed - gear C rotates through + 1 revolution	0	+ 1	$-\frac{T_C}{T_D}$	$-\frac{T_C}{T_D}\times\frac{T_E}{T_G}$
2.	Arm fixed - gear C rotates through $+x$ revolutions (i.e **Multiplied the 1st row by +x)**	0	$+x$	$-x\times\frac{T_C}{T_D}$	$-x\times\frac{T_C}{T_D}\times\frac{T_E}{T_G}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y-x\times\frac{T_C}{T_D}$	$y-x\times\frac{T_C}{T_D}\times\frac{T_E}{T_G}$

Since the gear G is fixed, therefore from the fourth row of the table,

$$y - x \times \frac{T_C}{T_D} \times \frac{T_E}{T_G} = 0 \qquad \text{or} \qquad y - x \times \frac{50}{20} \times \frac{35}{105} = 0$$

$$\therefore \qquad y - \frac{5}{6}x = 0 \qquad \ldots(i)$$

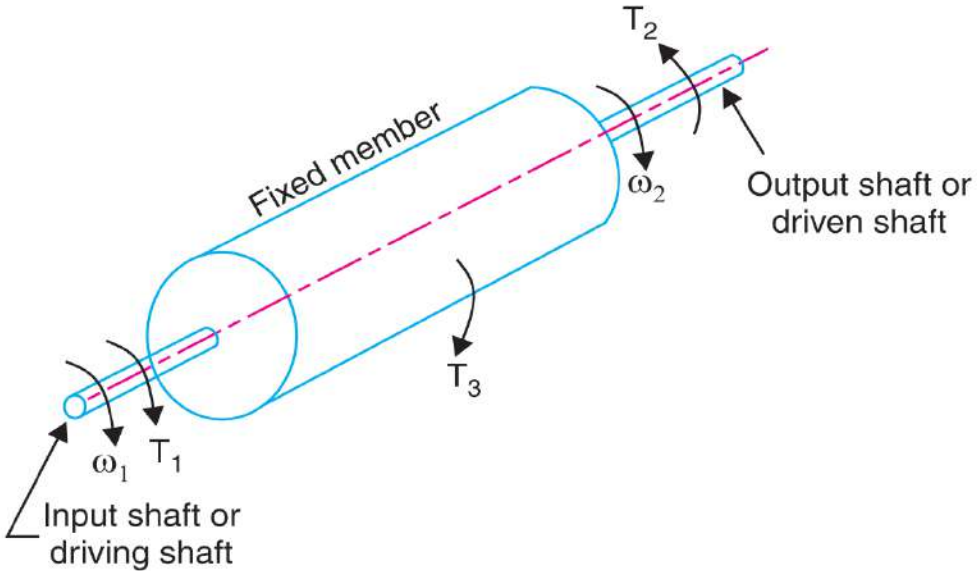
Since the gear C is rigidly mounted on shaft A, therefore speed of gear C and shaft A is same. We know that speed of shaft A is 110 r.p.m., therefore from the fourth row of the table,

$$x + y = 100 \qquad \ldots(ii)$$

From equations (*i*) and (*ii*), $x = 60$, and $y = 50$

$\therefore$ Speed of shaft B = Speed of arm = + y = 50 r.p.m. anticlockwise **Ans.**

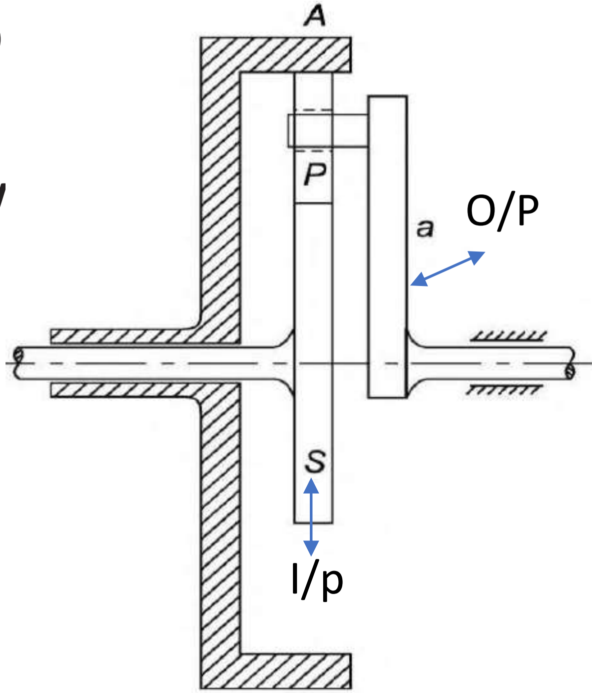
Torques in Epicyclic Gear Trains



When an epicyclic gear train transmits power , torques are transmitted from one element to another as shown in Fig. All the gears rotate with constant speed i.e angular acceleration is zero,

1. Input torque on the driving member (T1),
2. Output torque or resisting or load torque on the driven memb
3. Holding or braking or fixing torque on the fixed member (T3).

The net torque applied to the gear train must be zero. In other w



Under this condition geared system is held in equilibrium under the action of external torques acting on it.

$$T_1 + T_2 + T_3 = 0$$

$$\therefore \quad F_1.r_1 + F_2.r_2 + F_3.r_3 = 0$$

where F_1, F_2 and F_3 are the corresponding externally applied forces at radii r_1, r_2 and r_3.

Further, if ω_1, ω_2 and ω_3 are the angular speeds of the driving, driven and fixed members respectively, and the friction be neglected, then the net kinetic energy dissipated by the gear train must be zero, *i.e.* i/p power = o/p power

$$T_1.\omega_1 + T_2.\omega_2 + T_3.\omega_3 = 0 \quad \text{...}(iii)$$

But, for a fixed member, $\omega_3 = 0$

$$\therefore \quad T_1.\omega_1 + T_2.\omega_2 = 0 \quad \text{...}(iv)$$

Notes : 1. From equations **(*i*)** and **(*iv*),** the holding or braking torque T_3 may be obtained as follows :

$$T_2 = -T_1 \times \frac{\omega_1}{\omega_2} \quad \text{...[From equation } (iv)]$$

and

$$T_3 = -(T_1 + T_2) \quad \text{...[From equation } (i)]$$

$$= T_1\left(\frac{\omega_1}{\omega_2} - 1\right) = T_1\left(\frac{N_1}{N_2} - 1\right)$$

2. When input shaft (or driving shaft) and output shaft (or driven shaft) rotate in the same direction, then the input and output torques will be in opposite directions. Similarly, when the input and output shafts rotate in opposite directions, then the input and output torques will be in the same direction.

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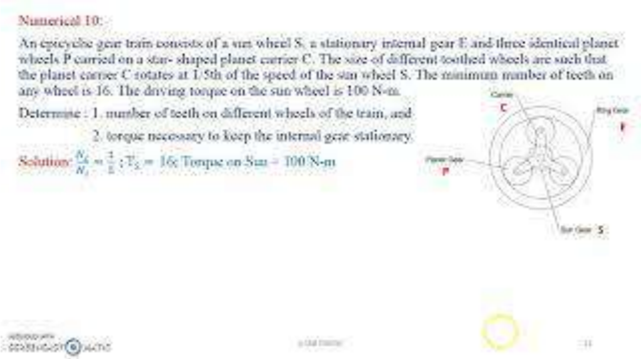

[TORQUE CALCULATION IN EPICYCLIC - SUN AND PLANET GEAR TRAINS (TABULAR METHOD) Numerical 9 & 10]

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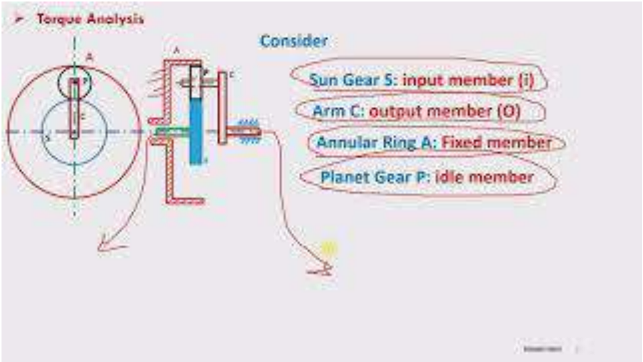

[Holding torque in epicyclic gear train]

Example 13.19. *Fig. 13.26 shows an epicyclic gear train. Pinion A has 15 teeth and is rigidly fixed to the motor shaft. The wheel B has 20 teeth and gears with A and also with the annular fixed wheel E. Pinion C has 15 teeth and is integral with B (B, C being a compound gear wheel). Gear C meshes with annular wheel D, which is keyed to the machine shaft. The arm rotates about the same shaft on which A is fixed and carries the compound wheel B, C. If the motor runs at 1000 r.p.m., find the speed of the machine shaft. Find the torque exerted on the machine shaft, if the motor develops a torque of 100 N-m.*

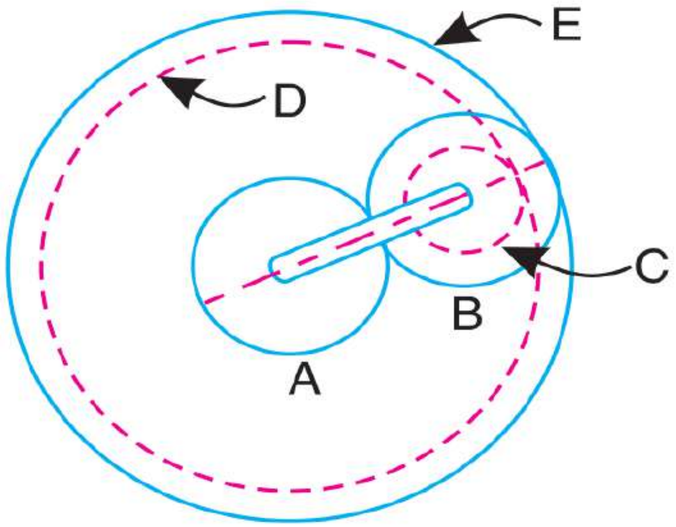

Fig. 13.26

Solution. Given : $T_A = 15$; $T_B = 20$; $T_C = 15$; $N_A = 1000$ r.p.m.; Torque developed by motor (or pinion A) = 100 N-m

First of all, let us find the number of teeth on wheels D and E. Let T_D and T_E be the number of teeth on wheels D and E respectively. Let d_A, d_B, d_C, d_D and d_E be the pitch circle diameters of wheels A, B, C, D and E respectively. From the geometry of the figure,

$$d_E = d_A + 2\,d_B \quad \text{and} \quad d_D = d_E - (d_B - d_C)$$

Since the number of teeth are proportional to their pitch circle diameters, therefore,

$$T_E = T_A + 2\,T_B = 15 + 2 \times 20 = 55$$

and

$$T_D = T_E - (T_B - T_C) = 55 - (20 - 15) = 50$$

Speed of the machine shaft

Step No.	Conditions of motion	Revolutions of elements: Arm	Pinion A	Compound wheel B-C	Wheel D	Wheel E
1.	Arm fixed-pinion A rotated through + 1 revolution (anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$	$-\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$-\frac{T_A}{T_B}\times\frac{T_B}{T_E}=-\frac{T_A}{T_E}$
2.	Arm fixed-pinion A rotated through + x revolutions	0	+ x	$-x\times\frac{T_A}{T_B}$	$-x\times\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$-x\times\frac{T_A}{T_E}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y	+ y	+ y
4.	Total motion	+ y	x + y	$y-x\times\frac{T_A}{T_B}$	$y-x\times\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$y-x\times\frac{T_A}{T_E}$

(i.e. **Multiplied the 1st row by +x)**

We know that the speed of the motor or the speed of the pinion A is 1000 r.p.m. Therefore

$$x + y = 1000 \quad \text{...(i)}$$

Also, the annular wheel E is fixed, therefore

$$y - x\times\frac{T_A}{T_E} = 0 \quad \text{or} \quad y = x\times\frac{T_A}{T_E} = x\times\frac{15}{55} = 0.273\,x \quad \text{...(ii)}$$

From equations (*i*) and (*ii*),

$$x = 786 \quad \text{and} \quad y = 214$$

$\therefore$ Speed of machine shaft = Speed of wheel *D*,

$$N_D = y - x \times \frac{T_A}{T_B} \times \frac{T_C}{T_D} = 214 - 786 \times \frac{15}{20} \times \frac{15}{50} = +37.15 \text{ r.p.m.}$$

$$= 37.15 \text{ r.p.m. (anticlockwise)} \ \textbf{Ans.}$$

Torque exerted on the machine shaft

We know that

Torque developed by motor × Angular speed of motor

= Torque exerted on machine shaft × Angular speed of machine shaft

or $100 \times \omega_A$ = Torque exerted on machine shaft $\times\ \omega_D$

$\therefore$ Torque exerted on machine shaft

$$= 100 \times \frac{\omega_A}{\omega_D} = 100 \times \frac{N_A}{N_D} = 100 \times \frac{1000}{37.15} = 2692 \text{ N-m} \quad \textbf{Ans.}$$

Example 13.20. *An epicyclic gear train consists of a sun wheel S, a stationary internal gear E and three identical planet wheels P carried on a star- shaped planet carrier C. The size of different toothed wheels are such that the planet carrier C rotates at 1/5th of the speed of the sunwheel S. The minimum number of teeth on any wheel is 16. The driving torque on the sun wheel is 100 N-m. Determine :* **1.** *number of teeth on different wheels of the train, and* **2.** *torque necessary to keep the internal gear stationary.*

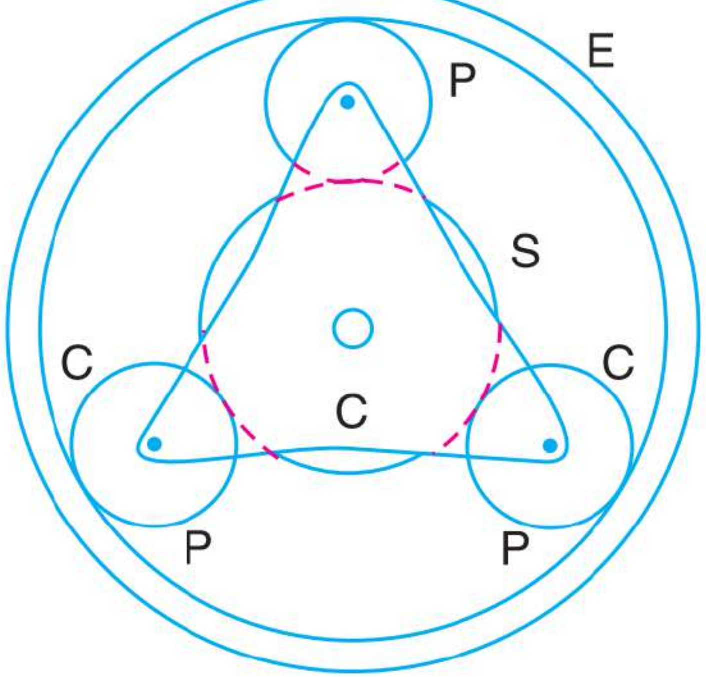

Fig. 13.27

Solution. Given : $N_C = \frac{N_S}{5}$

1. *Number of teeth on different wheels*

The arrangement of the epicyclic gear train is shown in Fig. 13.27. Let T_S and T_E be the number of teeth on the sun wheel *S* and the internal gear *E* respectively. The table of motions is given below :

Step No.	Conditions of motion	Revolutions of elements			
		Planet carrier C	Sun wheel S	Planet wheel P	Internal gear E
1.	Planet carrier C fixed, sunwheel S rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_S}{T_P}$	$-\frac{T_S}{T_P} \times \frac{T_P}{T_E} = -\frac{T_S}{T_E}$
2.	Planet carrier C fixed, sunwheel S rotates through $+x$ revolutions (i.e **Multiplied the 1st row by +x)**	0	$+x$	$-x \times \frac{T_S}{T_P}$	$-x \times \frac{T_S}{T_E}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_S}{T_P}$	$y - x \times \frac{T_S}{T_E}$

We know that when the sunwheel S makes 5 revolutions, the planet carrier C makes 1 revolution. Therefore from the fourth row of the table,

$$y = 1, \quad \text{and} \quad x + y = 5 \quad \text{or} \quad x = 5 - y = 5 - 1 = 4$$

Since the gear E is stationary, therefore from the fourth row of the table,

$$y - x \times \frac{T_S}{T_E} = 0 \quad \text{or} \quad 1 - 4 \times \frac{T_S}{T_E} = 0 \quad \text{or} \quad \frac{T_S}{T_E} = \frac{1}{4}$$

$$\therefore \quad T_E = 4T_S$$

Since the minimum number of teeth on any wheel is 16, therefore let us take the number of teeth on sunwheel, $T_S = 16$

$\therefore \quad T_E = 4\,T_S = 64$ **Ans.**

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Let d_S, d_P and d_E be the pitch circle diameters of wheels S, P and E respectively. Now from the geometry of Fig. 13.27,

$$d_S + 2\,d_P = d_E$$

Assuming the module of all the gears to be same, the number of teeth are proportional to their pitch circle diameters.

$$T_S + 2\,T_P = T_E \quad \text{or} \quad 16 + 2\,T_P = 64 \quad \text{or} \quad T_P = 24 \ \textbf{Ans.}$$

2. *Torque necessary to keep the internal gear stationary*

We know that

Torque on S × Angular speed of S

= Torque on C × Angular speed of C

$$100 \times \omega_S = \text{Torque on } C \times \omega_C$$

$$\therefore \quad \text{Torque on } C = 100 \times \frac{\omega_S}{\omega_C} = 100 \times \frac{N_S}{N_C} = 100 \times 5 = 500 \text{ N-m}$$

∴ Torque necessary to keep the internal gear stationary

= 500 – 100 = 400 N-m **Ans.**

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Example 13.21. *In the epicyclic gear train, as shown in Fig. 13.28, the driving gear A rotating in clockwise direction has 14 teeth and the fixed annular gear C has 100 teeth. The ratio of teeth in gears E and D is 98 : 41. If 1.85 kW is supplied to the gear A rotating at 1200 r.p.m., find :* **1.** *the speed and direction of rotation of gear E, and* **2.** *the fixing torque required at C, assuming 100 per cent efficiency throughout and that all teeth have the same pitch.*

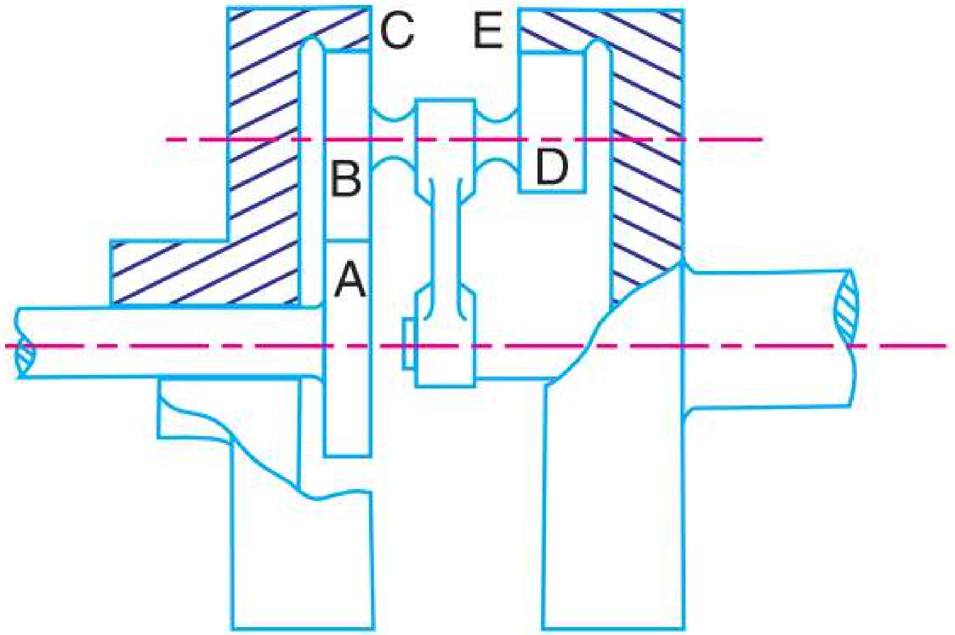


Fig. 13.28

Solution. Given : $T_A = 14$; $T_C = 100$; $T_E / T_D = 98 / 41$; $P_A = 1.85$ kW = 1850 W ; $N_A = 1200$ r.p.m.

Let d_A, d_B and d_C be the pitch circle diameters of gears A, B and C respectively. From Fig. 13.28,

$$d_A + 2\,d_B = d_C$$

Since teeth of all gears have the same pitch and the number of teeth are proportional to their pitch circle diameters, therefore

$$T_A + 2T_B = T_C \qquad \text{or} \qquad T_B = \frac{T_C - T_A}{2} = \frac{100 - 14}{2} = 43$$

Step No.	Conditions of motion	Revolutions of elements				
		Arm	Gear A	Compound gear B-D	Gear C	Gear E
1.	Arm fixed-Gear *A* rotated through – 1 revolution (*i.e.* 1 revolution clockwise)	0	– 1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_C}$ $=+\frac{T_A}{T_C}$	$+\frac{T_A}{T_B}\times\frac{T_D}{T_E}$
2.	Arm fixed-Gear *A* rotated through – *x* revolutions (i.e **Multiplied the 1st row by +x)**	0	$-x$	$+x\times\frac{T_A}{T_B}$	$+x\times\frac{T_A}{T_C}$	$+x\times\frac{T_A}{T_B}\times\frac{T_D}{T_E}$
3.	Add – *y* revolutions to all elements	$-y$	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-y-x$	$-y+x\times\frac{T_A}{T_B}$	$-y+x\times\frac{T_A}{T_C}$	$-y+x\times\frac{T_A}{T_B}\times\frac{T_D}{T_E}$

Since the annular gear C is fixed, therefore from the fourth row of the table,

$$-y + x \times \frac{T_A}{T_C} = 0 \qquad \text{or} \qquad -y + x \times \frac{14}{100} = 0$$

$$\therefore \qquad -y + 0.14\,x = 0 \qquad \ldots(i)$$

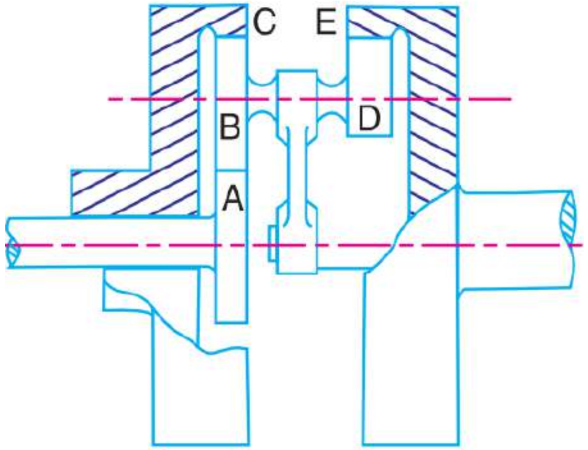
Also, the gear A is rotating at 1200 r.p.m., therefore

$$-x - y = 1200 \qquad \ldots(ii)$$

From equations *(i)* and *(ii)*, $x = -1052.6$, and $y = -147.4$

1. *Speed and direction of rotation of gear E*

From the fourth row of the table, speed of gear E,



$$N_E = -y + x \times \frac{T_A}{T_B} \times \frac{T_D}{T_E} = 147.4 - 1052.6 \times \frac{14}{43} \times \frac{41}{98}$$

$$= 147.4 - 143.4 = 4 \text{ r.p.m.}$$

$$= 4 \text{ r.p.m. (anticlockwise)}$$ **Ans.**

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2. Fixing torque required at C

We know that torque on $A = \dfrac{P_A \times 60}{2\pi N_A} = \dfrac{1850 \times 60}{2\pi \times 1200} = 14.7$ N-m

Since the efficiency is 100 per cent throughout, therefore the power available at E (P_E) will be equal to power supplied at A (P_A).

$$\therefore \text{Torque on } E = \frac{P_A \times 60}{2\pi \times N_E} = \frac{1850 \times 60}{2\pi \times 4} = 4416 \text{ N-m}$$

$\therefore$ Fixing torque required at C

$$= 4416 - 14.7 = 4401.3 \text{ N-m}$$ **Ans.**

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UNIT V BIT BANK

1. In a simple gear train, if the number of idle gears is odd, then the motion of driven gear will (a)

(a) be same as that of driving gear (b) be opposite as that of driving gear

(c) depend upon the number of teeth on the driving gear (d) none of the above

2. The train value of a gear train is (b)

(a) equal to velocity ratio of a gear train (b) reciprocal of velocity ratio of a gear train

(c) always greater than unity (d) always less than unity

3. When the axes of first and last gear are co-axial, then gear train is known as (c)

(a) simple gear train (b) compound gear train(c) reverted gear train(d) epicyclic gear train

4. In a clock mechanism, the gear train used to connect minute hand to hour hand, is (b)

(a) epicyclic gear train (b) reverted gear train(c) compound gear train (d) simple gear train

5. In a gear train, when the axes of the shafts, over which the gears are mounted, move relative to a fixed axis, is called (d)

(a) simple gear train (b) compound gear train (c) reverted gear train(d) epicyclic gear train

6. A differential gear in an automobile is a (b)

(a) simple gear train (b) epicyclic gear train

(c) compound gear train (d) none of these

7. A differential gear in automobilies is used to (d)

(a) reduce speed (b) assist in changing speed

(c) provide jerk-free movement of vehicle (d) help in turning

8. For a simple gear train, velocity ratio is the ratio of (b)

(a) Speed of driving shaft and speed of driven shaft
(b) Speed of driven shaft and speed of driving shaft
(c) Speed of driven shaft and (speed of driving shaft + speed of idler gears)
(d) Speed of driving shaft and (speed of driven shaft + speed of idler gears)

9. What is meant by an idle gear? (d)
(a) Gears between driver and driven gears
(b) Gears used when driver and driven gears move in same direction
(c) Both a. and b. (d) None of the above

10. Calculate speed of driving shaft in compound gear train, if the drivers have 50, 60, 80 and 100 teeth and followers have 18, 40, 60 and 80 teeth. Speed of driven shaft is 150 rpm (a)

(a) 21.73 rpm (b)30.23 rpm (c)19.77 rpm (d)None of the above

11. In simple gear trains, velocity ratio is independent of idle gears (a)

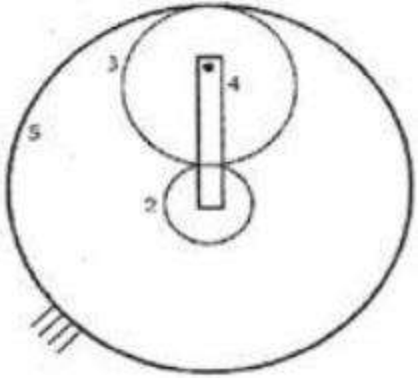
(a)True (b)False

12. Which type of gear train and gears are used in Humpage Gear Box? (a)

(a) Epicyclic gear train with bevel gears (b)Epicyclic gear train with worm gears

(c) Reverted gear train with bevel gears (d)Reverted gear train with worm gears

An epicyclic gear train is shown schematically in the adjacent figure



The sun gear2 on the input shaft is a 20 teeth external gear. The planet gear 3 is a 40 teeth external gear. The ring gear 5 is a 100 teeth internal gear. The ring gear 5 is fixed and the gear 2 is rotating at 60 rpm ccw (ccw=counter-clockwise and cw =clockwise).

Then arm attached to the output shaft will rotate at (a) **GATE-ME-2008**

(a) 10 rpm ccw (b) 10 rpm cw (c) 12 rpm cw (d) 12 rpm ccw

$T_2 = 20;\ T_3 = 40;\ T_s = 100$

Condition	Arm	Gear 2	Gear 3	Gear 4
1. Arm fixed-gear 2 rotates with	0	+1	$-\frac{T_2}{T_3}$	$\frac{T_2}{T_3} \times \frac{T_3}{T_5}$
2. Arm fixed gear 2 rotates with +x rotations	0	+x	$-x\frac{T_2}{T_3}$	$-x\frac{T_2}{T_5}$
3. Adding +y rotations	y	y+x	$y-x\frac{T_2}{T_3}$	$y-x\frac{T_2}{T_5}$

Given $y - x\frac{T_2}{T_5} = 0$

$\Rightarrow x = 5y \ldots\ldots\ldots (i)$

and $y + x = 60 \ldots\ldots\ldots.(ii)$

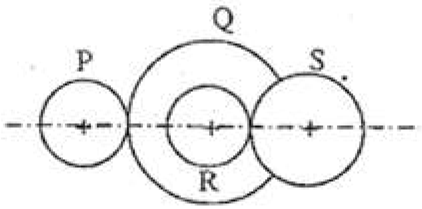
Solving (1) and (2), we get

$y + 5y = 60$

$\Rightarrow y = 10\ rpm\ ccw$

A compound gear train with gears P, Q, R and S has number of teeth 20, 40, 15 and 20 respectively. Gear Q and R are mounted on the same shaft as shown in the figure below. The diameter of the gear Q is twice that of gear R. if the module of the gear R is 2 mm, the centre distance in mm between gears P and S is (B)

(A) 40 (B) 80 (C) 120 (D) 160 **GATE-ME-2013**



Module of meshing gear pair are equal.

$$\frac{d_P}{T_p} = \frac{d_Q}{T_Q} \Rightarrow \frac{d_P}{20} = \frac{d_Q}{40}$$

$$\Rightarrow d_Q = 2d_P \dots\dots\dots(1)$$

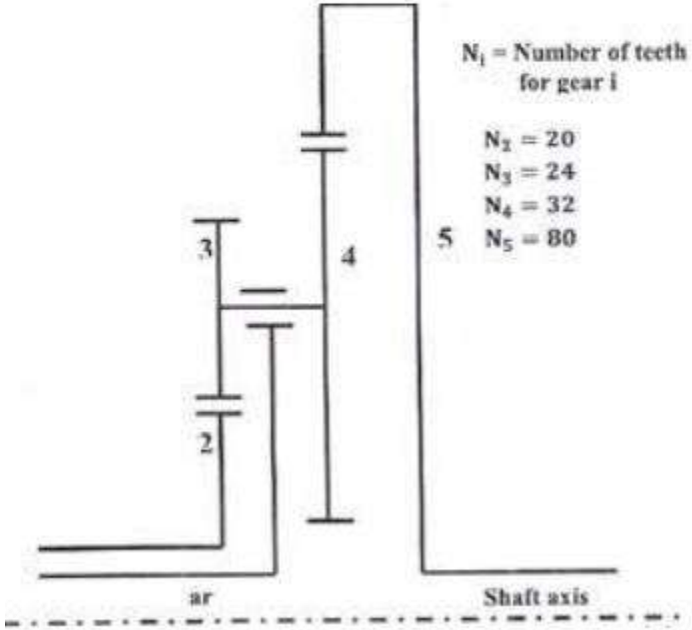
$$\frac{d_R}{T_R} = \frac{d_s}{T_s} = 2 \Rightarrow \frac{d_R}{15} = \frac{d_s}{20} = 2$$

$$\Rightarrow d_R = 30, d_s = 40 \dots\dots\dots(2)$$

For the epicyclic gear arrangement shown in the figure, ω_2=100 rad/s clockwise (cw) and ω_{arm}=80 rad/s counter clockwise (ccw). The angular velocity ω_5 (in rad/s) is (C)

GATE-ME-2010

(A) 0 (B) 70 cw (C) 140 ccw (D) 140 cw



	Arm	2	3	4	5
1. Give +x rotation CW to gear 2	0	+x	$\frac{-N_2}{N_3}x$	$\frac{-N_2}{N_3}x$	$\frac{-N_4}{N_5}\times\frac{-N_2}{N_3}x$
2. Give y rotation CW to arm	y	y	y	y	y
Total	y	y+x	$y-\frac{N_2}{N_3 x}$	$y-\frac{N_2}{N_3 x}$	$y-\frac{-N_4}{N_5}\times\frac{N_2}{N_3 x}$

$x + y = 100\ (CW)$

$y = -80$

Speed of gear 5,

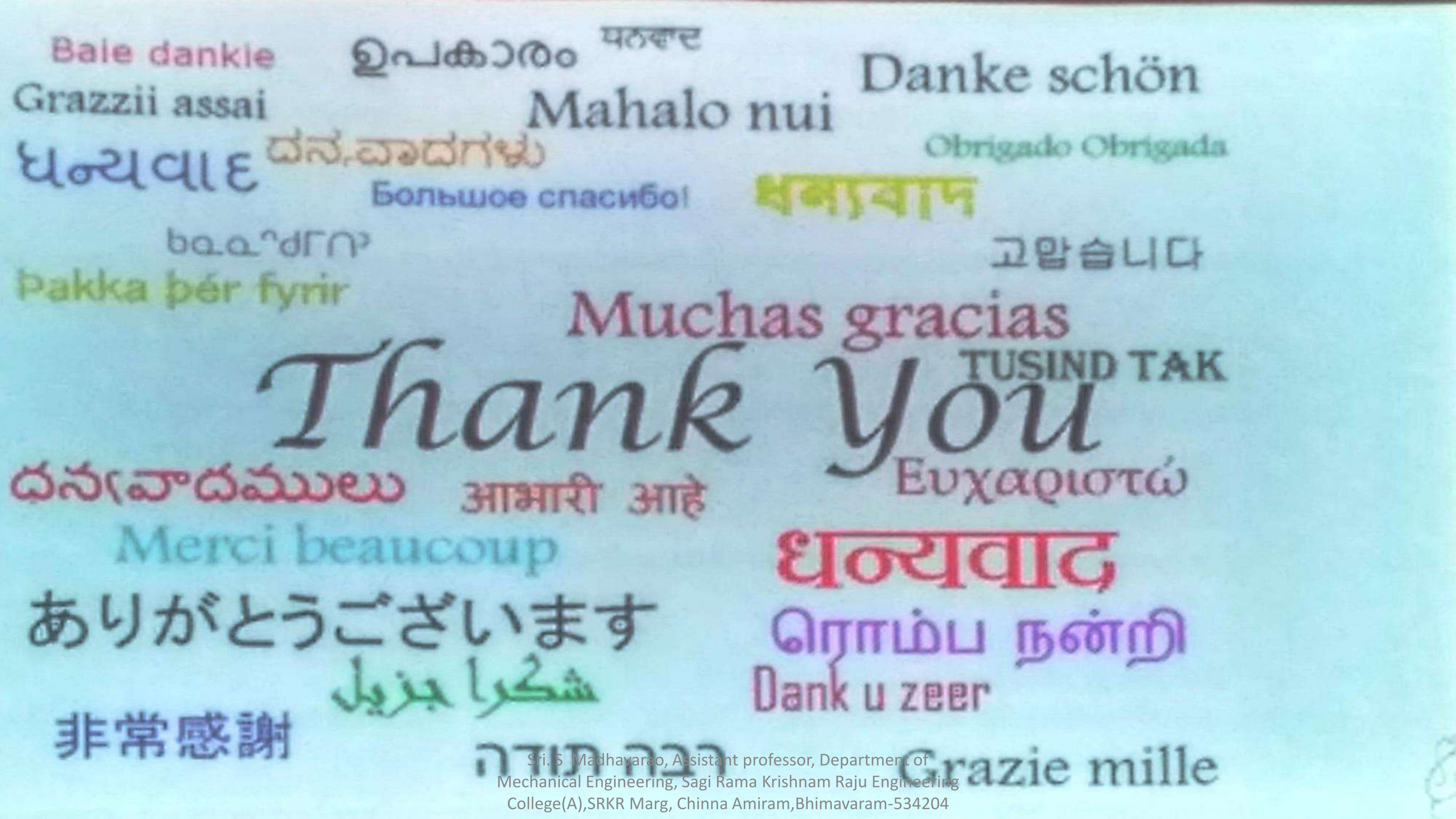
$$\omega_5 = -80 - \frac{32}{80}\times\frac{20}{24}\times 180$$

$$= -140 = 140\ CCW$$

References:

1. Theory of Machines, Rattan, Tata McGraw-Hill Education, 2009.

2. Theory of Machines, R S Kurmi, Eurasia Publishing House, 2005



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